

Elastic Flexure Models for Sputnik Planitia on Pluto

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Abstract

The 2015 New Horizons flyby of Pluto revealed an unexpected feature, Sputnik Planitia, a 1,300 km by 500 km elliptical basin with a depth of about 3 km. The question of the origin of Sputnik Planitia arose as soon as the basin was discovered, and thus far, two major hypotheses have been proposed. The first hypothesis is that an impactor created the basin and subsequent deposition of ice proceeded (Moore et al., 2016). The second hypothesis was that nitrogen ice accumulated at the surface of Pluto and a feedback between Pluto's orientation and nitrogen deposition produced an ice cap large enough to deform the crust and form a basin (Hamilton et al., 2016). Much of that nitrogen ice must be gone today. The ice cap hypothesis implies that the lithosphere of Pluto would have been flexed over a large distance. Therefore, we evaluate here whether the current topography of Sputnik Planitia and its surroundings shows evidence for the bulge that would have formed in the context of elastic flexure induced by a large nitrogen ice load inside Sputnik Planitia. First, an inversion scheme based on the superposition of numerous loads in a distributed area was tested using a synthetic model of elastic flexure and was determined to be able to extract flexural parameters that were used to find elastic thickness. Then, a set of 20 one-dimensional topographic profiles was extracted across the rim of Sputnik Planitia to evaluate a best-fitting flexural parameter, which is then converted to an elastic thickness estimate. Complications arise from defining the initial elevation of the surrounding plains and also the presence of craters and other degradation of the plains. Some profiles do not show evidence for a flexural signal, can be flat right up to the edge of Sputnik Planitia, or have an essentially regular slope over several hundred km. However, 13 out of the 20 profiles show a clear flexural bulge implying elastic thicknesses of 5 to 67 kilometers. The average elastic thickness is 29.9 kilometers from the weighted average flexural parameter of 108.5 kilometers. Most of these profiles are located across the eastern or western edges of Sputnik Planitia. The heat flux was calculated to be $29 \frac{\text{mW}}{\text{m}^2}$, indicating that Pluto was extremely active at the time the elastic flexure took place. The temperature gradient, 0.0039 K/m, was comparable to Europa, 0.0075 K/m, which means that when the elastic flexure took place, Pluto was as active as Europa is today. The high heat flux would most likely occur early in Pluto's history when more intense radioactive decay and accretion occurred.

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I. Introduction

Prior to the New Horizons flyby, the Hubble Space Telescope was the only technology able to capture images of Pluto, and these images were too poor in resolution to resolve anything but the coarsest information about its surface. This changed when in 2015, New Horizons flew by Pluto, capturing the first high-resolution images of Pluto and revealed striking surface features. One feature that stood out was a large heart-shaped feature formally named Tombaugh Regio. The large elliptical basin on the western side of the heart was named Sputnik Planitia and has been a recent area of interest to study due to its unique geological features and youthful surface. Whereas the crust of Pluto is composed dominantly of water ice (Grundy et al., 2016) and heavily cratered, Sputnik Planitia is covered by nitrogen ice and is devoid of craters. It is important to understand the origin of this enigmatic feature, if it has an endogenic or exogenic origin, and whether it implies that Pluto's interior is currently active.

Geologic Setting of Sputnik Planitia

Sputnik Planitia is a large nitrogen ice-covered basin centered at 20°N 180°E (Loff, 2015). It is teardrop shaped and is approximately 1300 kilometers by 900 kilometers wide (at its widest). The basin is estimated to be 3 to 4 kilometers deep, partially filled with nitrogen ice (Nimmo et al., 2016). Its location straddling the equator implies a positive mass anomalies most easily explained by a locally thin ice shell underlain by an ocean of liquid water (Nimmo et al., 2016). The western edge of Sputnik Planitia features angular blocky mountains proposed to have slid from the surrounding plains (O'Hara and Dombard, 2018). The plateau beyond the western rim of the Planitia contains numerous craters although the plain is sometimes characterized as "degraded" (Figure 1). The basin is limited on its eastern rim by pitted plains, where the pits may represent collapse into voids left by sublimation of CH₄ ice deposits (White et al., 2017). In the Northeastern rim are dark, trough-bounding plains. The southern edge of the basin lacks a well-defined rim. Instead, the basin-interior plains merge into deeply pitted plains (Figure 1). Inside the basin is a deposit of nitrogen ice that features irregularly, polygon-shaped, convection cells approximately 20 kilometers across, surrounded by shallow troughs (White et al., 2017). Analysis of these convection cells indicates that the N₂ ice is ~10 km thick (McKinnon et al., 2016 a; Trowbridge et al., 2016). Another striking discovery is the lack of craters inside Sputnik Planitia leading to the interpretation that the surface is younger than 10 million years old, probably because of constant resurfacing related to convection (Marchis and Trilling, 2016). The geological features of Sputnik Planitia are shown on the geological map in Figure 1 (White et al., 2017).

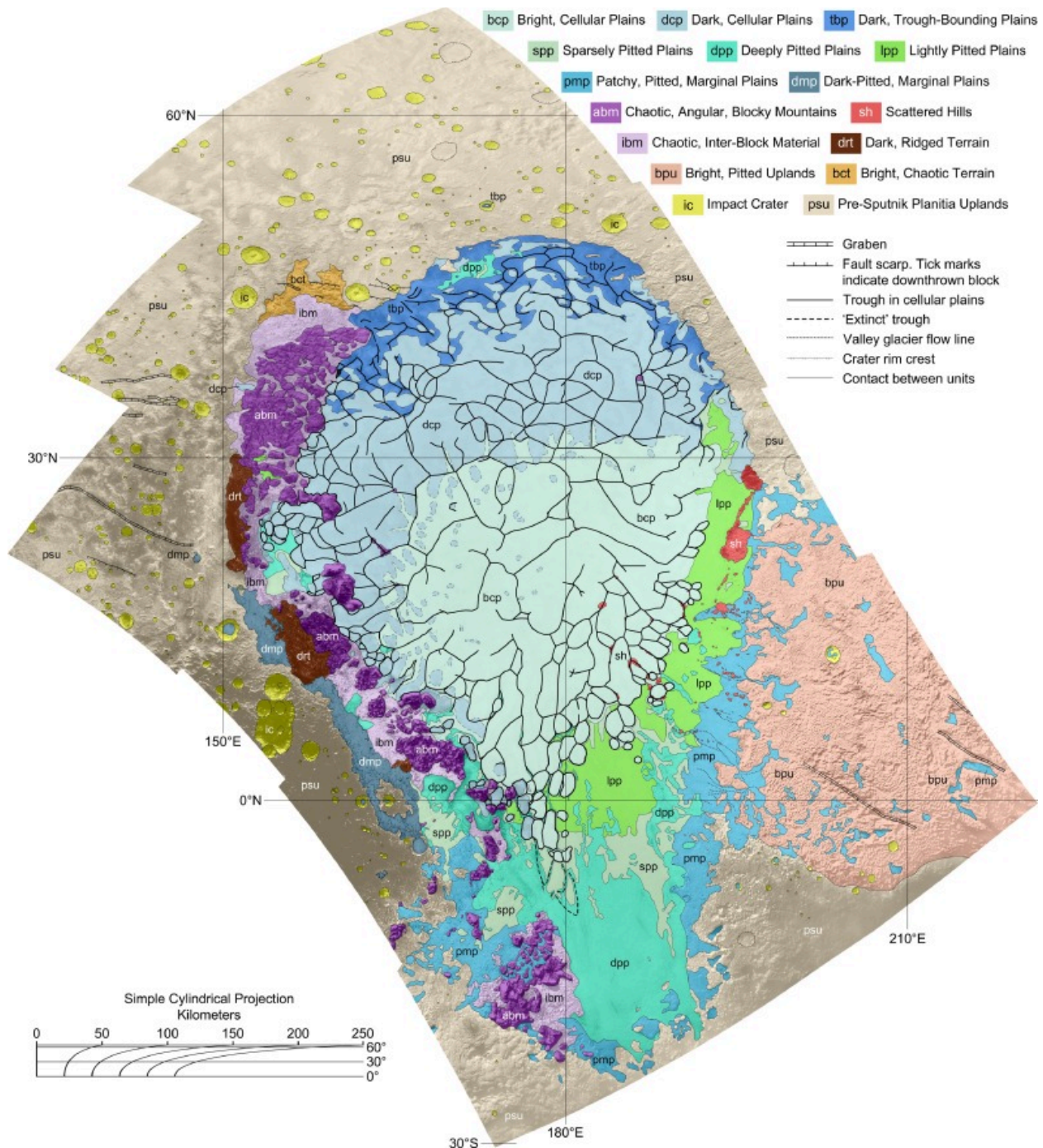


Fig. 1. Geological map of Sputnik Planitia displaying the different geological units imaged by the New Horizons spacecraft with a key of the different units and features (White et al., 2017).

Hypotheses for the Origin of Sputnik Planitia

Due to the unexpectedly young surface age of Sputnik Planitia, the question of how the basin originated and evolved immediately arose. One of the more popular origin theories is that Sputnik Planitia is located in an ancient impact basin created by an impactor that was at least 150 kilometers in diameter traveling at a low speed (McKinnon et al., 2016 b; Johnson et al., 2016). This theory is based on characteristics of impact basins that are recognized on the edge of

Sputnik Planitia, especially the elliptical shape of the basin. The plateau surrounding Sputnik Planitia can be interpreted as containing evidence for ejecta facies, a structural rim uplift, and multiple mountain rings. According to this model, the arrays of the blocky mountains on the western side of Sputnik Planitia are evidence for a raised rim although this feature is not present around the entire rim of the basin. Distant mountain chains form short arcs roughly separated by a factor of the $\sqrt{2}$, similar to lunar basin rings. The main area of support of an impact origin for the Sputnik Planitia basin is its elliptical shape, which is a characteristic of the largest impact basin in the solar system (Andrews-Hanna et al., 2008). The gravity signature of an impact basin could not be confirmed given that New Horizons spacecraft as there were no direct measurements of any gravity signatures (McKinnon et al., 2016 b) but its position straddling the equator is consistent with a thinned ice shell, as may be expected for an impact basin (Nimmo et al., 2016). With this theory, it is assumed that the formation of the basin would have N_2 ice naturally accumulate, possibly due to its low elevation. In this theory, the basin would have moved to its current position near the Pluto-Charon tidal axis through polar wandering, implying it features a positive mass anomaly (Keane et al., 2016; Nimmo et al., 2016).

Although Sputnik Planitia displays some of the characteristics of an impact basin, an alternative model posits that Sputnik Planitia has been at its current location since the time of its formation. In this theory, an albedo feedback effect would have resulted in the accumulation of a thick ice cap: nitrogen ice has a higher albedo than water ice so that the surface temperature of a nitrogen ice deposit is lower than that of surrounding region, which leads to further deposition of nitrogen ice (Hamilton et al., 2016). The feedback process is most likely to initiate at 30 degrees from the equator, consistent with the basin latitude today, and the positive mass of the ice cap would lock Sputnik Planitia at a longitude directly opposite to Charon. When Pluto and Charon locked, a permanent tidal bulge on Pluto would be created which further increases the gravity signature (Hamilton et al., 2016). As this process progressed, the accumulated ice would have caused the crust to bend due to the added weight thus creating a basin. This hypothesis is supported by that ice preferentially accumulated on Pluto near latitudes of 30 degrees north and south through models of the orbit-averaged incident solar energy flux at different latitudes (Hamilton et al., 2016). However, it should be noted that much of the ice cap must have disappeared as the basin is not currently completely filled with N_2 ice.

In my thesis, I test the second hypothesis by searching for evidence of the elastic flexure predicted to lead to the basin formation. Elastic flexure generates a bulge around a load at a distance related to the elastic thickness (Turcotte and Schubert, 2014). The topography of Pluto presents evidence for such a bulge, with high elevation regions lining Sputnik Planitia, especially to the East and West (Figure 2). Therefore, I hypothesize that the topography of Pluto around Sputnik Planitia matches the prediction of an elastic plate flexed by a large nitrogen ice load. The elastic flexure model constrains the thickness of the elastic layer of the lithosphere and the size of the load that is needed to create the observed topography. The values for elastic thickness are linked to the thermal structure of the water ice shell, which has an impact on the timing of the load. The amplitude of the load is linked to the initial ice deposit, although, as we will see, the progressive decay of flexural signals with distance from the load reduces the sensitivity of our analysis to load magnitude.

II. Methodology

I first visualized the digital elevation model of Pluto (Moore et al., 2016) to explore if the current topography of Sputnik Planitia and its surrounding show evidence for a bulge that formed

in the context of elastic flexure induced by a large nitrogen ice load. The topography data came from the Digital Elevation Maps (DEMs) published by Johns Hopkins' Applied Physics Laboratory and available through the Astrogeology Science Center¹. The topography map itself was created using Generic Mapping Tools (GMT)², an open source program that can manipulate geographic and Cartesian data sets to produce postscript images. The map is centered at 20°N, 175°E and uses a Lambert Equal Area projection to minimize distortion (Figure 2). The topography was color-coded using a Haxby color map, where the warm colors, reds and oranges, represent the highest topography while the cool colors, blues, represent the lowest topography. Green was representative of the zero-elevation surface (reference datum). The topography map shows negative topography in Sputnik Planitia and a surrounding bulge of positive topography that resembles the bulge generated by elastic flexure. The bulge is most easily visible to the West and East of Sputnik Planitia but is absent to the Northwest and South of the Planitia.

Twenty tracks were placed perpendicular to the edge of Sputnik Planitia around the entire basin. Each track is six hundred to eight hundred kilometers long, providing a sufficient length to show the topography low on the edge of the basin, the bulge surrounding it, and return to the background plateau elevation (Figure 2). Using GMT, these profiles are then extracted from the DEM into a text file that can be imported into Matlab. The output files consist of four columns: longitude, latitude, horizontal distance along track, and topography data. Only the third and fourth columns, the horizontal distance along the tracks and topography data, are used to create the models. Some of the tracks do not start exactly at the edge of Sputnik Planitia, so their horizontal coordinates are offset as shown in Table 2 and 3 so that the expected flexural signals are in the region of positive coordinates.

¹ https://astrogeology.usgs.gov/search/map/Pluto/NewHorizons/Pluto_NewHorizons_Global_DEM_300m_Jul2017

² <http://gmt.soest.hawaii.edu/>

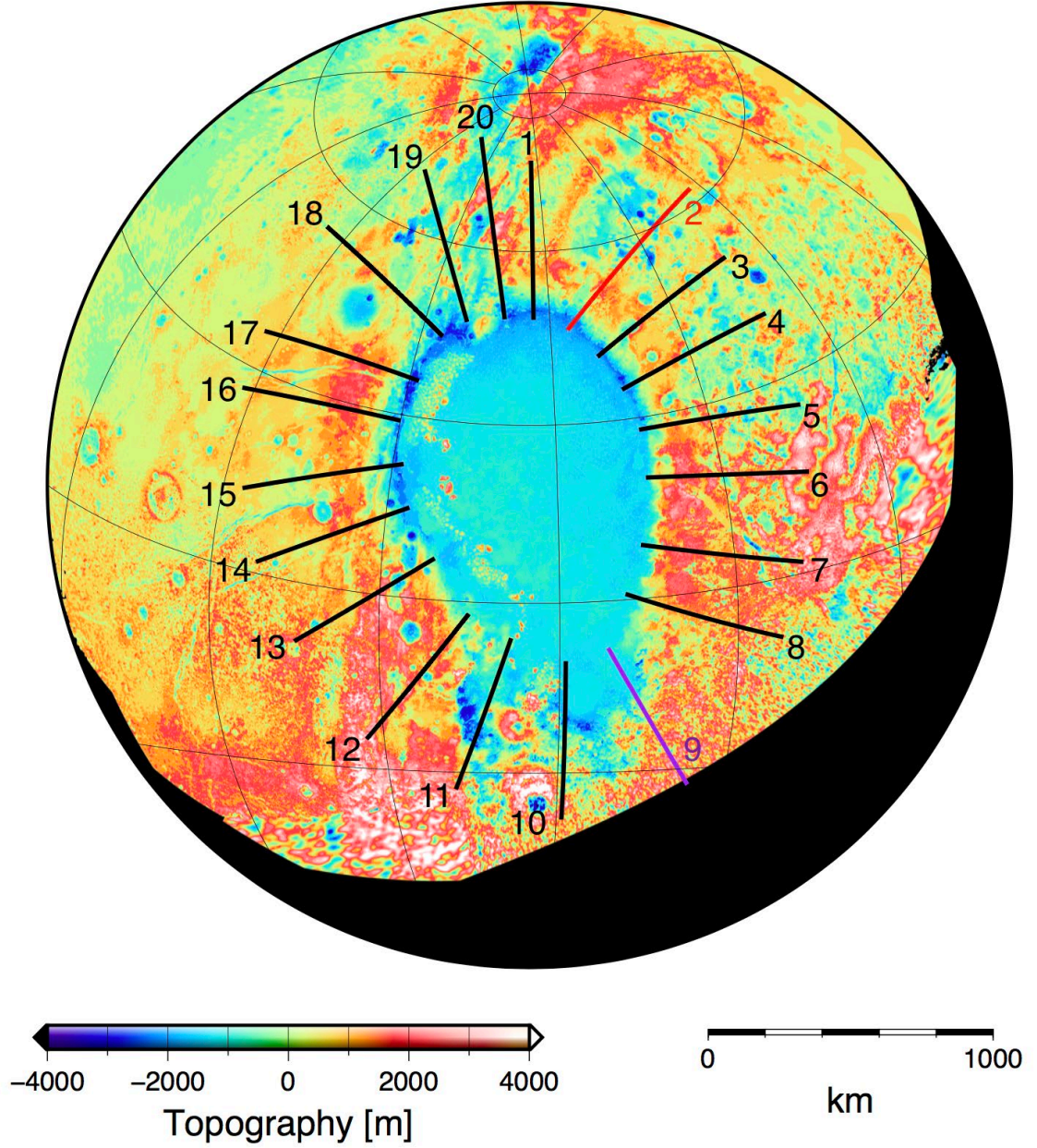


Fig. 2. Topography map of Pluto centered at 175°/20°, using the Lambert Equal Area projection. The map also includes 20 randomly selected tracks on the edge of Sputnik Planitia, highlighting two specific tracks that will be mentioned further in this paper. The black region is an area of no data.

