Supporting Information for "Bayesian inference of mantle viscosity from whole-mantle density models"

M.L. Rudolph¹, P. Moulik², and V. Lekić²

¹Department of Earth and Planetary Sciences, University of California, Davis CA ²Department of Geology, University of Maryland, College Park, MD

Contents of this file

1. Tables S1 to S2 $\,$

 $2.\ {\rm Figures}\ {\rm S1}\ {\rm to}\ {\rm S5}$

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SH Degrees	Median $(\%)$	Mean $(\%)$	Max (%)	stdev (%)
l=2-3	84	82	94	5.58
l=2+4	67	60	92	23.67
l=2-7	72	70	81	6.62
l=2-3	86	86	93	2.61
l=2+4	87	87	95	3.50
l=2-7	79	79	84	1.85
	$\begin{array}{c} \text{SH Degrees} \\ \hline l=2-3 \\ l=2+4 \\ l=2-7 \\ \hline l=2-3 \\ l=2+4 \\ l=2-7 \end{array}$	SH Degrees Median (%) l=2-3 84 l=2+4 67 l=2-7 72 l=2-3 86 l=2+4 87 l=2-7 79	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$

l=2-3 80 78918.27Model 167 l=2+474699218.29l=2-77473814.12l=2-387 85 91 3.98l=2+485 Model 17286 933.94l=2-77979821.57

Table S1. Mean, median, maximum, and standard deviation (stdev) of variance reductions (in percent) in the ensemble solutions for inversions using the posterior covariance matrix from the tomographic models (corresponding to Figure 3 and Figure S5).

Density Model	SH Degrees	Median (%)	Mean $(\%)$	Max (%)	stdev (%)
Model 155	l=2-3	84	83	93	5.01
	l=2+4	86	84	94	4.56
	l=2-7	79	78	82	1.23
Model 160	l=2-3	86	84	97	4.21
	l=2+4	88	87	97	3.00
	l=2-7	80	80	84	1.27
Model 167	l=2-3	79	77	93	8.31
	l=2+4	83	81	94	6.10
	1=2-7	78	78	83	1.70
Model 172	l=2-3	86	85	91	3.99
	l=2+4	86	86	93	3.43
	l=2-7	79	79	81	1.12

Table S2. Mean, median, maximum, and standard deviation (stdev) of variance reductions (in percent) in the ensemble solutions for inversions using a diagonal covariance matrix (corresponding to Figures S3-4).



Figure S1. Properties of end-member density models 155 (A-C), 160 (D-F), 167 (G-I), and 172 (J-L). For each model, we show the power spectrum of the density heterogeneity (expressed as percent deviation from the 1D average) by spherical harmonic degree (A,D,G,J). Next, we show the correlation coefficient between density and V_S variations as a function of depth and spherical harmonic degree (B,E,H,K). Finally, we show the effective value of $d \ln \rho/d \ln V_S$ by depth and spherical harmonic degree (C,F,I,L).



Figure S2. Convergence of covariance matrix eigenvalues with increasing number of samples.



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Figure S3. Posterior ensembles from viscosity inversions for density models 155 (A-C), 160 (D-F), 167 (G-I), and 172 (J-L). For each model, we show the distribution of the number of control points (A1, etc.), the variance reduced for the spherical harmonic degrees used to constrain the inversion (A2, etc.), and the probability density for viscosity in the ensemble solution (A3, etc.). The color scale for each ensemble solution is normalized so that a value of 1.0 corresponds to the maximum probability value in the ensemble.



Figure S4. Results from viscosity inversions using a diagonal covariance matrix (*i.e.* assuming that the data and forward modeling uncertainties associated with all spherical harmonic coefficients are equal and uncorrelated). The density models used in (A,C) have smaller misfit to the normal mode splitting measurements whereas the models in (B,D) have density variations that are strongly correlated with V_S variations, at the expense of fitting the normal mode constraints. The blue, grey, and red curves in each panel correspond to models constrained by spherical harmonic degrees l = 2 - 3, l = 2, 4, and l = 2 - 7 respectively. In each panel, the red, green, and blue colors show the probability distributions for each combination of spherical harmonic degrees. We also show the histogram of geoid variance reduction associated with each model.

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Figure S5. Posterior ensembles from viscosity inversions using a diagonal covariance matrix (*i.e.* assuming that the data and forward modeling uncertainties associated with all spherical harmonic coefficients are equal and uncorrelated). Posterior ensembles are shown for viscosity inversions using density models 155 (A-C), 160 (D-F), 167 (G-I), and 172 (J-L). For each model, we show the distribution of the number of control points (A1, etc.), the variance reduced for the spherical harmonic degrees used to constrain the inversion (A2, etc.), and the probability density for viscosity in the ensemble solution (A3, etc.). The color scale for each ensemble solution is normalized so that a value of 1.0 corresponds to the maximum probability value in the ensemble.