Seismic signature of small melt fraction atop the transition zone

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ABSTRACT

This article explores the combined effect of thermal, chemical, and melting anomalies of seismic velocities above the transition zone. While thermal and chemical effects influence the seismic velocities at subsolidus temperatures, the velocity structures are greatly modified in the presence of partial melting. We model the impedance contrast atop a low velocity layer at a depth of 350 km beneath the south Pacific, based on seismic observations of ScS wave reflectivity in the region. A compositionally distinct layer with varying basalt fraction fails to produce the observed average shear impedance contrast of −2.7%, for a range of potential temperatures between 1500 and 1700 K. A partially molten layer containing approximately 1 vol.% melt, explains the observed shear impedance. The melt fraction necessary to explain the observed shear impedance also trades off with the dihedral angle of the aggregate. For dihedral angles between 25° and 30°, between 1 to 1.1 vol.% melting is necessary to explain the observed impedance contrast. For such small volume fraction of melting, a near neutrally-buoyant melt can be redistributed by surface tension over the observed layer thickness of ~70 km. Due to the high frictional resistance and strong surface tensions prevalent at such small melt fractions, density-driven melt drainage will likely be inefficient.

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1. Introduction

In several regions of the Earth, a seismic low velocity layer has been identified atop the 410 km discontinuity (e.g., Courtier and Revenaugh, 2007; Gao et al., 2006; Jasbinsek and Dueker, 2007; Revenaugh and Sipkin, 1994; Tauzin et al., 2010; Vinnik and Farra, 2007). Recent evidence suggests that this layer may be global in nature, with a laterally varying thickness (Tauzin et al., 2010). While it has been suggested that this low velocity layer is likely partially molten, a comparative study between the seismic velocity anomalies arising due to temperature, solid-state chemical heterogeneity, and partial melting has not yet been performed. If partially molten, the dynamics of melt storage and efficiency of melt transport are also likely to be influenced by the degree of melting and the melt microstructure.

Seismic signals and depths of the transition zone discontinuities, in the subsolidus mantle depend on chemical composition and potential temperature. For example, the depth of olivine to wadsleyite transition (commonly referred to as the ‘410 km discontinuity’) and shear wave velocity are sensitive to the temperature variations as well as the basaltic component in the bulk mantle composition (Xu et al., 2008). Changes in the depth of olivine to wadsleyite transition arise from the C Lapeyron slope of the solid-state reaction and shrinking of olivine stability field towards lower pressure with an increase in the iron content. Shear wave velocity of the subsolidus mantle is sensitive to both an increase in the temperature and the fraction of garnet in the mantle. While garnet is seismically faster than olivine, it is slower than the transition zone polymorphs, thus affecting the effective elastic properties of the mantle rocks differently above and below the olivine-wadsleyite transition (Xu et al., 2008). Above the transition zone, increased temperature, lower garnet fraction, and partial melting can all reduce the seismic velocity. Due to the intricate interplay between these various free parameters, it is useful to identify an observable seismic signature that can be a quantitative predictor of the nature of the seismic anomaly, such as the impedance contrast of shear waves across the top of the low velocity layer.

While low velocity layers in the deep upper mantle have been detected in a variety of tectonic settings and with a suite of seismic frequencies, we focus on modeling the observations of Courtier and Revenaugh (2007). They observed a low velocity layer beneath the southwest Pacific (see Fig. 1), with the top of the layer occurring at an average depth of 350 km and with an average impedance contrast of −2.7%. The study used crossing paths of low-frequency ScS reverberations, which indicate that the feature is robust and regionally continuous, as opposed to significant small scale lateral variations that may be present when such features are detected with higher frequency methods. The regional coherency of the low-velocity layer observed in this particular area makes it an ideal starting place for integration of seismic observations with geodynamic modeling.