To quantitatively match the topography of Sputnik Planitia and its surroundings to the prediction of a basin with a surrounding bulge from elastic flexure induced by a load, the deflection from a single line load is given by Equation 1:

$$w = w_0 \left[\sin \frac{x-x_0}{\alpha} + \cos \frac{x-x_0}{\alpha} \right] \exp^{-\frac{x-x_0^i}{\alpha}} \quad \text{Equation 1 (Turcotte and Schubert, 2014)}$$

where w_0 is the maximum amplitude of the deflection (defined in Equation 2 and has units of m); x_0 is the position of the line load (in km); and α is the flexural parameter (defined in Equation 3 and has units of km).

The maximum amplitude of the deflection is defined as:

$$w_0 = \frac{V_0 \alpha^3}{8D} \quad \text{Equation 2 (Turcotte and Schubert, 2014)}$$

where V_0 is the magnitude of the applied line load (in $\frac{\text{kgm}}{\text{s}^2}$); α is the flexural parameter (defined in Equation 3 and has units of m); and D is the flexural rigidity (defined in Equation 4 and has units of Pa m^3).

The flexural parameter is defined as:

$$\alpha = \left[\frac{4D}{(\rho_m - \rho_c)g} \right]^{1/4} \quad \text{Equation 3 (Turcotte and Schubert, 2014)}$$

where D is the flexural rigidity (see Equation 4); ρ_m is the density (in $\frac{\text{kg}}{\text{m}^3}$) of the underlying layer of the ice shell; ρ_c is density above the elastic layer, here 0 $\frac{\text{kg}}{\text{m}^3}$; and g is the gravity (in $\frac{\text{m}}{\text{s}^2}$) on Pluto.

Flexural rigidity is defined as:

$$D = \frac{E * h^3}{12(1 - \nu^2)} \quad \text{Equation 4 (Turcotte and Schubert, 2014)}$$

where E is the Young's modulus (in GPa) of water ice; h is the elastic thickness (in km); and ν is the Poisson's ratio (unitless) of water ice.

| Physical Parameters | | |
|---------------------|--|-------------------------------------|
| Symbol | Name | Value/Unit |
| ρ_m | Density of material underlying the ice shell | 1030 $\frac{\text{kg}}{\text{m}^3}$ |
| ρ_c | Density of material above the ice shell | 0 $\frac{\text{kg}}{\text{m}^3}$ |
| g | Gravity | 0.62 $\frac{\text{m}}{\text{s}^2}$ |
| E | Young's modulus | 9 GPa |
| ν | Poisson's ratio | 0.3 |

Table 1. Rheological parameters used for both the one-dimensional and two-dimensional elastic flexure models. (Sources for parameters are from Johnson et al., 2016 and Nimmo et al., 2016.)

For application to the flexure surrounding Sputnik Planitia, we assume that there are is a set of loads, each at a position x_0^i , leading to a deflection amplitude w_0^i . Therefore, the total deflection of the elastic plate, assuming a single flexural parameter, α , is given by:

$$w = \sum_i^n w_0^i \left[\sin \frac{x - x_0^i}{\alpha} + \cos \frac{x - x_0^i}{\alpha} \right] \exp \frac{x - x_0^i}{\alpha} \quad \text{Equation 5}$$

An inversion is used to find the flexural parameter, an unknown variable, that produces the best fit deflection with the data. The inversion was created through a function called *invertelastic* which is found in Appendix B. The function is fed multiple inputs: the distance across a track, x , the associated topography on each point of the track, t , a range of flexural

parameters to test, $\{\alpha\}$, a vector containing the locations of each line load $\{x_0^i\}$, a mass variance used for regularization of the inversion V , and a topography variance σ . The topography variance σ is given by the standard variation of the final 200 m of the profile under the assumption that this section is far enough from the load that it includes no measurable flexural signal.

For each candidate flexural parameter value, the loads values $\{w_0^i\}$ that best fit the profile are determined using a least square method in a function called *lineloadinvert*, found in Appendix C. This function also returns the misfit, quantified by χ^2 (Equation 6), and the topography w predicted by the optimized load:

$$\chi^2 = \sum_i \frac{(t_i - w_i)^2}{\sigma^2} \quad \text{Equation 6 (Press et al., 2007)}$$

Essentially, *invertelastic* calls on multiple reiterations of *lineloadinvert*, so that there are χ^2 values for each of the flexural parameters the user decided to test.

Once the χ^2 associated with each candidate value of α has been recorded, the inversion result is given as the value of α that produces the minimum χ^2 . Uncertainty on the value of α is given by considering all the flexural parameter values that result in an acceptable value of χ^2 , that is, χ^2 less than a threshold value obtained as described below. The χ^2 threshold is calculated by summing the χ^2 minimum and the delta chi-square, $\Delta\chi^2$. The $\Delta\chi^2$ is the evaluation of the inverse incomplete gamma function, Equation 7:

$$x(P) = 2P^{-1} \left(\frac{\nu}{2}, P \right) \quad \text{Equation 7 (Press et al., 2007)}$$

where ν is the degrees of freedom which is difference of the number of data points and inverted parameters, and P is the confidence limit of 68% (the 1-sigma range).

With a range of acceptable flexural parameters, a range of elastic thickness can be found through the rearrangement of flexural rigidity, Equation 4, to create Equation 10:

$$h = \frac{12D(1-\nu^2)}{E^{1/3}} \quad \text{Equation 10}$$

where ν is Poisson's ratio; E is the Young's modulus; and D is the flexural rigidity, given in Equation 11:

$$D = \frac{\rho g \alpha^4}{4} \quad \text{Equation 11}$$

where ρ is the density of water ice and g is the gravity on Pluto. All parameter values are given in Table 1.

Corrections

For elastic flexure, at distances far from the load source, the topography trends towards zero (Equation 1). When evaluating the topography of Sputnik Planitia and its surrounding, the topography at long distances must be manipulated, so that the topography goes to zero at long distances. This was performed by taking the average of the last 200 kilometers of each track and subtracting it from the entire topographic profile.

Outside of Sputnik Planitia are numerous craters and pits which are not representative of the original topography and cause inaccurate measurements as seen in Table 2. The craters and pits were identified through visual inspection. They are then eliminated by removing the data that corresponded with the craters and pits. The removal of craters and pits correspond to

corrected profiles later in this paper, and the uncorrected profiles are profiles without this removal.

All of the relevant code used is included in Appendix VII.

III. Results

In Figure 2, the 20 tracks that were tested are presented. Highlighted on this figure are two tracks discussed in detailed here: Track 2 (red), which shows evidence for elastic flexure, and track 9 (magenta), which does not. All the other tracks and their fits are shown in Appendix k through bb.

For track 2, the first 39 km were eliminated as they were inside Sputnik Planitia, where the load is located, and the elastic flexure is only expressed outside of Sputnik Planitia. Also, the average topography subtracted from the data was -246.8 m and the topography variance was 749.5 m (Table 2). The range of flexural parameter, α , that was tested was 10 to 1000 km. The optimal α for profile 2 was 115 km to create the best fit (Figure 3), which corresponds to the minimum chi-square, χ^2 , value of 390.1. The range of acceptable values within the 1- sigma range gives $\alpha = 115_{-103}^{+184}$ km for the χ^2 threshold of 673.2 km (Figure 4). The acceptable range of elastic thickness is $32.4_{-30.8}^{+83.3}$ km.

In Figure 3, the topography predicted for the best fit α and the extrema of acceptable range of α are presented. The topography predicted for the minimum acceptable α bends sharply in the first 100 km of the profile whereas the topography predicted for the maximum acceptable α is subdued. For each extreme value of alpha, there are segments of the topographic profile that are systematically over-predicted or under-predicted, but the overall fit cannot be rejected due to the high topographic variance.

Figure 3 also shows the load amplitudes that correspond to the best fits. The largest amplitude in the load is the location of the line load which influences the elastic flexure most. However, because flexural deflection decreases exponentially with distance from the load (Equation 1), loads far from the edge of Sputnik Planitia do not have a large effect on the topographic profiles and are therefore poorly constrained. They are probably underpredicted because regularization of the least square inversion favors the null hypothesis $w_0 = 0$ in the absence of constrained. Therefore, the maximum extends and amplitude of the load distribution (Figure 3B) is not a reliable output of the inversion.

For profile 2, the load from 50 to 300 km inside Sputnik Planitia influences the elastic flexure the most for the optimal α value. The load distribution associated with the minimum acceptable α is greatest within 50 km of the edge of Sputnik Planitia, but it also features positive w_0 values, which would implicate the surface is lifted up instead of being down-dropped by the ice cap load. The load distribution associated with the maximum acceptable α extends to the greatest distances from the edge of Sputnik Planitia in which the track was taken from since the elastic flexure at this flexural parameter is sensitive by the topography at large distances. The large range of load values that produce the deflection that best fits the topographic profile within the acceptable range of α shows that load magnitude is not reliably constrained by my analysis.

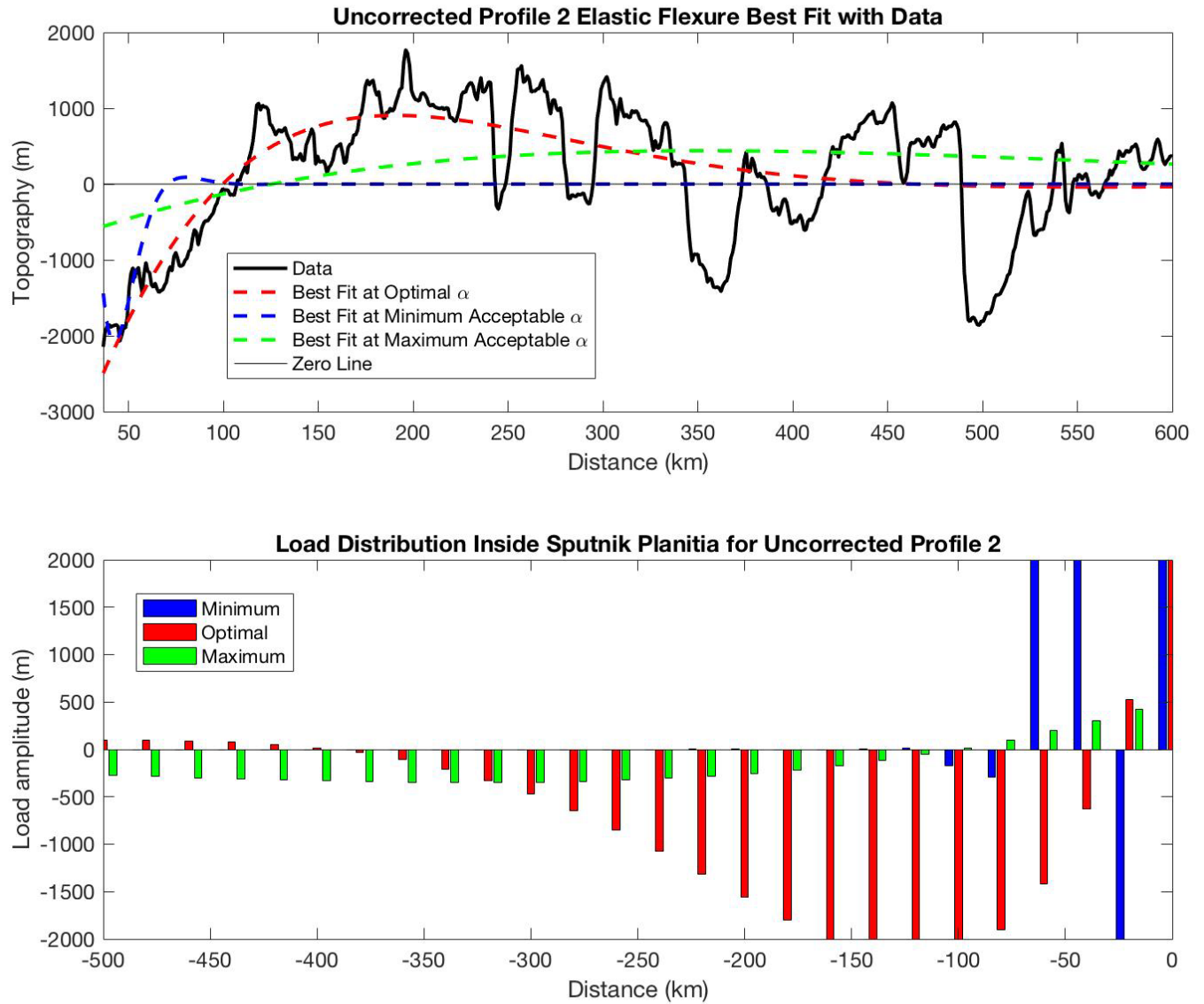


Fig. 3. Top panel: Track 2's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot still has craters present in the data at distance of 240 to 255, 275 to 304, 333 to 375, and 486 to 541. Lower panel: Profile 2's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

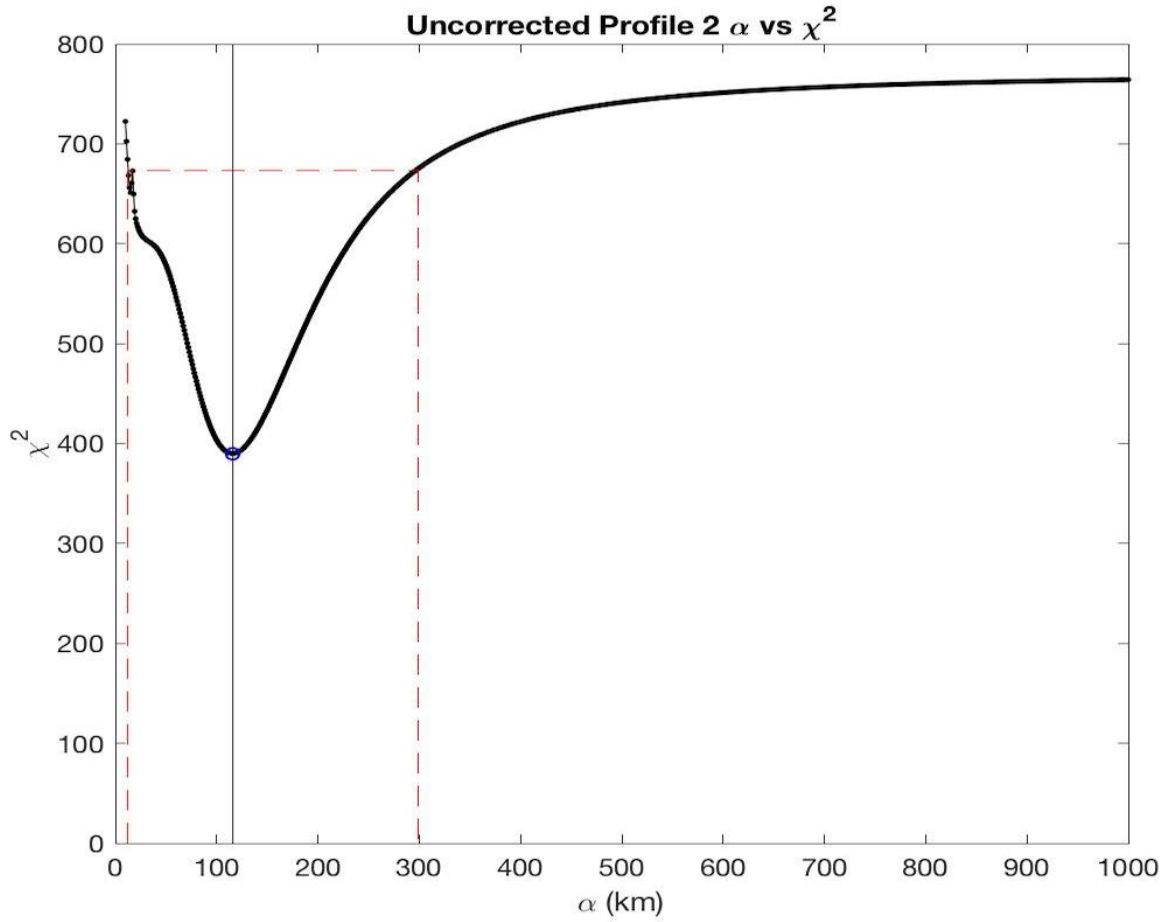


Fig.4. χ^2 versus flexural parameter plot of track 2 with craters where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

Surrounding Sputnik Planitia are craters and pits that influence the calculations of elastic flexure but are not representative of the flexure-induced topography. The craters and pits must be eliminated from the profiles if they exist. As seen in Figure 3, craters are located at the distances 240 to 255, 275 to 304, 333 to 375, and 486 to 541 km. I delete the topographic measurements that falls inside craters and pit and repeat the analysis for this corrected profile. The result is shown in Figure 5. For the corrected profile 2, the average topography at long-distance was 82.2 m, and the topography variance was 405.5 m. The acceptable range of α was 122^{+64}_{-55} km for the χ^2 range of 359.0 to 573.6 as shown in Figure 6. With the removal of craters, the acceptable α range compresses which improves confidence that the topography is well modeled by my elastic flexure model. Note also that none of the loads associated with the range of acceptable α require a positive load, so that the result is more consistent with the load being associated with an ice cap. The elastic thickness range calculated using the acceptable α range is $35.0^{+26.4}_{-19.3}$ km.

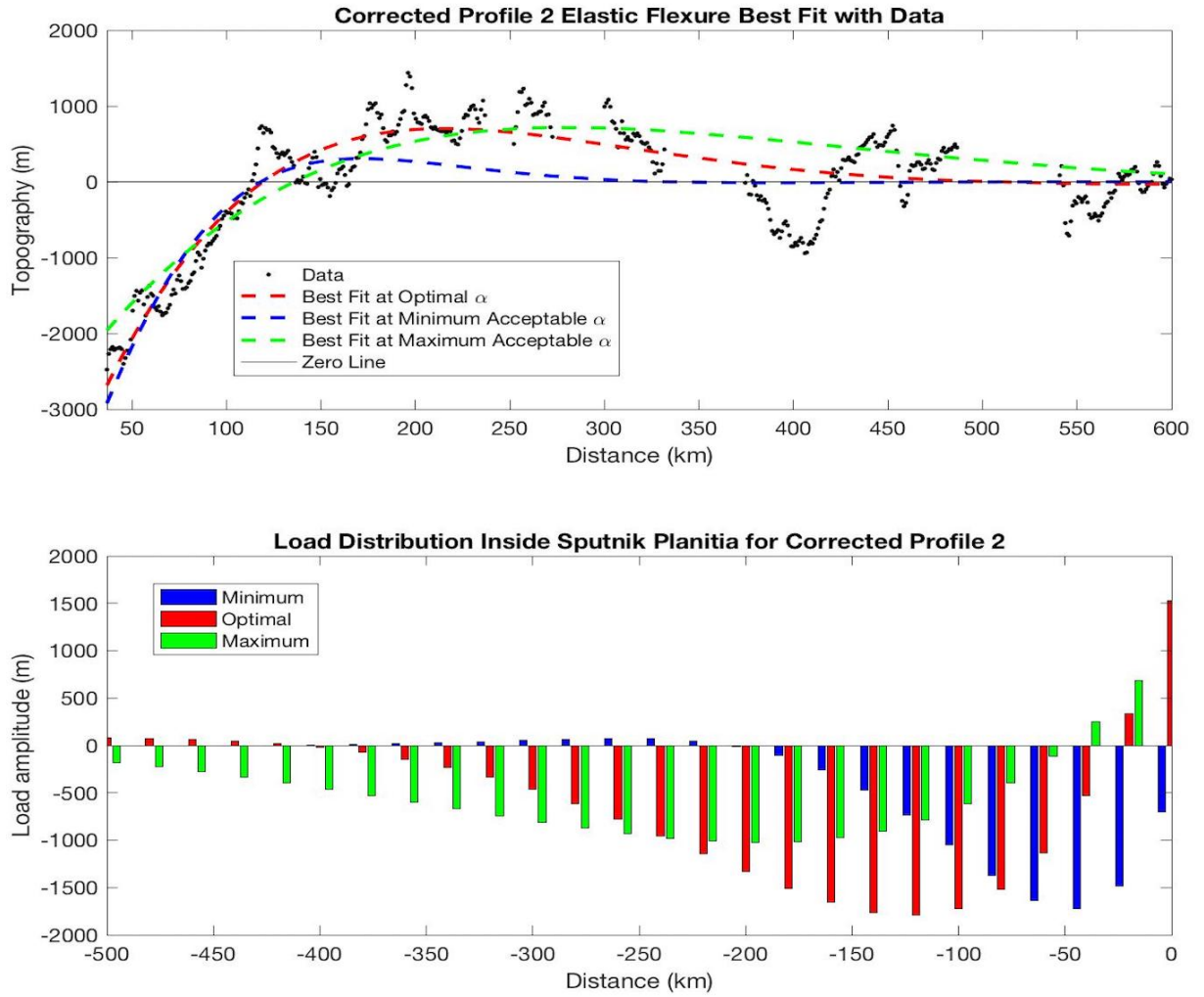


Fig. 5. Top panel: Track 2's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data at distance of 240 to 255, 275 to 304, 333 to 375, and 486 to 541. Lower panel: Profile 2's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

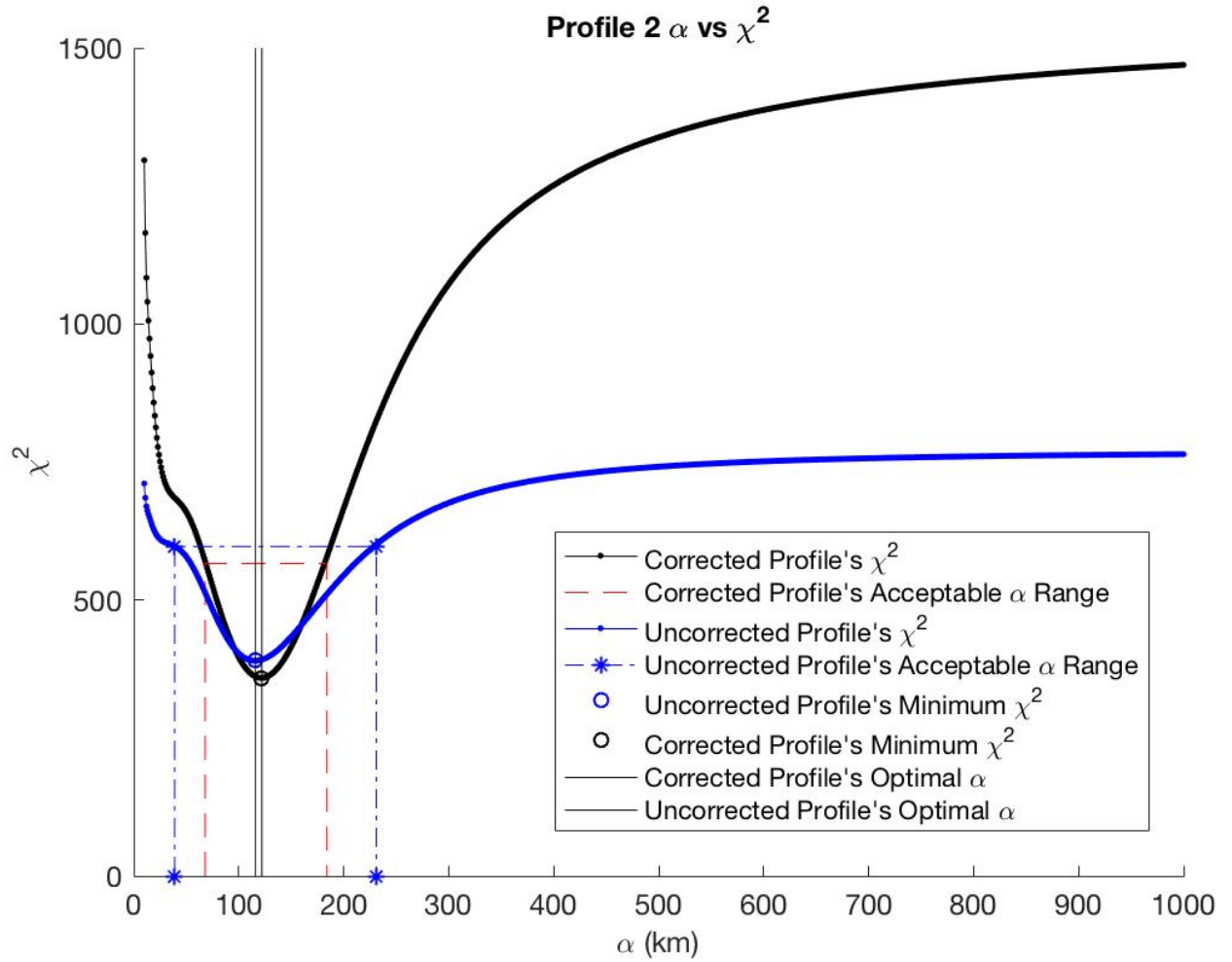


Fig.6. χ^2 versus flexural parameter plot of track 2 without craters where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized. Also on this figure is the acceptable flexural parameters for the uncorrected profile 2 to show that the range became narrower with the removal of craters along with its χ^2 curve.

A majority of the profiles taken around Sputnik Planitia (Figure 2) similarly depict elastic flexure, but seven profiles do not. A representative profile that displays no elastic flexure is profile 9 which is highlighted in magenta in Figure 2. Profile 9 comes from a track that is located in the Southeastern region of Sputnik Planitia which is dominated by pitted plains. The profile is relatively short due to the limited coverage of Pluto's DEM: the flyby mission did not return global images at sufficient resolution to conduct the kind stereo-imaging analysis from which the high-resolution DEM used here was derived. Due to the dominance of the pitted plains throughout the track, the profile shows large variance as seen in Figure 7. Since the topography varies greatly, the acceptable flexural parameter range can take extreme values. For this profile, the flexural parameter range was $1820_{-1760}^{+\infty}$ km which corresponds to the χ^2 range of 162.7 to 343.7 as shown in Figure 8. The elastic thickness range calculated from the flexural parameter range was $1285.7_{-1272.1}^{+\infty}$ km. Infinite values means the acceptable α range extends further than

what was used. These values are unreasonable as they far exceed the length of the profile and the flexural model ignores Pluto's curvature. Thus, this profile shows no evidence of elastic flexure. No corrections were done since the pitted plains were distributed through the entire track and would only result in discarding most of the data. The first 183 km were eliminated as they were inside Sputnik Planitia. The average topography subtracted from the data was 1767 m. The topography variance was 833.0 m. The flexural parameter, α , range that was tested was 10 to 2000 km.

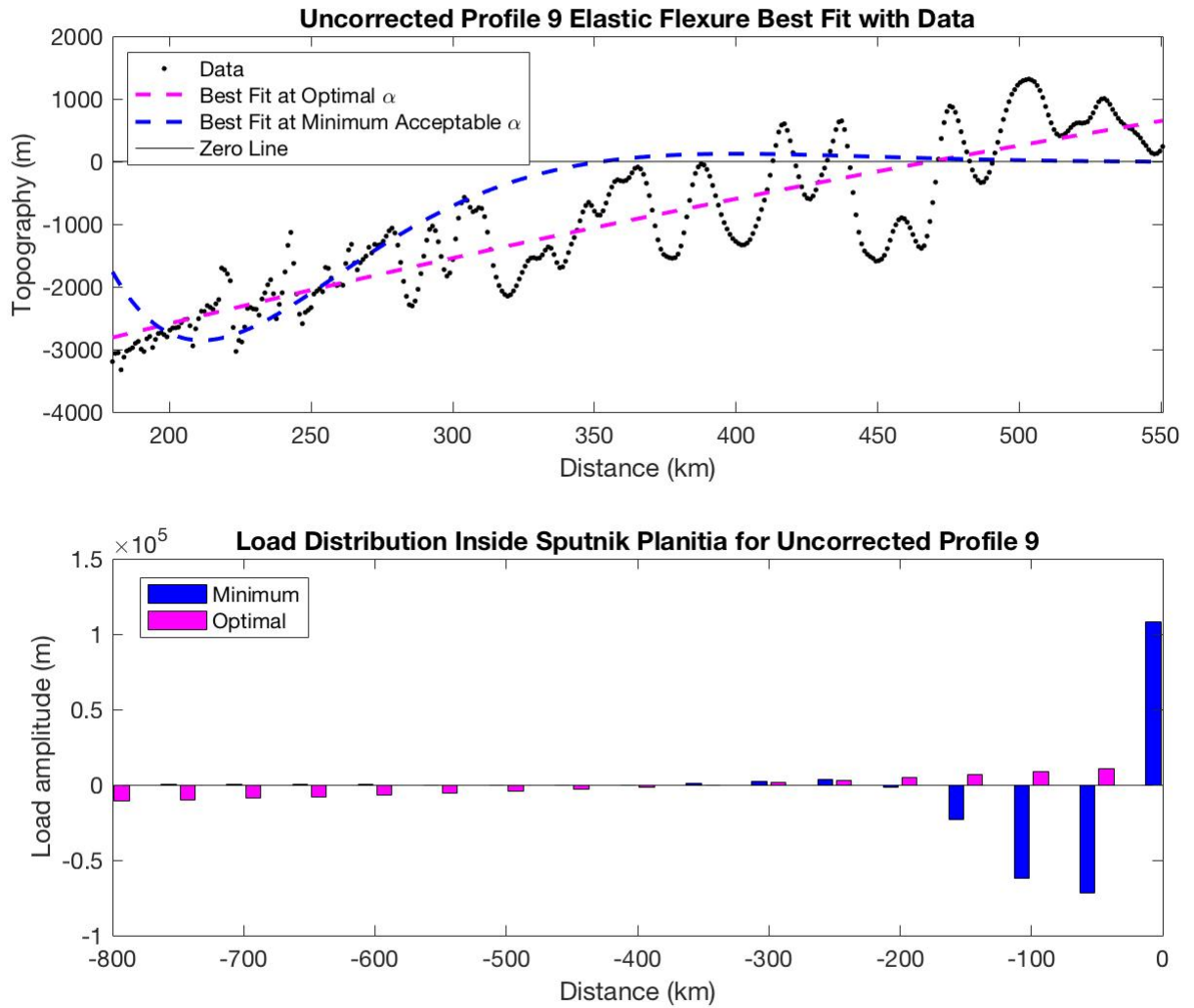


Fig.7. Top panel: Track 9's profile with the best fits from the acceptable range of α where the magenta dashed line is the best with the optimal α where χ^2 is minimized, and the blue dashed line is the best fit with the minimum acceptable α . Lower panel: Profile 9's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