Using information on the density and elastic moduli of the mineral and melt phases and information on the melt microstructure, such as

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dihedral angle and contiguity, the equilibrium geometry model of poroelasticity (Takei, 1998; 2002) can be used to predict the shear impedance contrast across the top boundary of the low velocity layer. In the absence of melting, the impedance contrast is defined by the shear wave velocities and densities of the solid phases. In the presence of partial melting, the melt-grain microstructure and melt volume fraction additionally influence the effective elastic moduli of the layer (Hier-Majumder, 2008; Hier-Majumder and Abbott, 2010; Takei, 1998; 2000; 2002; Yoshino et al., 2005). Two interdependent microstructural quantities are particularly important in this context; contiguity, the fractional area of intergranular contact, and dihedral angle at the grain-melt-grain triple junctions. Since contiguity depends on both melt fraction and dihedral angle, one can use both of these as input parameters for calculating the impedance contrast. The melt volume fraction in the partially molten region can be constrained by comparing the calculated impedance contrast with observed values for relevant dihedral angles.

Combining seismic observations, mineral physics data, and micromechanical models, one can quantify the extent of melting. Based on models of melt migration and storage, the estimated amount of melt allows investigation of the dynamic processes likely operative in this region. An important issue is the observed large vertical extent (thickness) of this low velocity layer. We discuss the likelihood of gravitational drainage and homogenization of melt-rich regions through a well-connected melt network and implications for the thickness of the low velocity layer.

2. Methods

The seismic signature of a partially molten aggregate is described by an effective constitutive relation derived from the known elastic properties of the solid and the melt. For a subsolidus aggregate, temperature and bulk chemical composition are the primary variables influencing the elastic properties. In this article, we use the data from Xu et al. (2008) for a number of different mantle potential temperatures and bulk compositions for the density and elastic moduli of the solid phase. For the physical property of the melt phase, we use experimentally determined data for basaltic melts from Ohtani and Maeda (2001). The mineral physics model of Xu et al. (2008) investigated the seismic velocity structure of the mantle up to a pressure of 40 GPa for two different assemblages termed ‘equilibrium assemblage’ and ‘mechanical mixture’. The bulk composition of the mantle consisted of harzburgite and basalt components. They varied the fraction of the basalt component from 0 to 1, and potential temperature varied between 1000 and 2000 K. Their recommended bulk mantle composition consists of 18% basaltic component. The plots in Fig. 2 depict the variation of density, seismic velocities, elastic moduli, and Poisson’s ratio from Xu et al. (2008) for mantle potential temperatures of 1000 K and 1500 K.

![Fig. 1. A summary of ScS observations in the study area from Courtier and Revenaugh (2007). The pink shaded region outlines the extent of the low velocity layer at depth. Polarity of regional subduction zones are indicated by triangles along the strikes of the trenches. The contours indicate the depth of the olivine-wadsleyite phase transition (‘410 km discontinuity’). Modified from Courtier and Revenaugh (2007).](image1)

![Fig. 2. Plot of density, seismic velocities, shear (G) and bulk (K) moduli, and Poisson’s ratio from the data of Xu et al. (2008). The elastic parameters were calculated from the wave velocities provided in their database. The blue curves correspond to a potential temperature of 1000 K and the red curves correspond to a potential temperature of 1500 K.](image2)
1500 K, respectively. Knowing these physical parameters, we use a micromechanical model to quantify the seismic velocities and impedance contrast. Details of the micromechanical model are provided below.

### 2.1. Micromechanical model

The micromechanical model calculates the effective bulk and shear moduli as a function of contiguity, the fractional area of intergranular contact in partially molten rocks. In a subsolidus aggregate, grains are surrounded entirely by neighboring grains and the contiguity of the aggregate is unity. In the presence of melt, contiguity varies from 0 to 1, as edges, corners, or boundaries of grains are coated by melt. At the disaggregation melt fraction, grain boundaries are covered entirely by melt and intergranular contact is lost. Typically the disaggregation melt fraction lies between 0.25 and 0.3 for partially molten mantle rocks (Scott and Kohlstedt, 2006). In aggregates containing melt fraction between zero and the disaggregation melt fraction, contiguity also depends on the dihedral angle (Hier-Majumder and Abbott, 2010; Takei, 2000; von Bargen and Waff, 1986). Typically, well-connected, low dihedral angle melts yield a low contiguity.