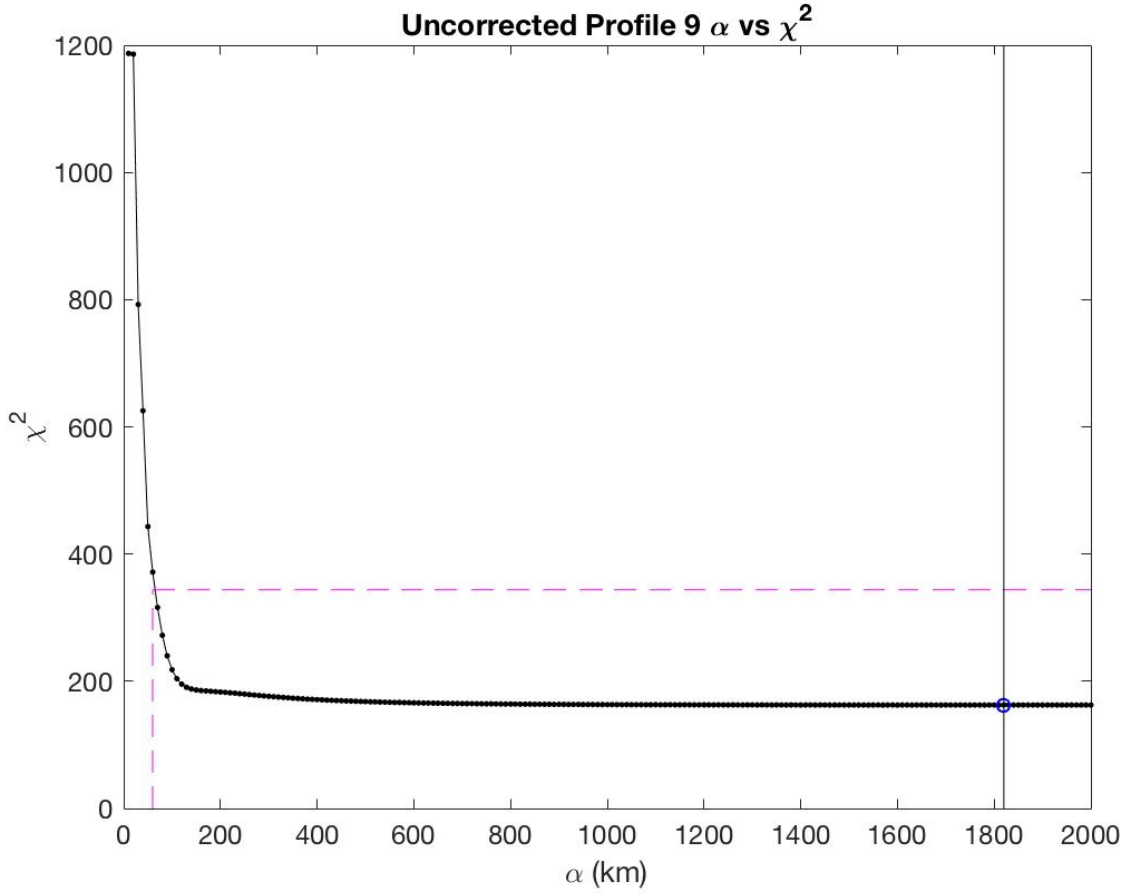


Fig. 8. χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

Profiles 1-2,4-5,7-8, 13-18, and 20 clearly show evidence of elastic flexure. Profile 1's acceptable α range was 56^{+69}_{-46} with an elastic thickness range of $12.4^{+23.8}_{-11.2}$ km without corrections. With corrections, Profile 1's acceptable α was 45^{+72}_{-35} km with an elastic thickness range of $9.3^{+23.8}_{-8.1}$ km. Profile 2's acceptable α range was 115^{+184}_{-103} with an elastic thickness range of $32.4^{+83.3}_{-30.8}$ km without corrections. With corrections, Profile 2's acceptable α was 122^{+64}_{-55} km with an elastic thickness range of $35.0^{+26.4}_{-19.3}$ km. Profile 4's acceptable α range was 75^{+49}_{-65} with an elastic thickness range of $18.3^{+17.5}_{-17.1}$ km without corrections. With corrections, Profile 4's acceptable α was $28^{+\infty}_{-18}$ km with an elastic thickness range of $4.9^{+\infty}_{-3.7}$ km. Profile 5's acceptable α range was $164^{+\infty}_{-122}$ with an elastic thickness range of $51.5^{+\infty}_{-43.3}$ km without corrections. With corrections, Profile 5's acceptable α was 181^{+364}_{-136} km with an elastic thickness range of $59.2^{+198.4}_{-49.9}$ km. Profile 7's acceptable α range was $149^{+\infty}_{-118}$ with an elastic thickness range of $45.7^{+\infty}_{-40.1}$ km without corrections. With corrections, Profile 7's acceptable α was $86^{+\infty}_{-\infty}$ km with an elastic thickness range of $22.0^{+\infty}_{-\infty}$ km. Profile 8's acceptable α range was $86^{+\infty}_{-\infty}$ with an elastic thickness range of $22.0^{+\infty}_{-\infty}$ km without corrections. With corrections, Profile 8's acceptable α was 128^{+42}_{-28} km with an elastic thickness range of $37.3^{+17.2}_{-10.4}$ km. Profile 13's acceptable α range was $28^{+\infty}_{-\infty}$ km with an elastic thickness range of $4.9^{+\infty}_{-\infty}$ km without

corrections. With corrections, Profile 13's acceptable α was 97^{+112}_{-81} km with an elastic thickness range of $25.8^{+46}_{-23.5}$ km. Profile 14's acceptable α range was 92^{+59}_{-76} with an elastic thickness range of $24.0^{+22.5}_{-21.7}$ km without corrections. With corrections, Profile 14's acceptable α was 92^{+40}_{-39} km with an elastic thickness range of $24.0^{+14.9}_{-12.5}$ km. Profile 15's acceptable α range was 113^{+42}_{-35} with an elastic thickness range of $31.6^{+16.6}_{-12.3}$ km without corrections. With corrections, Profile 15's acceptable α was 115^{+46}_{-36} km with an elastic thickness range of $32.4^{+18.3}_{-12.8}$ km. Profile 16's acceptable α range was 94^{+27}_{-22} with an elastic thickness range of $24.7^{+9.9}_{-7.4}$ km without corrections. With corrections, Profile 16's acceptable α was 104^{+25}_{-22} km with an elastic thickness range of $29.3^{+9.4}_{-7.7}$ km. Profile 17's acceptable α range was 104^{+16}_{-14} with an elastic thickness range of $28.3^{+5.9}_{-5.0}$ km without corrections. With corrections, Profile 17's acceptable α was 102^{+15}_{-13} km with an elastic thickness range of $27.6^{+5.5}_{-4.6}$ km. Profile 18's acceptable α range was 148^{+108}_{-108} with an elastic thickness range of $45.3^{+37.4}_{-37.4}$ km without corrections. With corrections, Profile 18's acceptable α was 145^{+835}_{-97} km with an elastic thickness range of $44.1^{+519.1}_{-34.0}$ km. Profile 20's acceptable α range was 219^{+84}_{-50} with an elastic thickness range of $76.4^{+41.4}_{-22.3}$ km without corrections. With corrections, Profile 20's acceptable α was 198^{+70}_{-77} km with an elastic thickness range of $66.8^{+33.2}_{-32.2}$ km. The remaining profiles with the best fits and corresponding load amplitudes are found in the Appendix IX. Also, in the Appendix IX, is the χ^2 versus α plots for all profiles. The associated χ^2 range for the acceptable α 's, topography variance, background elevation subtracted value, and zero-offset values for each profile is found in Table 2 and 3.

Profiles 3, 6, 9-12, and 19 do not show clear evidence of elastic flexure. Profile's 3 acceptable α range was 17^{+100}_{-7} with an elastic thickness range of 2.5^{+100}_{-0} km without corrections. With corrections, Profile 3's acceptable α was 74^{+31}_{-64} km with an elastic thickness range of $18.0^{+10.7}_{-16.8}$ km. Profile 6's acceptable α range was 53^{+100}_{-0} with an elastic thickness range of 11.5^{+100}_{-0} km without corrections. With corrections, Profile 6's acceptable α was 432^{+100}_{-396} km with an elastic thickness range of $189.0^{+100}_{-182.1}$ km. Profile 9's acceptable α range was 1820^{+100}_{-1760} with an elastic thickness range of $1285.7^{+100}_{-1272.1}$ km without corrections. Profile 10's acceptable α range was 7810^{+100}_{-7700} with an elastic thickness range of $8966^{+100}_{-8893.5}$ km without corrections. Profile 11's acceptable α range was 87^{+100}_{-27} with an elastic thickness range of $22.3^{+100}_{-8.7}$ km without corrections. With corrections, Profile 11's acceptable α was 88^{+113}_{-11} km with an elastic thickness range of $189.0^{+100}_{-182.1}$ km. Profile 12's acceptable α range was 346^{+100}_{-330} with an elastic thickness range of $140.5^{+100}_{-138.2}$ km without corrections. With corrections, Profile 12's acceptable α was 117^{+100}_{-101} km with an elastic thickness range of $33.1^{+100}_{-30.8}$ km. Profile 19's acceptable α range was 554^{+100}_{-0} with an elastic thickness range of 263.3^{+100}_{-0} km without corrections. Profiles 9, 10, and 19 were not corrected because their topography had too many craters or pits or that they showed no evidence for elastic flexure which is evident in their large values for α . The remaining profiles with the best fits and corresponding load amplitudes are found in the Appendix IX. Also, in the Appendix IX, is the χ^2 versus α plots for all profiles. The associated χ^2 range for the acceptable α 's, topography variance, background elevation subtracted value, and zero-offset values for each profile is found in Table 2 and 3.

| Uncorrected Profiles | Zero Offset (km) | Background Elevation (m) | Topography Variance (m) | χ^2 Minimum | χ^2 Threshold | α (km) | Elastic Thickness (km) |
|----------------------|------------------|--------------------------|-------------------------|------------------|--------------------|---------------------------|------------------------------|
| Profile 1 | 32 | 599.1 | 466.0 | 651.3 | 927.9 | 56^{+69}_{-46} | $12.4^{+23.8}_{-11.2}$ |
| Profile 2 | 39 | -246.8 | 748.5 | 390.1 | 673.2 | 115^{+184}_{-103} | $32.4^{+83.3}_{-30.8}$ |
| Profile 3 | 24 | 229.6 | 683.8 | 584.7 | 865.3 | $17^{+\infty}_{-7}$ | $2.5^{+\infty}_{-\infty}$ |
| Profile 4 | 54 | 119.5 | 442.4 | 909.0 | 1174 | 75^{+49}_{-65} | $18.3^{+17.5}_{-17.1}$ |
| Profile 5 | 36 | 64.7 | 662.6 | 542.4 | 816.9 | $164^{+\infty}_{-122}$ | $51.5^{+\infty}_{-43.3}$ |
| Profile 6 | 8 | 1918 | 848.2 | 497.3 | 793.1 | $53^{+\infty}_{-\infty}$ | $11.5^{+\infty}_{-\infty}$ |
| Profile 7 | 46 | 278.2 | 1170 | 259.3 | 535.8 | $149^{+\infty}_{-118}$ | $45.7^{+\infty}_{-40.1}$ |
| Profile 8 | 45 | 1164 | 761.9 | 280.5 | 555.5 | $86^{+\infty}_{-\infty}$ | $22.0^{+\infty}_{-\infty}$ |
| Profile 9 | 183 | 1767 | 833.0 | 162.7 | 343.7 | $1820^{+\infty}_{-1760}$ | $1285.7^{+\infty}_{-1272.1}$ |
| Profile 10 | 223 | -806.6 | 980.3 | 240.5 | 428.1 | $7810^{+\infty}_{-7700}$ | $8966^{+\infty}_{-8893.5}$ |
| Profile 11 | 409 | 896.6 | 540.1 | 160.9 | 245.4 | $87^{+\infty}_{-27}$ | $22.3^{+\infty}_{-8.7}$ |
| Profile 12 | 21 | 1830 | 787.8 | 452.5 | 741.7 | $346^{+\infty}_{-330}$ | $140.5^{+\infty}_{-138.2}$ |
| Profile 13 | 32 | 817.3 | 830.8 | 792.5 | 1076 | $28^{+\infty}_{-\infty}$ | $4.9^{+\infty}_{-\infty}$ |
| Profile 14 | 34 | 728.2 | 328.5 | 601.5 | 871.9 | 92^{+59}_{-76} | $24.0^{+22.5}_{-21.7}$ |
| Profile 15 | 24 | 339.1 | 298.5 | 1186 | 1473 | 113^{+42}_{-35} | $31.6^{+16.6}_{-12.3}$ |
| Profile 16 | 29 | 795.6 | 236.6 | 1519 | 1804 | 94^{+27}_{-22} | $24.7^{+9.9}_{-7.4}$ |
| Profile 17 | 21 | 739.5 | 125.2 | 2974 | 3263 | 104^{+16}_{-14} | $28.3^{+5.9}_{-5.0}$ |
| Profile 18 | 10 | 406.2 | 529.6 | 315.2 | 610.0 | $148^{+\infty}_{-108}$ | $45.3^{+\infty}_{-37.4}$ |
| Profile 19 | 3 | -146.9 | 584.9 | 604.0 | 902.3 | $554^{+\infty}_{-\infty}$ | $263.3^{+\infty}_{-\infty}$ |
| Profile 20 | 57 | -547.9 | 498.7 | 1979 | 2300 | 219^{+84}_{-50} | $76.4^{+41.4}_{-22.3}$ |

Table 2. Uncorrected profiles summary of the values for the offset on the beginning of the tracks to begin at the edge of Sputnik Planitia, the background topography removed, the topography variance, χ^2 minimum, the χ^2 threshold, the range of acceptable flexural parameters, and acceptable range of elastic thickness.

| Corrected Profiles | Zero Offset (km) | Background Topography (m) | Topography Variance (m) | χ^2 Minimum | χ^2 Threshold | α (km) | Elastic Thickness (km) |
|--------------------|------------------|---------------------------|-------------------------|------------------|--------------------|-------------------------------------|---|
| Profile 1 | 32 | 742.1 | 394.8 | 360.8 | 570.8 | 45 ⁺⁷² ₋₃₅ | 9.3 ^{+23.8} _{-8.1} |
| Profile 2 | 39 | 82.20 | 405.5 | 359.0 | 573.6 | 122 ⁺⁶⁴ ₋₅₅ | 35.0 ^{+26.4} _{-19.3} |
| Profile 3 | 24 | 670.6 | 233.8 | 460.0 | 740.6 | 74 ⁺³¹ ₋₆₄ | 18.0 ^{+10.7} _{-16.8} |
| Profile 4 | 54 | 1085.6 | 345.8 | 141.0 | 406.4 | 28 ^{+∞} ₋₁₈ | 4.9 ^{+∞} _{-3.7} |
| Profile 5 | 36 | 279.7 | 541.2 | 386.9 | 668.5 | 181 ⁺³⁶⁴ ₋₁₃₆ | 59.2 ^{+198.4} _{-49.9} |
| Profile 6 | 8 | 2294 | 581.8 | 545.1 | 774.9 | 432 ^{+∞} ₋₃₉₆ | 189.0 ^{+∞} _{-182.1} |
| Profile 7 | 46 | 988.1 | 991.0 | 174.8 | 451.4 | 86 ^{+∞} _{-∞} | 22.0 ^{+∞} _{-∞} |
| Profile 8 | 45 | 726.8 | 190.1 | 1040 | 1315 | 128 ⁺⁴² ₋₂₈ | 37.3 ^{+17.2} _{-10.4} |
| Profile 9* | 183 | 1767 | 833.0 | 162.7 | 343.7 | 1820 ^{+∞} ₋₁₇₆₀ | 1285.7 ^{+∞} _{-1272.1} |
| Profile 10* | 223 | -806.6 | 980.3 | 240.5 | 428.1 | 7810 ^{+∞} ₋₇₇₀₀ | 8966 ^{+∞} _{-8893.5} |
| Profile 11 | 409 | 1404 | 300.9 | 206.5 | 291.0 | 88 ⁺¹¹³ ₋₁₁ | 22.6 ^{+45.5} _{-3.6} |
| Profile 12 | 21 | 1830 | 787.8 | 325.7 | 615.0 | 117 ^{+∞} ₋₁₀₁ | 33.1 ^{+∞} _{-30.8} |
| Profile 13 | 32 | 1088 | 521.3 | 442.6 | 660.7 | 97 ⁺¹¹² ₋₈₁ | 25.8 ⁺⁴⁶ _{-23.5} |
| Profile 14 | 34 | 747.6 | 286.7 | 503.0 | 738.4 | 92 ⁺⁴⁰ ₋₃₉ | 24.0 ^{+14.9} _{-12.5} |
| Profile 15 | 24 | 364.5 | 284.9 | 624.3 | 912.0 | 115 ⁺⁴⁶ ₋₃₆ | 32.4 ^{+18.3} _{-12.8} |
| Profile 16 | 29 | 836.8 | 186.2 | 1160 | 1391 | 104 ⁺²⁵ ₋₂₂ | 29.3 ^{+9.4} _{-7.7} |
| Profile 17 | 21 | 734.5 | 126.7 | 2225 | 2379 | 102 ⁺¹⁵ ₋₁₃ | 27.6 ^{+5.5} _{-4.6} |
| Profile 18 | 10 | 480.8 | 490.7 | 267.9 | 536.8 | 145 ⁺⁸³⁵ ₋₉₇ | 44.1 ^{+519.1} _{-34.0} |
| Profile 19* | 3 | -146.9 | 584.9 | 604.0 | 902.3 | 554 ^{+∞} _{-∞} | 263.3 ^{+∞} _{-∞} |
| Profile 20 | 57 | -547.9 | 498.7 | 453.6 | 671.3 | 198 ⁺⁷⁰ ₋₇₇ | 66.8 ^{+33.2} _{-32.2} |

Table 3. Corrected profiles summary of the values for the offset on the beginning of the tracks to begin at the edge of Sputnik Planitia, the background topography removed, the topography variance, χ^2 minimum, the χ^2 threshold, the range of acceptable flexural parameters, and acceptable range of elastic thickness. * means no removal of craters and pits were done.

The graphical representation of the flexural parameters for each profile is summarized in Figure 9 and 10 found in the discussion section. These figures show the flexural parameter plotted with the profile's azimuth. The bottom most point is profile 1 and increases to profile 2 as azimuth increases. The profiles that do have flexure are profiles 1-2,4-5,7-8, 13-18, and 20 while those that do not have clear elastic flexure are profiles 3,6,9-12, and 19. In these plots, the weighted average flexural parameter, 108.5 kilometers, is plotted as blue vertical line.

IV. Discussion

Overall, 13 out of the 20 profiles show a clear flexural bulge implying elastic thicknesses of 5 to 67 kilometers (corrected profiles). These profiles are located primarily in Western and Eastern edges of Sputnik Planitia, and their corresponding flexural parameter values are between 17 to 198 for the uncorrected profiles and 28 and 554 for the corrected profiles as seen in Figure

9 and 10, not including profiles 9,10, and 19. The remaining six profiles do not show evidence for elastic flexure. While the topography can be fitted using our flexure model, the range of α values is not bounded, which means I cannot reject the null hypothesis that the topography does not contain a flexural signal at all.

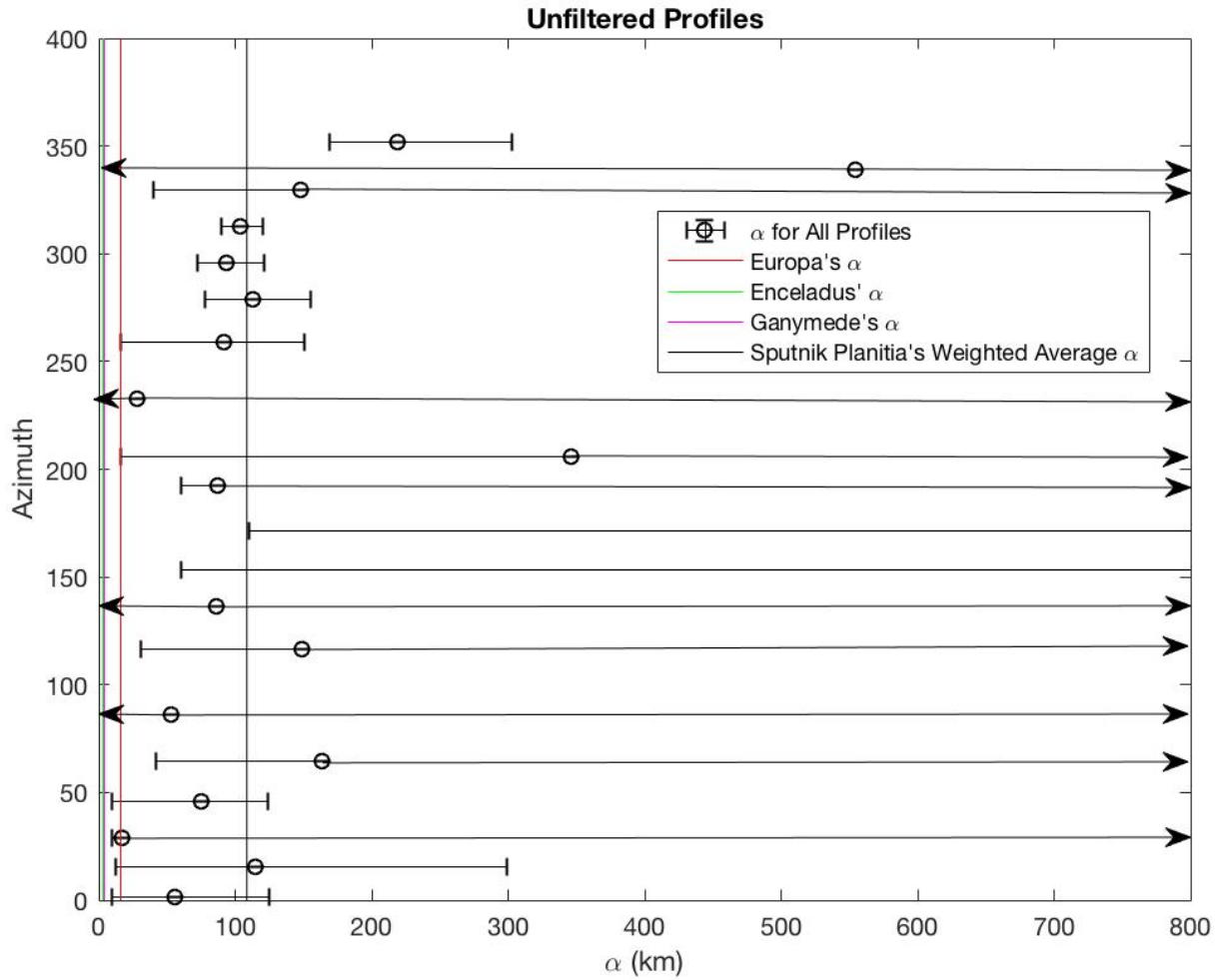


Figure 9. Azimuth versus α plot for profiles with craters and pits, showing that the flexural parameter range for most profiles are between 17 and 554 with two outlier values for Profiles 9 and 10 which are past the bounds of 800 km. The arrows indicate infinite values that are past the bounds of 0 to 800 km. The α values for Europa, Enceladus, and Ganymede are plotted in red, green, and magenta respectively. Sputnik Planitia's weighted average α is in black.

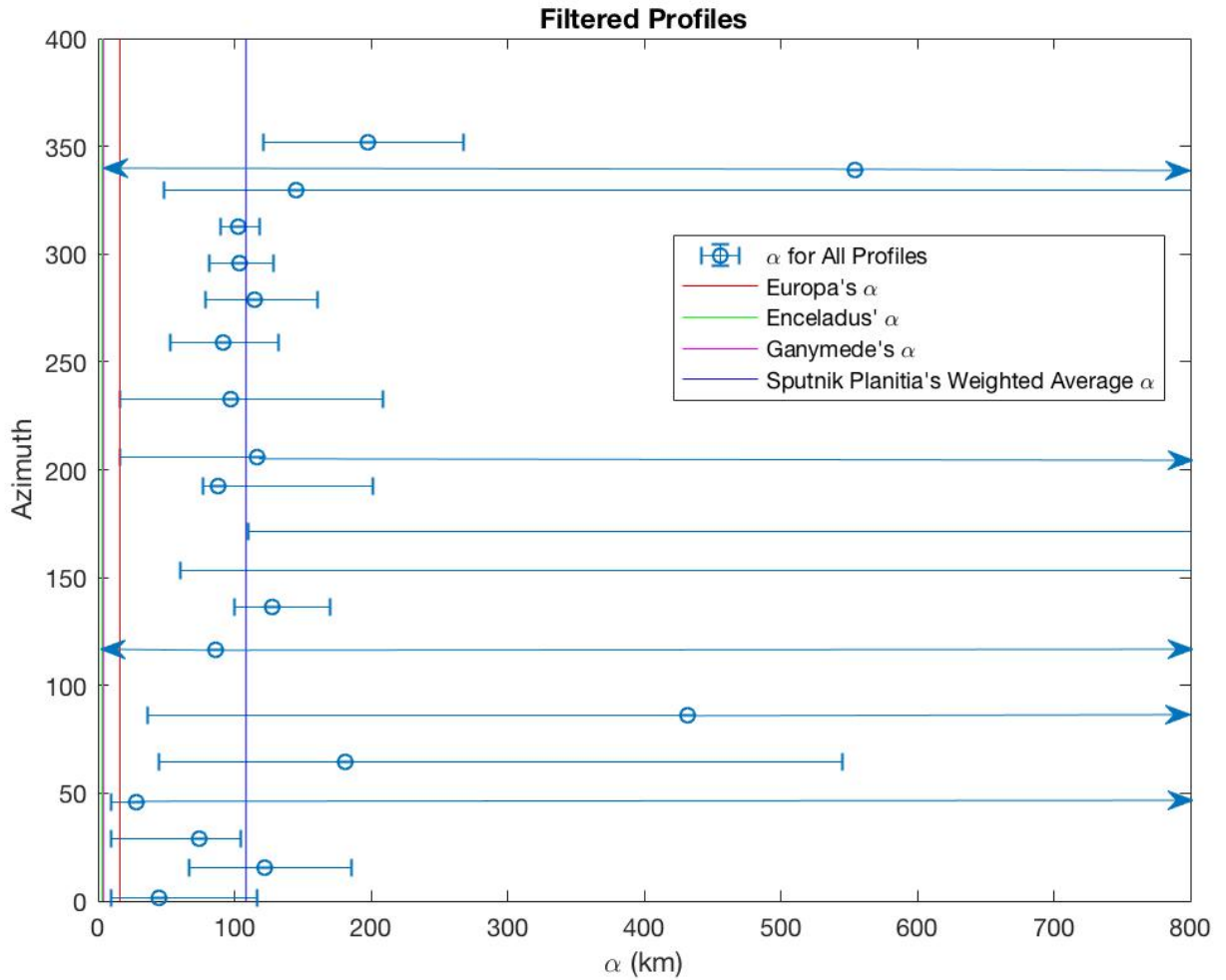


Figure 10. Azimuth versus α plot for profiles with craters and pits, showing that the flexural parameter range for most profiles are between 28 and 554 with two outlier values for Profiles 9 and 10 which are past the bounds of 800 km. The arrows indicate infinite values that are past the bounds of 0 to 800 km. The α values for Europa, Enceladus, and Ganymede are plotted in red, green, and magenta respectively. Sputnik Planitia's weighted average α is in black.

As seen in Figures 9 and 10, the α value for Europa, Enceladus, and Ganymede are plotted with the range of α for both the uncorrected and corrected profiles. These three bodies were chosen to compare to because they are all icy crust with topographic information from which elastic thicknesses have been derived. Also, Europa, Enceladus, and Ganymede also contain an underlying ocean that Pluto may have too. The flexural parameters for these three bodies were calculated using the parameters used to calculate effective elastic thicknesses. For Europa, Nimmo et al. (2003) calculated an effective elastic thickness of 6 kilometers which I used to calculate the flexural parameter to be 16.1 kilometers. For Enceladus, the effective elastic thickness calculated was 0.3 kilometers which I used to find that the flexural parameter was 3.1 kilometers (Giese et al., 2008). Then, Ganymede's effective elastic thickness was 1 kilometer which I then used to calculate the flexural parameter to be 4.0 kilometers (Nimmo et al., 2002). Comparing the flexural parameters to one another, they are all relatively similar in

their value especially Enceladus and Ganymede. When comparing Pluto's weighted average flexural parameter of 108.5 kilometers to these icy bodies, it is much larger. Only a few profile have the minimum acceptable flexural parameters that overlap with Europa's flexural parameter. Pluto's flexural parameter may be expected to be higher than that of Europa, Ganymede, and Enceladus because its surface temperature is the coldest out of the bodies compared. However, it is important to understand whether the difference in surface temperature is sufficient to explain the thick elastic lithosphere inferred for Pluto or if the interior also has to be colder than in the other icy satellites.

To compare the icy moons to Pluto, I estimated the temperature gradient and heat flux compatible with the flexural signal. Heat flux, related to the temperature gradient, indicates how active the planet was at time the elastic flexure took place. It may be expected that the heat flux was highest early on in the planet's history due to more intense radioactive decay in the interior rocky core and energy associated with accretion. However, models of thermal evolution of Pluto, including tidal interaction between Pluto and Charon, put an upper bound of 6 mW/m³ on the expected heat flux on Pluto. The elastic flexure inferred here provides a rare observational constraint on the heat flux.

The temperature gradient was calculated by taking the difference of the temperature of ice and the surface temperature divided by the elastic thickness. The calculation can be found in Appendix VII AA. The temperature gradients were calculated because it can tell us at the time that the flexure took place and if Pluto was as active as Europa, Enceladus, and Ganymede. The temperature gradient for Europa, Enceladus, and Ganymede were 0.0075 K/m, 0.2833 K/m, and 0.03 K/m respectively. Pluto's temperature gradient, using the weighted average flexural parameter's corresponding elastic thickness of 29.9 kilometers, was 0.0039 K/m. Pluto's temperature gradient is the smallest, but it is closest to that of Europa which is a body similar in size to Pluto. This means that Pluto may have been as active as Europa is today, so the flexure must have taken place earlier in Pluto's history near 100 million years after the formation of CAIs (Hammond et al., 2016). This is also at the time in which the tidal evolution of Pluto and Charon is completed (Hammond et al., 2016). The timing of the flexure is consistent with Hamilton et al., 2016 in that Sputnik Planitia formed shortly after Charon did, within a few hundred thousand years.

At the time in which the flexure took place, the interior's heat flux is also of value because it indicates if the interior was very hot or not. To evaluate the heat flux of Pluto, Equation 12 was used:

$$Q = \frac{-567 \ln(T_E/T_S)}{z_E - z_0} \quad \text{Equation 12}$$

where T_E is the temperature constant of ice which is 150 K (Nimmo et al., 2003); T_S is the surface temperature of Pluto which is 33 K (Trowbridge et al., 2016); z_E is the elastic thickness which is 30 km; and z_0 is the initial elastic thickness which is 0.

The heat flux for Pluto, using the elastic thickness from the weighted average flexural parameter is 30 kilometers, is 29 $\frac{\text{mW}}{\text{m}^2}$. This value of heat flux is very large compared to previous estimates of the heat flux of Pluto which on average was 3-4 $\frac{\text{mW}}{\text{m}^2}$ (McKinnon et al., 2016 and Trowbridge et al., 2016). The high heat flux calculated for Pluto means that the interior was very hot early in Pluto's history. This high heat flux is viable due to the presence of extensional faults (Moore et al., 2016). The presence of extensional faults indicates that there was global volume expansion, and this means that no ice II formed. The prevention of ice II formation can come from either keeping Pluto warm enough for the ocean to survive today, or the silicate core is less than 2.9 $\frac{\text{g}}{\text{cm}^3}$

and ice shell is less than 260 kilometers (Hammond et al., 2016). With high heat flux early in Pluto's history may have sustained enough heat for such a scenario with the survival of an ocean.

The source of the early heat can only be speculated. Sources of the heating may be from accretion, radioactive decay, or the large impact that created Charon. The heat from accretion as found by Robuchon and Nimmo, 2011, was only 5.70×10^{27} J compared to radioactive decay which was 1.30×10^{27} J. The radioactive decay energy was released between 30 million years and 4.5 billion years after CAI formation (Robuchon and Nimmo, 2011). Also, the effect of tidal heating should be considered from Charon due to the impact that created it. On Earth, a large object struck Pluto to form Charon (Barr and Collins, 2015). As Charon formed and became gravitational bound to Pluto, the tides may have created significant heat. Tides depend on eccentricity, and a high eccentricity causes significant tidal heating (Hamilton, 2018). One can only speculate about the early eccentricity of Charon, and that it may have been high as it tries to enter into equilibrium with Pluto.

I. Conclusion

If Sputnik Planitia started as a large nitrogen ice cap, and the weight of the load caused a deflection as exhibited by the current basin, then the expected flexure signal would be an elastic flexure signal of a line load. 13 out of the 20 profiles examined showed that the topography around Sputnik Planitia is representative of an elastic flexure signal. These profiles were found on the Western and Eastern edges of Sputnik Planitia. The range of elastic thicknesses for these profiles are 5 to 67 kilometers, excluding the anomalous values. When averaged using the weighted average flexural parameter of 108.5 kilometers, the elastic thickness is 29.9 kilometers. When comparing the flexural parameters and temperature gradient of Enceladus, Europa, and Ganymede to Pluto, Europa had the most similarity in values. This means that Pluto was once as active as Europa is today, and that the flexure took place early in Pluto's history. This conclusion is consistent with the Hamilton et al., 2016 hypothesis.

The elastic thickness for Pluto is thin which means that the interior was warm. After further investigation through the calculation of the heat flux of Pluto, I further confirmed that the interior was very warm early in Pluto's history with a value of $29 \frac{\text{mW}}{\text{m}^2}$. This heat flux is significantly higher than previous estimates of $4 \frac{\text{mW}}{\text{m}^2}$. The heat source for such a high heat flux is of interest and can only be speculated. Some possibilities for early heat sources are accretion, radioactive decay, and the impact that struck Pluto to create Charon. The heat source that could contribute the greatest heat is the impact that struck Pluto if the assumption of Charon's eccentricity early on was high to cause significant tidal heating.