First, we evaluate the contiguity \( \psi \) for a melt fraction \( \phi \) and dihedral angle \( \theta \) using the formulation of von Bargen and Waff (1986),

\[
\psi = f(\theta, \phi).
\]

The contiguity from the model of von Bargen and Waff (1986) was calculated using the corrections outlined by Takei (2000). The nondimensional grain–grain contact area from von Bargen and Waff (1986) was calculated by \( n - A_v \cdot d \) (where \( A_v \cdot d \) is given in their Eq. (8)). The effective elastic moduli can be expressed in terms of contiguity \( \psi \) as

\[
N = \mu (1-\psi) g(\psi)
\]

\[
K_v = K \left[ (1-\psi) h(\psi) + \frac{(1-(1-\psi)h(\psi))^2}{1-\psi(1-h(\psi))} + \phi K_m/K_v \right],
\]

where \( K \) and \( K_m \) are the bulk moduli of the solid and the melt, \( \mu \) is the shear modulus of the solid, and \( K_v \) and \( N \) are the effective bulk and shear moduli, respectively. The functions \( g(\psi) \) and \( h(\psi) \) are given by,

\[
g(\psi) = 1 - (1-\psi)^n,
\]

\[
h(\psi) = 1 - (1-\psi)^m,
\]

where the exponents \( n \) and \( m \) also depend on the contiguity \( \psi \) (Takei, 2002, Appendix A). According to the equilibrium geometry model, the ratio between the S and P wave velocities through the partially molten aggregate and the solid is given by

\[
\frac{V_{P}}{V_{S0}} = \sqrt{\frac{(N/\mu)}{(\mu/\rho)}}
\]

and

\[
\frac{V_{P}}{V_{P0}} = \sqrt{\frac{K_v/K + 4\beta/3(N/\mu)}{1 + 4\beta/3(\mu/\rho)}},
\]

where \( \beta = \mu/K \). The quantity \( \rho \) is the volume averaged density of the aggregate. Knowing the shear wave velocity and density of the partially molten aggregate, the impedance contrast is directly calculated.

### 3. Compositional, thermal, and melting anomalies

To test the influence of melting, we introduced a 70 km-thick, partially molten layer atop the 410 km discontinuity with 1 and 5 vol.% melt (marked on Fig. 3). The seismic velocities and the elastic moduli corresponding to these two cases are displayed in Fig. 3. The blue and red curves correspond to subsolidus assemblages at potential temperatures of 1000 and 1500 K, respectively. The reduction in seismic velocities indicates that 1 vol.% melting influences seismic wave velocities by the same extent as increasing the temperature by 500 K.

The equilibrium geometry model (Takei, 2002) is tested with respect to the theoretical bounds. The plots in Fig. 4 display the Reuss (lower) and Voigt (upper) bounds (Mavko et al., 2003) of the elastic moduli for

![Fig. 3. Profiles of (a) seismic velocities and (b) elastic moduli for a dry assemblage, and for assemblages containing melt volume fractions of 0.01 (black) and 0.05 (pink). As in the previous plot, the blue and red curves correspond to potential temperatures of 1000 K and 1500 K, respectively.](image-url)
the melting configuration discussed above. The Voigt–Reuss–Hill average is also displayed. The curve labeled 'Xu' indicates the properties of the solid assemblage. Results from the equilibrium geometry model, labeled 'EG' in the plot, is bound quite well between the limits. A detailed comparison between the equilibrium geometry and other inclusion like models is presented by Takei (2002).

In the following sections, we describe the influence of the dihedral angle on the seismic properties and a comparison between the different sources of seismic anomalies.

3.1. Influence of melt geometry

We used the formulation of von Bargen and Waff (1986) to test the influence of dihedral angle on the seismic properties, through the contiguity of the aggregate. The influence of changing dihedral angle from 10° to 40° is depicted on the velocity profile of the partially molten layer in Fig. 5. Small dihedral angle melts reduce the shear and compressional wave velocities more than larger dihedral angles, mimicking the influence of a greater extent of melting. The variation of dihedral angle arises from the chemical composition of the melt (Yoshino et al., 2005). For example, carbonate melt in olivine subtends a dihedral angle between 25° and 30° (Minarik and Watson, 1995), while hydrous basalt subtends an angle of 28° (Mei et al., 2002).

3.2. Comparison with observed shear impedance

The calculated velocity anomalies due to compositional and thermal variations or melting can be tested against the seismic observations.

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**Fig. 4.** (a) Reduction of bulk modulus due to 1% melting by volume. The profiles are displayed for several different averaging schemes. Bulk modulus for the melt (yellow) was determined from the empirical results of Ohtani and Maeda (2001). The curve labeled ‘EG’ indicates the equilibrium geometry model of Takei (2002). Also displayed are the Voigt and Reuss bounds, and the Voigt–Reuss–Hill average of the bulk and shear moduli. (b) An enlarged view of the plot in (a).

**Fig. 5.** (a) Shear and (b) compressional wave velocity profiles for melt fractions of 0.01 (black) and 0.05 (pink). Two different curves are demonstrated for dihedral angles of 10° and 40°. Velocity of the unmelted aggregates are based on the data from Xu et al. (2008).
from Courtier and Revenaugh (2007). The results from their work reported a decrease in shear impedance occurring at different depths across the study region. The plot in Fig. 6(a) summarizes the observed values and depths of shear impedance from their study. An average value of shear impedance contrast of $-2.7\%$ was reported for the top of the low velocity layer, which was observed at an average depth of 350 km.

The observed decrease in the shear impedance is explained more readily by a small amount of melting rather than by the presence of a warm or compositionally distinct layer. The plots in Fig. 6(b) display results for both cases. The square outline indicates the range of the observed impedance contrasts, while the cyan bar indicates the average value. Broken curves correspond to the impedance contrast of a compositionally distinct layer with a varying basalt fraction, while the mantle has a prescribed average basalt fraction of 18%. Different values on the dashed curves indicate different potential temperatures. For a basalt depleted layer at 350 km, a small negative value of impedance contrast is observed. This variation in impedance is caused by the fact that the basalt poor layer is depleted in garnet, which is seismically faster than olivine. As the basalt fraction increases, so does the amount of garnet in the bulk composition of the rock. This decreases the impedance contrast, becoming zero at a basalt fraction of 0.18, and then becoming positive. The influence of different potential temperatures, as indicated by the different curves, is also rather modest.

Melting, on the other hand, has a more drastic influence on the impedance contrast atop the layer. With an increase in the melt volume fraction, the impedance contrast decreases sharply. Contiguity of the partially molten rocks decreases with an increase in the melt fraction following a power law (von Bargen and Waff, 1986), leading to a sharp reduction in the seismic velocity. The influence of the variation in dihedral angle is also marked in the plots by the various curves with annotated dihedral angles. For a given melt volume fraction, a low dihedral angle melt displays a lower impedance than a higher dihedral angle melt. With an increase in melt fraction, the shear impedance in the aggregate drops drastically, increasing the magnitude of the impedance contrast. Contiguity of most partially molten aggregates relevant to the Earth’s mantle becomes zero around a melt fraction of 0.3. The underlying model of contiguity starts to break down under such high melt volume fractions. The plot in Fig. 6(b) extends up to a melt fraction of 0.03. It is quite obvious that the observed impedance contrast can be explained by an extent of melting much smaller than the disaggregation melt fraction, within which the geometric model is well justified.

We examine the trade-off between dihedral angle and melting from a number of model results. We calculate impedance contrast as a function of melt fraction for various dihedral angles. From the model runs, we select values of shear impedance that are within $\pm 0.12\%$ of the observed average shear impedance contrast of $-2.7\%$. The corresponding values of dihedral angle and melt volume fraction are displayed in Fig. 7. The vertical bar depicts the observed range of dihedral angles, for carbonate or basaltic melts. Within this range of dihedral angles, the observed shear impedance contrast can be explained by a melt volume fraction of 0.01. If the melt subtends a lower dihedral angle, then an even smaller amount of melt volume fraction will be required to explain the observed shear impedance contrast.

4. Discussion

4.1. Dynamics of melt segregation

An interesting feature of the observed low velocity layer is the relatively large thickness, about 70 km, suggested by Courtier and Revenaugh (2007) and globally varying on the order of 30–90 km, suggested by Tauzin et al. (2010). The inferred modest amount of melting can influence the dynamics of storage and transport of the partial melt in two different ways.