Due to the high heat flux of Pluto early in its history, further research would be of interest to test the significance of the heat generated from tidal heating induced by Charon. Since elastic thickness of Pluto being a reasonable value, further study would be of interest as well. The modeling itself would need to consider more realistic rheologies, such as viscoelastic flow. Also, the model has no time-dependent parameters or direct way to explore possible thermal gradients which would be of interest to study the evolution of Sputnik Planitia over time. Also, it would be useful to compare the results of elastic flexure to that of an impact crater to find if one is a better fit for the topography over the other.

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II. Code Appendix

A. GMT Code for Colored Topography Map with 20 Tracks

```
#####ProfileComposite#####
#!/bin/bash
gmt set PROJ_ELLIPSOID pluto

PS=LambertProjectionPlutoV5.ps
RR=-R130/220/-20/54    #Creates the grid size
#RR=-R130/-20/235/40r  #Creates the grid size
#JJ=-JT175/18/6i
JJ=-JA175/20/6i
GG=PlutoDEM.grd
CC=temp.cpt

PP1=prof1oc.xyz
PP2=prof2oc.xyz
PP3=prof3oc.xyz
PP4=prof4oc.xyz
PP5=prof5oc.xyz
PP6=prof6oc.xyz
PP7=prof7oc.xyz
PP8=prof8oc.xyz
PP9=prof9oc.xyz
PP10=prof10oc.xyz
PP11=prof11oc.xyz
PP12=prof12oc.xyz
PP13=prof13oc.xyz
PP14=prof14oc.xyz
PP15=prof15oc.xyz
PP16=prof16oc.xyz
PP17=prof17oc.xyz
PP18=prof18oc.xyz
PP19=prof19oc.xyz
PP20=prof20oc.xyz

grdtrack -E176/48+d+a+i1k+l600k -G$GG > $PP1
grdtrack -E184/46+d+a42+i1k+l700k -G$GG > $PP2
grdtrack -E190/41+d+a57+i1k+l600k -G$GG > $PP3
grdtrack -E194/35+d+a69+i1k+l600k -G$GG > $PP4
grdtrack -E196/28+d+a88+i1k+l600k -G$GG > $PP5
grdtrack -E196/20+d+a95+i1k+l600k -G$GG > $PP6
grdtrack -E194/9+d+a103+i1k+l600k -G$GG > $PP7
grdtrack -E191/1+d+a112+i1k+l600k -G$GG > $PP8
grdtrack -E188/-8+d+a152+i1k+l600k -G$GG > $PP9
grdtrack -E181/-10+d+a180+i1k+l600k -G$GG > $PP10
grdtrack -E172/-6+d+a198+i1k+l800k -G$GG > $PP11
```



```

grdtrack -E165/-2+d+a216+i1k+l600k -G$GG > $PP12
grdtrack -E159/7+d+a235+i1k+l600k -G$GG > $PP13
grdtrack -E154/15+d+a245+i1k+l600k -G$GG > $PP14
grdtrack -E152/22+d+a255+i1k+l600k -G$GG > $PP15
grdtrack -E150/29+d+a273+i1k+l600k -G$GG > $PP16
grdtrack -E152/36+d+a280+i1k+l600k -G$GG > $PP17
grdtrack -E155/44+d+a305+i1k+l600k -G$GG > $PP18
grdtrack -E160/47+d+a335+i1k+l600k -G$GG > $PP19
grdtrack -E169/48+d+a349+i1k+l700k -G$GG > $PP20

```

```

grdimage $RR $JJ $GG -P -K -I+ -C$CC -Y3i > $PS
psxy $PP1 $JJ $RR -K -O -W2p >> $PS
psxy $PP2 $JJ $RR -K -O -W2p,red >> $PS
psxy $PP3 $JJ $RR -K -O -W2p >> $PS
psxy $PP4 $JJ $RR -K -O -W2p >> $PS
psxy $PP5 $JJ $RR -K -O -W2p >> $PS
psxy $PP6 $JJ $RR -K -O -W2p >> $PS
psxy $PP7 $JJ $RR -K -O -W2p >> $PS
psxy $PP8 $JJ $RR -K -O -W2p >> $PS
psxy $PP9 $JJ $RR -K -O -W2p,purple >> $PS
psxy $PP10 $JJ $RR -K -O -W2p >> $PS
psxy $PP11 $JJ $RR -K -O -W2p >> $PS
psxy $PP12 $JJ $RR -K -O -W2p >> $PS
psxy $PP13 $JJ $RR -K -O -W2p >> $PS
psxy $PP14 $JJ $RR -K -O -W2p >> $PS
psxy $PP15 $JJ $RR -K -O -W2p >> $PS
psxy $PP16 $JJ $RR -K -O -W2p >> $PS
psxy $PP17 $JJ $RR -K -O -W2p >> $PS
psxy $PP18 $JJ $RR -K -O -W2p >> $PS
psxy $PP19 $JJ $RR -K -O -W2p >> $PS
psxy $PP20 $JJ $RR -K -O -W2p >> $PS

```

```

psscale -C$CC -Dx0i/-0.51i+w3i/0.15i+e+h -Bxa2000f1000g1000+l"Topography [m]" -I -O -K
>>$PS

```

```

psbasemap $RR $JJ -Ba10g30 -Lx5i/0i+c20+w1000k+f+l+ab+o0/-0.42i -O >> $PS

```

B. Matlab Function to Test Multiple Flexural Parameters and Find Minimum χ^2 (written by Laurent Montesi and Vedran Leckic)

```

function [alpha_est,q_est,misfit_min,misfit,Imin]=InvertElastic(x,w,alpha,xq,V,o);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% [alpha_est,q_est,misfit_min]=InvertElastic(x,w,alpha,xq,V);
% input
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% x : x-coordinate (sampling vector)

```

```

% w : topography (observation)
% alpha : SET of values of alpha to consider
% xq: where the load might exist
% V : mass variance
% o: topography variance (m)
% output
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% alpha_est : estimated value of alpha
% q_est : estimated load vector
% misfit: misfit
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Testing multiple alphas to find misfits; no graphical output
for iA=1:numel(alpha);
    [q_est,misfit(iA)]=lineloadinvert(x,w,alpha(iA),xq,V,o,0);
end

%% Finding the best alpha through inspection of lowest misfit
% Quantative way of finding lowest misfit
[Mmin,Imin]=min(misfit);
alpha_est=alpha(Imin);
disp(sprintf('Minimum misfit of %g obtain for alpha = %g',Mmin,alpha_est));

%% Plot alpha vs misfit to find the best alpha
figure(2); clf; hold on;
plot(alpha,misfit,'k.-')
plot(alpha_est,Mmin,'ob')
box on;
xlabel('\alpha (km)','fontsize',12)
ylabel('\chi^2','fontsize',12)
title('\alpha vs \chi^2','fontsize',12)
set(gca,'FontSize',12)

% Illustrate the best fit.
[q_est,misfit_min]=lineloadinvert(x,w,alpha_est,xq,V,o,1);

```

C. Matlab Function to Calculate Line Load Deflection and χ^2 /Misfit (Written by Laurent Montesi and Vedran Lekic)

```

function [q_est,misfit]=lineloadinvert(x,w,alpha,xq,V,o,flagplot);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% [q_est,L2norm]=lineloadinvert(x,w,alpha,xq,V,flagplot);

```

```

% input
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% x : x-coordinate (sampling vector)
% w : topography (observation)
% a : flexural wavelength
% xq: where the load might exist
% V : mass variance
% o: topography variance
% flagplot: set to non-zero if want graphical output
% output
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% q_est : estimated load vector
% misfit: misfit
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% General flexure solution
wg=@(x,x0)(sin((x-x0)./alpha)+cos((x-x0)./alpha)).*exp(-(x-x0)./alpha);
% x : x-coordinate
% x0 : position of line load
% w0 : amplitude of deflection
% a : alpha: flexural wavelength

% Create a matrix M so that the deformation "observed" at the "sampling"
% locations x would be given by M * q;
M=NaN(numel(x),numel(xq));
% clear M;
for iq=1:numel(xq) %loop over all the loads
    M(:,iq)=wg(x,xq(iq)); % Create matrix M
end

% Introduce some prior information
Cm = (V^2)*eye(numel(xq)); % Here, variance on mass is 10

% Now, let's INVERT the problem so that we estimate the mass distribution
% q_est that best fits the "observed" deflections w.
q_est = (transpose(M)*M + Cm)\(transpose(M)*w);

%Calculating predicted deflection
wp=M*q_est;
%Calculating residuals
res=w-wp';
%Calculating Least Squares
misfit= sum(res.^2)/o^2;

```

```

% Graphical output
if flagplot~=0
    figure(1); clf;
    plot(x,w,'linewidth',2);
    plot(x,M*q_est,'r--','linewidth',2);
    set(gca,'box','on','fontsize',12);
    xlabel('Distance (km)','fontsize',18);
    ylabel('Topography (m)','fontsize',18);
    xlim([min(x),max(x)]);
    plot([0,0],get(gca,'ylim'),'k');
    plot(get(gca,'xlim'),[0,0],'k');
end

```

D. Function to Find Elastic Thickness

```

function he=alpha2thickness(alpha);
%% Best alpha: 390
pm=1030; % density of underlying layer in kg/m^3..water ice
% pc=920; %density of shell in kg/m^3...N2 ice
pc=0; %No overlying material;
g=0.62; %gravity of pluto in m/s^2

E=9e9; %Young's modulus in Pa
v=0.3; %Poisson's ratio of water ice.
% Assumes alpha in km; so *1000 to express in m
% Flexural rigidity
D=((alpha*1000)^4)*(pm-pc)*g/4;
he=((D*12*(1-v^2)/E)^(1/3))/1000;

disp(sprintf('Elastic thickness = %g',he))

```

E. Profile 2 Uncorrected Code

```

load prof2oc.xyz
indexmin2=find(min(prof2oc(:,4)) == prof2oc(:,4));
xp2 = transpose(prof2oc(indexmin2:end,3));
t2 = transpose(prof2oc(indexmin2:end,4));
z=find(round(xp2) == 400);
z1=find((xp2(end)) == xp2);
sd=std(t2(z:z1));
averagedtopo2=mean(t2(z:z1));
newprof2=t2-averagedtopo2;

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-500:50:0]; %Load vector

```

```

[A2,Q,M,misfit2,Imin]=InvertElastic(xp2,newprof2,Atest,xq,0.01,sd);
H2=alpha2thickness(A2);
% Finding delta chi-square
nu=numel(xp2)-(numel(A2)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683 %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);

%% Plotting acceptable alpha values and misfit threshold
%Calculations of alpha range and misfit threshold
%deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit2(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit2(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A2; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');

%% Calculating elastic thickness for acceptable alpha range
Arange=[Amin:Amax];%Acceptable alpha range
H2=alpha2thickness(A2); %Optimal elastic thickness at misfit minimum
H2min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H2max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

F. Profile 2 Corrected Code

```

load prof2oc.xyz
indexmin2=find(min(prof2oc(:,4)) == prof2oc(:,4));
xp2 = transpose(prof2oc(indexmin2:end,3));
t2 = transpose(prof2oc(indexmin2:end,4));
tn2=[t2(1:240-indexmin2),t2(255-indexmin2:275-indexmin2),t2(302-indexmin2:333-
indexmin2),t2(375-indexmin2:486-indexmin2),t2(541-indexmin2:end)];

```

```

xpn2=[xp2(1:240-indexmin2),xp2(255-indexmin2:275-indexmin2),xp2(302-indexmin2:333-
indexmin2),xp2(375-indexmin2:486-indexmin2),xp2(541-indexmin2:end)];
z=find((round(xpn2) == 400));
z1=find((xpn2(end) == xpn2));
sd=std(tn2(z:z1));
averagedtopo2=mean(tn2(z:z1));
newprof2=tn2-averagedtopo2;

% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-500:50:0]; %Load vector

[A2h,Q,M,misfit2h,Imin]=InvertElastic(xpn2,newprof2,Atest,xq,0.01,sd);

%% Finding delta chi-square
nu=numel(xpn2)-(numel(A2h)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683;
deltachisquare=gammaincinv(P,nu/2);

%% Plotting acceptable alpha values and misfit threshold
%Calculations of alpha range and misfit threshold
%deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit2h(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit2h(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A2h; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r-');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',18)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;
return
print(1,'-dpdf','Fit test 1');
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H2h=alpha2thickness(A2h); %Optimal elastic thickness at misfit minimum
H2hmin=alpha2thickness(Amin); %Minimum acceptable elastic thickness

```

```
H2hmax=alpha2thickness(Amax); %Maximum acceptable elastic thickness
```

G. Profile 9 Uncorrected Code

```
load prof9oc.xyz
indexmin9=183;
xp9 = transpose(prof9oc(indexmin9:550,3));
t9 = transpose(prof9oc(indexmin9:550,4));
z=find(round(xp9) == 400);
z1=find(xp9(end) == xp9);
sd=std(prof9oc(z:z1,4));
averagedtopo9=mean(prof9oc(z:z1,4));
newprof9=t9-averagedtopo9; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:10:2000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A9,Q,M,misfit9,Imin]=InvertElastic(xp9,newprof9,Atest,xq,0.01,sd);
H9=alpha2thickness(A9);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp9)-(numel(A9)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit9(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit9(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A9; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)),'m--')
plot([Amin,Amin],[0,1]*Mmax,'m--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
```

```
figure(1); orient tall;
```

```
return
```

```
print(1,'-dpdf','Fit test 1');
```

```
%% Finding Range of Acceptable Elastic Thicknesses
```

```
Arange=[Amin:Amax];%Acceptable alpha range
```

```
H9=alpha2thickness(A9); %Optimal elastic thickness at misfit minimum
```

```
H9min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
```

```
H9max=alpha2thickness(Amax); %Maximum acceptable elastic thickness
```

H. Profile 1

a. Uncorrected Code

```
load profloc.xyz
```

```
indexmin1=find(min(profloc(:,4)) == profloc(:,4)); %Finding the edge of Sputnik Planitia
```

```
xp1 = transpose(profloc(indexmin1:end,3)); % Starting the profile at the edge of Sputnik Planitia
```

```
t1 = transpose(profloc(indexmin1:end,4)); %Starting the topography data at the edge of profile just defined
```

```
z=find(round(xp1) == 400);
```

```
z1=find((xp1(end)) == xp1);
```

```
sd=std(t1(z:z1));
```

```
averagedtopo1=mean(t1(z:z1));
```

```
newprof1=t1-averagedtopo1; %Reconstruction of topography without uncertainty
```

```
%% Variables
```

```
Atest=[10:1:1000]; %Alpha values to be tested
```

```
xq=[-300:10:0]; %Load vector
```

```
%% Calculation
```

```
[A1,Q,M,misfit1,Imin,wp]=InvertElastic(xp1,newprof1,Atest,xq,0.01,sd);
```

```
H1=alpha2thickness(A1);
```

```
%% Plotting acceptable alpha values and misfit threshold
```

```
% Finding delta chi-square
```

```
nu=numel(xp1)-(numel(A1)+numel(Q)); %Degrees of freedom: Number of data points - number of parameters being inverted
```

```
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of delta chi square
```

```
deltachisquare=gammaincinv(P,nu/2);
```

```
%Calculations of alpha range and misfit threshold
```

```
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
```

```
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite values)
```

```
Amin=Atest(max(find(misfit1(1:Imin)>Mmax))); %Finding minimum alpha value within the threshold
```

```
Amax=Atest(min(find(misfit1(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value within the threshold
```



```

% Creating the plots
LE=M; %elastic wavelength
AE=A1; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H1=alpha2thickness(A1); %Optimal elastic thickness at misfit minimum
H1min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H1max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

b. Corrected Code

```

load profloc.xyz
%% Creating Intial Profiles
indexmin1=find(min(profloc(:,4)) == profloc(:,4)); %Finding the edge of Sputnik Planitia
xp1 = transpose(profloc(indexmin1:end,3)); % Starting the profile at the edge of Sputnik
Planitia
t1 = transpose(profloc(indexmin1:end,4)); %Starting the topography data at the edge of profile
just defined
%% Reconstructing Profiles without Craters or Pits
tn1=[t1(1:335-indexmin1),t1(467-indexmin1:end)]; %Pit located between 357-467
xpn1=[xp1(1:335-indexmin1),xp1(467-indexmin1:end)];
z=find((round(xpn1)) == 467);
z1=find((xpn1(end)) == xpn1);
sd=std(tn1(z:z1));
averagedtopo1=mean(tn1(z:z1));
newprof1=tn1-averagedtopo1;%Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-300:10:0]; %Load vector
%% Calculation
[A1,Q,M,misfit1,Imin]=InvertElastic(xpn1,newprof1,Atest,xq,0.01,sd);
H1=alpha2thickness(A1);
%% Plotting acceptable alpha values and misfit threshold