First, the mobility of a dense or buoyant melt, at a given melt viscosity, depends strongly on the melt volume fraction. For dihedral angles less than 60°, the frictional resistance is inversely proportional to the melt fraction squared (Hier-Majumder, 2011). Consequently, the drainage efficiency of both buoyant and dense melts is reduced at lower melt fractions. Ricard et al. (2001) discuss the drainage of melt under various conditions of applied stress. Following their model of forced compaction of a one-dimensional column of partially molten aggregates, we present a simple calculation in Appendix A. Maximum fluid velocity obtained from Eq. (A.6) in this model is plotted as a function of the melt volume percent in the column in Fig. 8(a). The calculated melt velocity for 1% melt volume fraction, with a 5% density contrast between the melt and the matrix, is less than 1 mm/yr. Assuming a characteristic mantle upwelling velocity of 1 cm/yr, such sluggish rate of melt extraction will likely lead to long term retention

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Fig. 6. (a) Observed distribution of impedance contrast as a function of depth from the work of Courtier and Revenaugh (2007). (b) Percentage variation in impedance contrast as a function of basalt fraction (broken curves) and melt (solid curves). Labels on the curves indicate changes in potential temperatures and dihedral angles, respectively.
of melt in the low velocity layer. Recent experimental measurements indicate that carbonate-rich melts atop the transition zone are neutrally buoyant, precluding any gravitational drainage (Ghosh et al., 2007). The frictional resistance offered by the matrix is also plotted as a function of the melt volume percent in the low velocity layer. The sharp, two orders of magnitude increase in the frictional resistance with a 1% decrease in the melt fraction indicates that melt drainage out of the partially molten zone will be rather sluggish.

Secondly, at small melt fractions, surface tension arising from grain boundaries likely reduces the efficiency of gravitational drainage. Strong tension on intergranular contacts reduces the dihedral angle at the melt-grain triple junctions (von Bargen and Waff, 1986; Wray, 1976), establishing a well-connected network. Despite the presence of this well connected pathway, a larger force is required to counter the strong capillary tension and segregate melt from the matrix, especially at small melt fractions (Hier-Majumder et al., 2006). Numerical models indicate that surface tension, coupled with compaction–decompaction, can influence buoyancy-driven melt drainage over length scales several times larger than the matrix compaction length (Hier-Majumder et al., 2006; Takei and Hier-Majumder, 2009). Thus, if the melt in the low velocity layer has a density contrast with the matrix, for the inferred 1% melt volume fraction, drainage of such melt out of the low velocity layer can be inefficient.

In the absence of a density contrast between the melt and the matrix, surface tension can homogenize local peaks of melt over several compaction length scales (Hier-Majumder et al., 2006; Stevenson, 1986; Takei and Hier-Majumder, 2009). Numerical simulations of a one dimensional melting column indicate that a background melt fraction of 1% and a peak melt fraction of 11% can be homogenized over a length of tens of kilometers in approximately 1 Ma (Hier-Majumder et al., 2006). Such a homogenization mechanism can help explain the relatively large vertical extent of the layer. One issue that complicates the relatively simple picture of capillary homogenization, is the likely variation of density contrast between melt and the matrix with depth. In addition, an increase in melt viscosity by devolatilization will also likely restrict the mobility.

4.2. Influence of melt composition

As the plot in Fig. 7 demonstrates, the melt fraction necessary to explain the observed shear impedance contrast can be lower for lower dihedral angles. Recent experimental evidence of melting of carbonate bearing rocks suggests that the likelihood of carbonate-rich melts in a ridge geotherm can take place at a depth of 350 km (Dasgupta and Hirschmann, 2010). At pressures atop the transition zone, such a melt becomes neutrally buoyant (Ghosh et al., 2007) and can be redistributed via homogenization. Variable melt composition and volatile content can influence the dihedral angle and hence our estimate of the extent of melting. Another important influence on the dihedral angle is the effect of pressure, since the dihedral angle generally decreases with pressure (Yoshino et al., 2007). Finally, liquid immiscibility of silicate and carbonate melts can provide an additional mechanism for trapping of
carbonate bearing melts (Minarik, 1998). Further high resolution measurements of dihedral angle and experimental determination of three dimensional contiguity (Zhu et al., 2011) will provide powerful insights relevant to this study.