```

```

% Finding delta chi-square
nu=numel(xpn1)-(numel(A1)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit1(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amin=Atest(max(find(misfit1(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit1(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A1; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H1=alpha2thickness(A1); %Optimal elastic thickness at misfit minimum
H1min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H1max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

I. Profile 3

a. Uncorrected Code

```

load prof3oc.xyz
indexmin3=24;
xp3 = transpose(prof3oc(indexmin3:end,3));
t3 = transpose(prof3oc(indexmin3:end,4));
z=find((round(xp3) == 400));
z1=find((xp3(end) == xp3));
sd=std(t3(z:z1));

```

```

averagedtopo3=mean(t3(z:z1));
newprof3=t3-averagedtopo3; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-300:10:0]; %Load vector
%% Calculation
[A3,Q,M,misfit3,Imin]=InvertElastic(xp3,newprof3,Atest,xq,0.01,sd);
H3=alpha2thickness(A3);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp3)-(numel(A3)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit3(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
%Amin=Atest(max(find(M))); %There is no misfit values before the minimum, so this is the
alternative Amin
Amax=Atest(min(find(misfit3(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A3; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',18)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax]; %Acceptable alpha range
H3=alpha2thickness(A3); %Optimal elastic thickness at misfit minimum
H3min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
%H3max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

b. Corrected Code

```
load prof3oc.xyz
indexmin3=24;
xp3 = transpose(prof3oc(indexmin3:end,3));
t3 = transpose(prof3oc(indexmin3:end,4));
%Craters at 61 to 224, 411 to 539, 589
tn3=[t3(1:84-indexmin3),t3(210-indexmin3:411-indexmin3), t3(539-indexmin3:589-
indexmin3)];
xpn3=[xp3(1:84-indexmin3),xp3(210-indexmin3:411-indexmin3), xp3(539-indexmin3:589-
indexmin3)];
z=find((round(xpn3) == 400));
z1=find((xpn3(end) == xpn3));
sd=std(prof3oc(z:z1,4));
averagedtopo3=mean(prof3oc(z:z1,4));
newprof3=tn3-averagedtopo3; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-300:10:0]; %Load vector
%% Calculation
[A3,Q,M,misfit3,Imin]=InvertElastic(xpn3,newprof3,Atest,xq,0.01,sd);
H3=alpha2thickness(A3);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp3)-(numel(A3)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit3(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amin=Atest(max(find(M))); %There is no misfit values before the minimum, so this is the
alternative Amin
Amax=Atest(min(find(misfit3(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A3; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
```

```

set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H3=alpha2thickness(A3); %Optimal elastic thickness at misfit minimum
H3min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H3max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

J. Profile 4

a. Uncorrected Code

```

load prof4oc.xyz
indexmin4=54;%=find(min(prof4u(:,4)) == prof4u(:,4));
xp4 = transpose(prof4oc(indexmin4:end,3));
t4 = transpose(prof4oc(indexmin4:end,4));
z=find(round(xp4) == 400);
z1=find((xp4(end)) == xp4);
sd=std(prof4oc(z:z1,4));
averagedtopo4=mean(prof4oc(z:z1,4));
newprof4=t4-averagedtopo4; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-300:10:0]; %Load vector
%% Calculation
[A4,Q,M,misfit4,Imin]=InvertElastic(xp4,newprof4,Atest,xq,0.01,sd);
H4=alpha2thickness(A4);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp4)-(numel(A4)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)

```

```

Amin=Atest(max(find(misfit4(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit4(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A4; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H4=alpha2thickness(A4); %Optimal elastic thickness at misfit minimum
H4min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H4max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

b. Corrected Code

```

load prof4oc.xyz
indexmin4=54;%=find(min(prof4u(:,4)) == prof4u(:,4));
xp4 = transpose(prof4oc(indexmin4:end,3));
t4 = transpose(prof4oc(indexmin4:end,4));
xpn4=[xp4(1:223-indexmin4)];
tn4=[t4(1:223-indexmin4)];
z=find((round(xpn4)) == 100);
z1=find((xpn4(end)) == xpn4);
sd=std(tn4(z:z1));
averagedtopo4=mean(tn4(z:z1));
newprof4=tn4-averagedtopo4; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-300:10:0]; %Load vector
%% Calculation
[A4,Q,M,misfit4,Imin]=InvertElastic(xpn4,newprof4,Atest,xq,0.01,sd);
H4=alpha2thickness(A4);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square

```

```

nu=numel(xp4)-(numel(A4)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
%Amin=Atest(max(find(misfit4(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amin=Atest(max(find(M))); %There is no misfit values before the minimum, so this is the
alternative Amin
Amax=Atest(min(find(misfit4(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A4; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)),'r--')
plot([Amin,Amin],[0,1]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H4=alpha2thickness(A4); %Optimal elastic thickness at misfit minimum
H4min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H4max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

K. Profile 5

a. Uncorrected Code

```

load prof5oc.xyz
indexmin5=find(min(prof5oc(:,4)) == prof5oc(:,4));
xp5 = transpose(prof5oc(indexmin5:end,3));
t5 = transpose(prof5oc(indexmin5:end,4));
z=find((round(xp5) == 400));

```

```

z1=find(xp5(end) == xp5);
sd=std(t5(z:z1));
averagedtopo5=mean(t5(z:z1));
newprof5=t5-averagedtopo5; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-600:20:0]; %Load vector
%% Calculation
[A5,Q,M,misfit5,Imin]=InvertElastic(xp5,newprof5,Atest,xq,0.01,sd);
H5=alpha2thickness(A5);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp5)-(numel(A5)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit5(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit5(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A5; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)), 'r--')
plot([Amin,Amin],[0,1]*Mmax, 'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range

```



```
H5=alpha2thickness(A5); %Optimal elastic thickness at misfit minimum
H5min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H5max=alpha2thickness(Amax); %Maximum acceptable elastic thickness
```

b. Corrected Code

```
load prof5oc.xyz
indexmin5=find(min(prof5oc(:,4)) == prof5oc(:,4));
xp5 = transpose(prof5oc(indexmin5:end,3));
t5 = transpose(prof5oc(indexmin5:end,4));
xpn5=[xp5(1:225-indexmin5),xp5(245-indexmin5:254-indexmin5),xp5(278-indexmin5:377-
indexmin5),xp5(435-indexmin5:454-indexmin5),xp5(470-indexmin5:end)];
tn5=[t5(1:225-indexmin5),t5(245-indexmin5:254-indexmin5),t5(278-indexmin5:377-
indexmin5),t5(435-indexmin5:454-indexmin5),t5(470-indexmin5:end)];
z=find((round(xpn5)) == 435);
z1=find((xpn5(end)) == xpn5);
sd=std(tn5(z:z1));
averagedtopo5=mean(tn5(z:z1));
newprof5=tn5-averagedtopo5; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A5,Q,M,misfit5,Imin]=InvertElastic(xpn5,newprof5,Atest,xq,0.01,sd);
H5=alpha2thickness(A5);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp5)-(numel(A5)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit5(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit5(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A5; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
```

```

set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H5=alpha2thickness(A5); %Optimal elastic thickness at misfit minimum
H5min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H5max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

L. Profile 6

a. Uncorrected Code

```

load prof6oc.xyz
indexmin6=8;
xp6 = transpose(prof6oc(indexmin6:end,3));
t6 = transpose(prof6oc(indexmin6:end,4));
z=find((round(xp6)) == 400);
z1=find((xp6(end)) == xp6);
sd=std(t6(z:z1));
averagedtopo6=mean(t6(z:z1));
newprof6=t6-averagedtopo6;

% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector

[A6,Q,M,misfit6,lmin]=InvertElastic(xp6,newprof6,Atest,xq,0.01,sd);
H6=alpha2thickness(A6);
%% Finding delta chi-square
nu=numel(xp6)-(numel(A6)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683;
deltachisquare=gammaincinv(P,nu/2);

%% Plotting acceptable alpha values and misfit threshold
%Calculations of alpha range and misfit threshold
%deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)

```

```
Amin=Atest(max(find(misfit6(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit6(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
```

b. Corrected Code

```
load prof6oc.xyz
indexmin6=8;
xp6 = transpose(prof6oc(indexmin6:end,3));
t6 = transpose(prof6oc(indexmin6:end,4));
xpn6=[xp6(1:261-indexmin6),xp6(300-indexmin6:450-indexmin6),xp6(499-indexmin6:527-
indexmin6),xp6(572-indexmin6:end)];
tn6=[t6(1:261-indexmin6),t6(300-indexmin6:450-indexmin6),t6(499-indexmin6:527-
indexmin6),t6(572-indexmin6:end)];
z=find((round(xpn6)) == 400);
z1=find((xpn6(end)) == xpn6);
sd=std(tn6(z:z1));
averagedtopo6=mean(tn6(z:z1));
newprof6=tn6-averagedtopo6;

% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector

[A6,Q,M,misfit6,Imin]=InvertElastic(xpn6,newprof6,Atest,xq,0.01,sd);

%% Finding delta chi-square
nu=numel(xpn6)-(numel(A6)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683;
deltachisquare=gammaincinv(P,nu/2);
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit6(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit6(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A6; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)),'r--')
plot([Amin,Amin],[0,1]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
```

```

set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H6=alpha2thickness(A6); %Optimal elastic thickness at misfit minimum
H6min=alpha2thickness(Amin); %Minimum acceptable elastic thickness

%% Plotting acceptable alpha values and misfit threshold
%Calculations of alpha range and misfit threshold
%deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit6(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit6(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A6; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)), 'r--')
plot([Amin,Amin],[0,1]*Mmax, 'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H6=alpha2thickness(A6); %Optimal elastic thickness at misfit minimum
H6min=alpha2thickness(Amin); %Minimum acceptable elastic thickness

```

M. Profile 7

a. Uncorrected Code

```
load prof7oc.xyz
indexmin7=46;
xp7 = transpose(prof7oc(indexmin7:end,3));
t7 = transpose(prof7oc(indexmin7:end,4));
z=find((round(xp7)) == 400);
z1=find(xp7(end) == xp7);
sd=std(t7(z:z1));
averagedtopo7=mean(t7(z:z1));
newprof7=t7-averagedtopo7; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A7,Q,M,misfit7,Imin]=InvertElastic(xp7,newprof7,Atest,xq,0.01,sd);
H7=alpha2thickness(A7);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp7)-(numel(A7)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit7(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit7(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A7; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)), 'r--')
plot([Amin,Amin],[0,1]*Mmax, 'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
```

```

print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amin];%Acceptable alpha range
H7=alpha2thickness(A7); %Optimal elastic thickness at misfit minimum
H7min=alpha2thickness(Amin); %Minimum acceptable elastic thickness

```

b. Corrected Code

```

load prof7oc.xyz
indexmin7=46;
xp7 = transpose(prof7oc(indexmin7:end,3));
t7 = transpose(prof7oc(indexmin7:end,4));
xpn7=[xp7(1:435-indexmin7),xp7(537-indexmin7:end)];
tn7=[t7(1:435-indexmin7),t7(537-indexmin7:end)];
z=find((round(xpn7)) == 400);
z1=find((xpn7(end)) == xpn7);
sd=std(tn7(z:z1));
averagedtopo7=mean(tn7(z:z1));
newprof7=tn7-averagedtopo7; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A7,Q,M,misfit7,Imin]=InvertElastic(xpn7,newprof7,Atest,xq,0.01,sd);
H7=alpha2thickness(A7);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp7)-(numel(A7)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit7(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit7(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots

```

```

LE=M; %elastic wavelength
AE=A7; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)), 'r--')
plot([Amin,Amin],[0,1]*Mmax, 'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',18)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H7=alpha2thickness(A7); %Optimal elastic thickness at misfit minimum
H7min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H7max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

N. Profile 8

a. Uncorrected Code

```

load prof8oc.xyz
indexmin8=45;
xp8 = transpose(prof8oc(indexmin8:end,3));
t8 = transpose(prof8oc(indexmin8:end,4));
z=find(round(xp8) == 400);
z1=find(xp8(end) == xp8);
sd=std(t8(z:z1));
averagedtopo8=mean(t8(z:z1));
newprof8=t8-averagedtopo8; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-400:20:0]; %Load vector
%% Calculation
[A8,Q,M,misfit8,Imin]=InvertElastic(xp8,newprof8,Atest,xq,0.01,sd);
H8=alpha2thickness(A8);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp8)-(numel(A8)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted

```

```

P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit8(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit8(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A8; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H7=alpha2thickness(A7); %Optimal elastic thickness at misfit minimum
H7min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H7max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

b. Corrected Code

```

load prof8oc.xyz
indexmin8=45;
xp8 = transpose(prof8oc(indexmin8:end,3));
t8 = transpose(prof8oc(indexmin8:end,4));
xpn8=[xp8(1:411-indexmin8)];
tn8=[t8(1:411-indexmin8)];
z=find((round(xpn8)) == 400);
z1=find((xpn8(end)) == xpn8);
sd=std(tn8(z:z1));
averagedtopo8=mean(tn8(z:z1));
newprof8=tn8-averagedtopo8; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested

```



```

xq=[-400:20:0]; %Load vector
%% Calculation
[A8,Q,M,misfit8,Imin]=InvertElastic(xpn8,newprof8,Atest,xq,0.01,sd);
H8=alpha2thickness(A8);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp8)-(numel(A8)+numel(Q)); %Degrees of freedom: Number of data points - number
of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit8(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit8(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A8; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H8=alpha2thickness(A8); %Optimal elastic thickness at misfit minimum
H8min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H8max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

O. Profile 10

a. Uncorrected Code

```

load prof10oc.xyz
indexmin10=find(min(prof10oc(:,4)) == prof10oc(:,4));
xp10 = transpose(prof10oc(indexmin10:end,3));
t10 = transpose(prof10oc(indexmin10:end,4));
z=find(round(xp10) == 400);

```

```

z1=find(xp10(end) == xp10);
sd=std(t10(z:z1));
averagedtopo10=mean(t10(z:z1));
newprof10=t10-averagedtopo10; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:100:10000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A10,Q,M,misfit10,Imin]=InvertElastic(xp10,newprof10,Atest,xq,0.01,sd);
H10=alpha2thickness(A10);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp10)-(numel(A10)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit10(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit10(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A10; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)),'r--')
plot([Amin,Amin],[0,1]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H10=alpha2thickness(A10); %Optimal elastic thickness at misfit minimum

```

```
H10min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H10max=alpha2thickness(Amax); %Maximum acceptable elastic thickness
```

P. Profile 11

a. Uncorrected Code

```
load prof11oc.xyz
indexmin11=409;
xp11 = transpose(prof11oc(indexmin11:end,3));
t11 = transpose(prof11oc(indexmin11:end,4));
z=find(round(xp11) == 500); %Need to take a longer profile
z1=find(xp11(end) == xp11);
sd=std(t11(z:z1));
averagedtopo11=mean(t11(z:z1));
newprof11=t11-averagedtopo11; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-600:20:0]; %Load vector
%% Calculation
[A11,Q,M,misfit11,Imin]=InvertElastic(xp11,newprof11,Atest,xq,0.01,sd);
H11=alpha2thickness(A11);
% Finding delta chi-square
nu=numel(xp11)-(numel(A11)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683;
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
%deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit11(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit11(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A11; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)),'r--')
plot([Amin,Amin],[0,1]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
```

```

print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H11=alpha2thickness(A11); %Optimal elastic thickness at misfit minimum
H11min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H11max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

b. Corrected Code

```

load prof1loc.xyz
indexmin11=409;
xp11 = transpose(prof1loc(indexmin11:end,3));
t11 = transpose(prof1loc(indexmin11:end,4));
xpn11=[xp11(1:507-indexmin11),xp11(578-indexmin11:end)];
tn11=[t11(1:507-indexmin11),t11(578-indexmin11:end)];
z=find(round(xpn11) == 578); %Need to take a longer profile
z1=find(xpn11(end) == xpn11);
sd=std(tn11(z:z1));
averagedtopo11=mean(tn11(z:z1));
newprof11=tn11-averagedtopo11; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-600:20:0]; %Load vector
%% Calculation
[A11,Q,M,misfit11,Imin]=InvertElastic(xpn11,newprof11,Atest,xq,0.01,sd);
H11=alpha2thickness(A11);
% Finding delta chi-square
nu=numel(xp11)-(numel(A11)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683;
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
%deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit11(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit11(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A11; % convert to flexural parameter alpha