In addition to the impedance contrast of shear waves, melt geometry influenced by melt composition, may have a modest influence on anelastic attenuation (Karato, 2008, Ch 11.4). Anelastic attenuation can be caused either by ‘melt squirt’ through an interconnected network (Mavko and Nur, 1975) or enhanced grain boundary sliding by unpinning of triple grain junctions (Faul et al., 2004). It is possible, however, to observe changes in the shear wave velocity without any corresponding changes in attenuation if the melt is isolated in pockets (Karato, 2008, Ch 11.4). Since the inherent microstructural elements in the anelastic attenuation models are different from our current model, a direct quantification of anelastic attenuation from our model is beyond the scope of this article. For a comparison of the velocity reduction upon melting, predicted by different models, see Takei (2002). For a small amount of melt as inferred from our model, the anelastic attenuation is likely to be rather modest (Gribb and Cooper, 2000).

Electrical conductivity of the partially molten low velocity layer can also be influenced by melting and melt composition. A number of studies indicate regional variations in the electrical conductivity atop the transition zone. Beneath the North Pacific and French Alps, electrical conductivity in the depth range of our interest varies between 0.001 and 0.01 S/m (Tarits et al., 2004; Utada et al., 2003). In contrast, electrical conductivity values between 0.05 and 0.1 S/m were recorded atop the transition zone in NE China (Ichiki et al., 2001). An earlier study by (Lizarralde et al., 1995) also indicated an electrical conductivity of 0.05 to 0.1 S/m between 150 and 400 km depth in the North Pacific.

Using the calculated melt volume fraction, we can estimate the effective electrical conductivity, \( \sigma \), from Archie’s law, \( \sigma = C \sigma_m \phi^{0.6} \) (ten Grotenhuis et al., 2005; Yoshino et al., 2010). Using values from ten Grotenhuis et al. (2005), for the constant \( C = 1.47 \), melt conductivity \( \sigma_{\text{melt}} = 7.5 \) S/m, and the exponent \( n = 1.3 \), we obtain an effective conductivity of 0.03 S/m, for \( \phi = 0.01 \) of basaltic melt. For the same melt volume fraction, using the values of \( C = 0.67, \sigma_{\text{melt}} = 10 \) S/m, and \( n = 0.89 \) from the result of Yoshino et al. (2010), we obtain an effective conductivity of 0.11 S/m for olivine-MORB aggregates. Yoshino et al. (2010) also suggest, for carbonate melt in an olivine-rich rock, \( C = 0.97, \sigma_{\text{melt}} = 100 \) S/m, and \( n = 1.14 \), leading to an effective electrical conductivity of 0.51 S/m, higher than the high values reported by Ichiki et al. (2001) and Lizarralde et al. (1995). Only 0.35% carbonate melt can produce an effective conductivity of 0.13 S/m (Yoshino et al., 2010). This melt fraction, however, is too low to explain the observed shear impedance contrast for the dihedral angle for carbonate melt reported by Minarik and Watson (1995), as indicated in Fig. 7. Lower pressure measurements of electrical conductivity by Gaillard et al. (2008), yield an even higher conductivity for carbonate melts, requiring an even smaller melt fraction in the low velocity layer. Thus, based on our estimates of melt volume fraction, the high electrical conductivity of the low velocity layer is best explained by 1 vol.% MORB melting using the data of Yoshino et al. (2010). The plot in Fig. 9 compares these three laboratory measurements with the observed values of conductivity atop the transition zone. The calculated conductivity values are plotted as a function of the melt volume fraction. As the plot and the estimates above indicate, the effective electrical conductivity of a carbonate melt bearing aggregate is much higher than the observed conductivity, for the melt volume fractions that match the shear wave impedance contrast. New studies on high resolution measurement of dihedral angle in carbonate melt bearing aggregates can help reconcile the apparent contradiction between these two sets of observations.

5. Conclusions

The combined model of thermal, compositional, and melting anomalies studied in this work indicate that the shear wave impedance contrast observed atop the low velocity layer above the ‘410 km discontinuity’ must be caused by melting, rather than a solid, compositionally distinct layer or a thermal anomaly. Based on the currently available experimental measurements of dihedral angles, we estimate that melting of approximately 1% by volume, causes the seismic anomaly observed beneath the southwest Pacific by Courtier and Revenaugh (2007). Importantly, a zero-degree dihedral angle, as proposed by Courtier and Revenaugh (2007), is not necessary to maintain the observed thickness of the low velocity layer. For the small melt volume fraction, gravitational separation is inefficient due to low permeability and melt retention due to capillary tension at the grain–grain contacts. For a neutrally buoyant melt, homogenization by compaction dec ompaction can redistribute the melt over the entire thickness of the observed low velocity layer.

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Appendix A. Melt percolation in a column

In this section, we consider the percolation of melt in a one-dimensional column in the steady state. This calculation follows the analysis outlined by Ricard et al. (2001) for one-dimensional forced compaction (see Section 2.1 in their article). The mass and momentum conservation equations are similar to Hier-Majumder et al. (2006). Consider a one-dimensional column filled with a constant melt volume fraction \( \phi_0 \). In the steady state, for a constant melt fraction, conservation...
of momentum for this system can be written by rearranging Eq. (13) of Hier-Majumder et al. (2006) as

$$\frac{4\mu_m}{3} \left(1 - \phi_0^2\right) \frac{d^2v_m}{dz^2} - \left(1 - \phi_0^2\right) \Delta\rho g = 0,$$  \hspace{1cm} (A.1)

where $\mu_m$ is the matrix viscosity, $v_m$ is the matrix velocity, $z$ is the height within the column (0 at the base), $\Delta\rho$ is the density contrast between the matrix and the melt, $g$ is gravity, and $c$ is the frictional resistance. The frictional resistance, inverse of melt mobility, depends on the melt fraction. Using a model of melt distributed in an isotropic tubular network (Hier-Majumder, 2011), this can be expressed as a function of grain size $a$, melt or fluid viscosity, $\mu_f$ and $\phi_0$ as Hier-Majumder (2011),

$$c = \frac{72\pi}{a^3\phi_0^3}.$$  \hspace{1cm} (A.2)

To solve for the matrix velocity profile within the layer, we nondimensionalize (A.1) with the following nondimensionalization scheme,

$$z = \delta \sqrt{\phi_0 a(1-\phi_0) z'}, \hspace{1cm} v_m = \frac{4\mu_m}{3c} \frac{v_{m'}}{\phi_0},$$  \hspace{1cm} (A.3)

where compaction length $\delta = \sqrt{4\mu_m a(1-\phi_0)/3c}$, $\mu_m$ is the density of the matrix and the primed quantities are dimensionless. The nondimensional second order linear ordinary differential equation in $v_{m'}$ is given by

$$\frac{d^2v_{m'}}{dz'^2} - \left[\frac{3\Delta\rho}{4\mu_m} \phi_0^2 (1-\phi_0) + v_{n'}\right] = 0.$$  \hspace{1cm} (A.4)

We impose the boundary conditions $v_{n'} = \tau$ at $z' = 0.1/\sqrt{\phi_0 a(1-\phi_0)}$ and integrate (A.4) to obtain the solution for the matrix velocity given by

$$v_{m'} = \frac{\left[1 - \cosh z_0 \sinh z' - \cosh z'\right]}{\sinh z_0} \left[\frac{3\Delta\rho}{4\mu_m} \phi_0^2 (1-\phi_0) + \tau\right] - \frac{3\Delta\rho}{4\mu_m} \phi_0^2 (1-\phi_0),$$  \hspace{1cm} (A.5)

where $z_0 = 1/\sqrt{\phi_0 a(1-\phi_0)}$. Velocity of the fluid phase, $v_f$, can be obtained from the combined mass conservation equation for the melt and the matrix phases (Hier-Majumder et al., 2006, Eq. (11)), yielding

$$v_f = \frac{1 - \phi_0}{\phi_0} v_m,$$  \hspace{1cm} (A.6)

where the velocity of the matrix is given by Eq. (A.5).

The maximum value of the melt velocity was obtained from Eq. (A.6) as an upper limit of the melt velocity within the compacting layer. The velocity field was calculated using $\tau = 0$ in Eq. (A.5). To obtain the dimensional value of the maximum melt velocity and the frictional resistance we used the constants listed in Table A.1.

### Table A.1

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<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
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<tr>
<td>$\mu_m$</td>
<td>Matrix viscosity</td>
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References