```

```

figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H11=alpha2thickness(A11); %Optimal elastic thickness at misfit minimum
H11min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H11max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

Q. Profile 12

a. Uncorrected Code

```

load prof12oc.xyz
indexmin12=21;
xp12 = transpose(prof12oc(indexmin12:end,3));
t12 = transpose(prof12oc(indexmin12:end,4));
z=find(round(xp12) == 400);
z1=find((xp12(end)) == xp12);
sd=std(t12(z:z1));
averagedtopo12=mean(t12(z:z1));
newprof12=t12-averagedtopo12; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A12,Q,M,misfit12,Imin]=InvertElastic(xp12,newprof12,Atest,xq,0.01,sd);
H12=alpha2thickness(A12);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp12)-(numel(A12)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)

```

```

Amin=Atest(max(find(misfit12(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit12(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A12; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)), 'r--')
plot([Amin,Amin],[0,1]*Mmax, 'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');

```

```

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax]; %Acceptable alpha range
H12=alpha2thickness(A12); %Optimal elastic thickness at misfit minimum
H12min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H12max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

b. Corrected Code

```

load prof12oc.xyz
indexmin12=21;
xp12 = transpose(prof12oc(indexmin12:end,3));
t12 = transpose(prof12oc(indexmin12:end,4));
xpn12=[xp12(1:184-indexmin12),xp12(206-indexmin12:239-indexmin12),xp12(285-
indexmin12:end)];
tn12=[t12(1:184-indexmin12),t12(206-indexmin12:239-indexmin12),t12(285-indexmin12:end)];
z=find(round(xpn12) == 400);
z1=find((xpn12(end)) == xpn12);
sd=std(tn12(z:z1));
averagedtopo12=mean(tn12(z:z1));
newprof12=tn12-averagedtopo12; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A12,Q,M,misfit12,Imin]=InvertElastic(xpn12,newprof12,Atest,xq,0.01,sd);

```

```

H12=alpha2thickness(A12);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp12)-(numel(A12)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit12(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit12(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A12; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)),'r--')
plot([Amin,Amin],[0,1]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H12=alpha2thickness(A12); %Optimal elastic thickness at misfit minimum
H12min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H12max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

R. Profile 13

a. Uncorrected Code

```

load prof13oc.xyz
indexmin13=32;
xp13 = transpose(prof13oc(indexmin13:end,3));
t13 = transpose(prof13oc(indexmin13:end,4));
z=find(round(xp13) == 400);

```

```

z1=find((xp13(end)) == xp13);
sd=std(t13(z:z1));
averagedtopo13=mean(t13(z:z1));
newprof13=t13-averagedtopo13; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A13,Q,M,misfit13,Imin]=InvertElastic(xp13,newprof13,Atest,xq,0.01,sd);
H13=alpha2thickness(A13);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp13)-(numel(A13)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit13(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit13(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A13; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H13=alpha2thickness(A13); %Optimal elastic thickness at misfit minimum
H13min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H13max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```


b. Corrected Code

```
load prof13oc.xyz
indexmin13=32;
xp13 = transpose(prof13oc(indexmin13:end,3));
t13 = transpose(prof13oc(indexmin13:end,4));
xpn13=[xp13(1:110-indexmin13),xp13(203-indexmin13:484-indexmin13),xp13(522-
indexmin13:end)];
tn13=[t13(1:110-indexmin13),t13(203-indexmin13:484-indexmin13),t13(522-indexmin13:end)];
z=find(round(xpn13) == 400);
z1=find((xpn13(end)) == xpn13);
sd=std(tn13(z:z1));
averagedtopo13=mean(tn13(z:z1));
newprof13=tn13-averagedtopo13; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A13,Q,M,misfit13,Imin]=InvertElastic(xpn13,newprof13,Atest,xq,0.01,sd);
H13=alpha2thickness(A13);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xpn13)-(numel(A13)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit13(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit13(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A13; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;
```

```

return
print(1,'-dpdf','Fit test 1');

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H13=alpha2thickness(A13); %Optimal elastic thickness at misfit minimum
H13min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H13max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

S. Profile 14

a. Uncorrected Code

```

load prof14oc.xyz
indexmin14=find(min(prof14oc(:,4)) == prof14oc(:,4));
xp14 = transpose(prof14oc(indexmin14:end,3));
t14 = transpose(prof14oc(indexmin14:end,4));
z=find(round(xp14) == 400);
z1=find((xp14(end)) == xp14);
sd=std(t14(z:z1));
averagedtopo14=mean(t14(z:z1));
newprof14=t14-averagedtopo14; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-400:10:0]; %Load vector
%% Calculation
[A14,Q,M,misfit14,Imin]=InvertElastic(xp14,newprof14,Atest,xq,0.01,sd);
H14=alpha2thickness(A14);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp14)-(numel(A14)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit14(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit14(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A14; % convert to flexural parameter alpha
figure(2); hold on;

```

```

plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H14=alpha2thickness(A14); %Optimal elastic thickness at misfit minimum
H14min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H14max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

b. Corrected Code

```

load prof14oc.xyz
indexmin14=find(min(prof14oc(:,4)) == prof14oc(:,4));
xp14 = transpose(prof14oc(indexmin14:end,3));
t14 = transpose(prof14oc(indexmin14:end,4));
xpn14=[xp14(1:103-indexmin14),xp14(131-indexmin14:169-indexmin14),xp14(196-
indexmin14:586-indexmin14)];
tn14=[t14(1:103-indexmin14),t14(131-indexmin14:169-indexmin14),t14(196-indexmin14:586-
indexmin14)];
z=find(round(xpn14) == 400);
z1=find((xpn14(end)) == xpn14);
sd=std(tn14(z:z1));
averagedtopo14=mean(tn14(z:z1));
newprof14=tn14-averagedtopo14; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-400:10:0]; %Load vector
%% Calculation
[A14,Q,M,misfit14,Imin]=InvertElastic(xpn14,newprof14,Atest,xq,0.01,sd);
H14=alpha2thickness(A14);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xpn14)-(numel(A14)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out

```

```

Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit14(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit14(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A14; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');

```

```

%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H14=alpha2thickness(A14); %Optimal elastic thickness at misfit minimum
H14min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H14max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

T. Profile 15

a. Uncorrected Code

```

load prof15oc.xyz
indexmin15=find(min(prof15oc(:,4)) == prof15oc(:,4));
xp15 = transpose(prof15oc(indexmin15:end,3));
t15 = transpose(prof15oc(indexmin15:end,4));
z=find(round(xp15) == 400);
z1=find((xp15(end)) == xp15);
sd=std(t15(z:z1));
averagedtopo15=mean(t15(z:z1));
newprof15=t15-averagedtopo15; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A15,Q,M,misfit15,Imin]=InvertElastic(xp15,newprof15,Atest,xq,0.01,sd);
H15=alpha2thickness(A15);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square

```

```

nu=numel(xp15)-(numel(A15)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit15(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit15(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A15; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H15=alpha2thickness(A15); %Optimal elastic thickness at misfit minimum
H15min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H15max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

b. Corrected Code

```

load prof15oc.xyz
indexmin15=find(min(prof15oc(:,4)) == prof15oc(:,4));
xp15 = transpose(prof15oc(indexmin15:end,3));
t15 = transpose(prof15oc(indexmin15:end,4));
xpn15=[xp15(1:69-indexmin15),xp15(109-indexmin15:160-indexmin15),xp15(211-
indexmin15:333-indexmin15),xp15(364-indexmin15:481-indexmin15),xp15(502-
indexmin15:end)];
tn15=[t15(1:69-indexmin15),t15(109-indexmin15:160-indexmin15),t15(211-indexmin15:333-
indexmin15),t15(364-indexmin15:481-indexmin15),t15(502-indexmin15:end)];
z=find(round(xpn15) == 400);
z1=find((xpn15(end)) == xpn15);

```

```

sd=std(tn15(z:z1));
averagedtopo15=mean(tn15(z:z1));
newprof15=tn15-averagedtopo15; %%Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %%Alpha values to be tested
xq=[-800:50:0]; %%Load vector
%% Calculation
[A15,Q,M,misfit15,Imin]=InvertElastic(xpn15,newprof15,Atest,xq,0.01,sd);
H15=alpha2thickness(A15);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp15)-(numel(A15)+numel(Q)); %%Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %%Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %%Taking infinity values out
Mmax=max(deltachisquare)+M; %%Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit15(1:Imin)>Mmax))); %%Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit15(Imin:end)>Mmax))+Imin+1); %%Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %%elastic wavelength
AE=A15; %% convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%%Acceptable alpha range
H15=alpha2thickness(A15); %%Optimal elastic thickness at misfit minimum
H15min=alpha2thickness(Amin); %%Minimum acceptable elastic thickness
H15max=alpha2thickness(Amax); %%Maximum acceptable elastic thickness

```

U. Profile 16

a. Uncorrected Code

```
load prof16oc.xyz
indexmin16=find(min(prof16oc(:,4)) == prof16oc(:,4));
xp16 = transpose(prof16oc(indexmin16:end,3));
t16 = transpose(prof16oc(indexmin16:end,4));
z=find(round(xp16) == 400);
z1=find((xp16(end)) == xp16);
sd=std(t16(z:z1));
averagedtopo16=mean(t16(z:z1));
newprof16=t16-averagedtopo16; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A16,Q,M,misfit16,Imin]=InvertElastic(xp16,newprof16,Atest,xq,0.01,sd);
H16=alpha2thickness(A16);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp16)-(numel(A16)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit16(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit16(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A16; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
```

```

print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H16=alpha2thickness(A16); %Optimal elastic thickness at misfit minimum
H16min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H16max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

b. Corrected Code

```

load prof16oc.xyz
indexmin16=find(min(prof16oc(:,4)) == prof16oc(:,4));
xp16 = transpose(prof16oc(indexmin16:end,3));
t16 = transpose(prof16oc(indexmin16:end,4));
xpn16=[xp16(1:64-indexmin16),xp16(75-indexmin16:240-indexmin16),xp16(299-
indexmin16:340-indexmin16),xp16(357-indexmin16:422-indexmin16),xp16(436-
indexmin16:535-indexmin16),xp16(544-indexmin16:end)];
tn16=[t16(1:64-indexmin16),t16(75-indexmin16:240-indexmin16),t16(299-indexmin16:340-
indexmin16),t16(357-indexmin16:422-indexmin16),t16(436-indexmin16:535-
indexmin16),t16(544-indexmin16:end)];
z=find(round(xpn16) == 400);
z1=find((xpn16(end)) == xpn16);
sd=std(tn16(z:z1));
averagedtopo16=mean(tn16(z:z1));
newprof16=tn16-averagedtopo16; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A16,Q,M,misfit16,Imin]=InvertElastic(xpn16,newprof16,Atest,xq,0.01,sd);
H16=alpha2thickness(A16);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xpn16)-(numel(A16)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit16(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit16(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold

```



```

% Creating the plots
LE=M; %elastic wavelength
AE=A16; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H16=alpha2thickness(A16); %Optimal elastic thickness at misfit minimum
H16min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H16max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

V. Profile 17

a. Uncorrected Code

```

load prof17oc.xyz
indexmin17=find(min(prof17oc(:,4)) == prof17oc(:,4));
xp17 = transpose(prof17oc(indexmin17:end,3));
t17 = transpose(prof17oc(indexmin17:end,4));
z=find(round(xp17) == 400);
z1=find((xp17(end)) == xp17);
sd=std(t17(z:z1));
averagedtopo17=mean(t17(z:z1));
newprof17=t17-averagedtopo17; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A17,Q,M,misfit17,Imin]=InvertElastic(xp17,newprof17,Atest,xq,0.01,sd);
H17=alpha2thickness(A17);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp17)-(numel(A17)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);

```

```

%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit17(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit17(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A17; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H17=alpha2thickness(A17); %Optimal elastic thickness at misfit minimum
H17min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H17max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

b. Corrected Code

```

load prof17oc.xyz
indexmin17=find(min(prof17oc(:,4)) == prof17oc(:,4));
xp17 = transpose(prof17oc(indexmin17:end,3));
t17 = transpose(prof17oc(indexmin17:end,4));
xpn17=[xp17(1:66-indexmin17),xp17(84-indexmin17:463-indexmin17),xp17(498-
indexmin17:end)];
tn17=[t17(1:66-indexmin17),t17(84-indexmin17:463-indexmin17),t17(498-indexmin17:end)];
z=find(round(xpn17) == 400);
z1=find((xpn17(end)) == xpn17);
sd=std(tn17(z:z1));
averagedtopo17=mean(tn17(z:z1));
newprof17=tn17-averagedtopo17; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector

```

```

%% Calculation
[A17,Q,M,misfit17,Imin]=InvertElastic(xpn17,newprof17,Atest,xq,0.01,sd);
H17=alpha2thickness(A17);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xpn17)-(numel(A17)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit17(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit17(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A17; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H17=alpha2thickness(A17); %Optimal elastic thickness at misfit minimum
H17min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H17max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

W. Profile 18

a. Uncorrected Code

```

load prof18oc.xyz
indexmin18=find(min(prof18oc(:,4)) == prof18oc(:,4));
xp18 = transpose(prof18oc(indexmin18:end,3));
t18 = transpose(prof18oc(indexmin18:end,4));
z=find(round(xp18) == 400);

```

```

z1=find((xp18(end)) == xp18);
sd=std(t18(z:z1));
averagedtopo18=mean(t18(z:z1));
newprof18=t18-averagedtopo18; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A18,Q,M,misfit18,Imin]=InvertElastic(xp18,newprof18,Atest,xq,0.01,sd);
H18=alpha2thickness(A18);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp18)-(numel(A18)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit18(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit18(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A18; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)), 'r--')
plot([Amin,Amin],[0,1]*Mmax, 'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range

```

```
H18=alpha2thickness(A18); %Optimal elastic thickness at misfit minimum
H18min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H18max=alpha2thickness(Amax); %Maximum acceptable elastic thickness
```

b. Corrected Code

```
load prof18oc.xyz
indexmin18=find(min(prof18oc(:,4)) == prof18oc(:,4));
xp18 = transpose(prof18oc(indexmin18:end,3));
t18 = transpose(prof18oc(indexmin18:end,4));
xpn18=[xp18(1:66-indexmin18),xp18(84-indexmin18:463-indexmin18),xp18(498-
indexmin18:end)];
tn18=[t18(1:66-indexmin18),t18(84-indexmin18:463-indexmin18),t18(498-indexmin18:end)];
z=find(round(xpn18) == 400);
z1=find((xpn18(end)) == xpn18);
sd=std(tn18(z:z1));
averagedtopo18=mean(tn18(z:z1));
newprof18=tn18-averagedtopo18; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A18,Q,M,misfit18,Imin]=InvertElastic(xpn18,newprof18,Atest,xq,0.01,sd);
H18=alpha2thickness(A18);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xpn18)-(numel(A18)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit18(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit18(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A18; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
```

```

orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H18=alpha2thickness(A18); %Optimal elastic thickness at misfit minimum
H18min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H18max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

X. Profile 19

a. Uncorrected Code

```

load profil9oc.xyz
indexmin19=find(min(profil9oc(:,4)) == profil9oc(:,4));
xp19 = transpose(profil9oc(indexmin19:end,3));
t19 = transpose(profil9oc(indexmin19:end,4));
z=find(round(xp19) == 400);
z1=find((xp19(end)) == xp19);
sd=std(t19(z:z1));
averagedtopo19=mean(t19(z:z1));
newprofil9=t19-averagedtopo19; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A19,Q,M,misfit19,Imin]=InvertElastic(xp19,newprofil9,Atest,xq,0.01,sd);
H19=alpha2thickness(A19);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xp19)-(numel(A19)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit19(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold

```

```

Amax=Atest(min(find(misfit19(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A19; % convert to flexural parameter alpha
figure(2); hold on;
x=Amin:max(Atest);
y=Mmax;
plot(x,y*ones(size(x)), 'r--')
plot([Amin,Amin],[0,1]*Mmax, 'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H19=alpha2thickness(A19); %Optimal elastic thickness at misfit minimum
H19min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H19max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

Y. Profile 20

a. Uncorrected Code

```

load prof20oc.xyz
indexmin20=57;
xp20 = transpose(prof20oc(indexmin20:end,3));
t20 = transpose(prof20oc(indexmin20:end,4));
z=find(round(xp20) == 600);
z1=find((xp20(end)) == xp20);
sd=std(t20(z:z1));
averagedtopo20=mean(t20(z:z1));
newprof20=t20-averagedtopo20; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A20,Q,M,misfit20,Imin]=InvertElastic(xp20,newprof20,Atest,xq,0.01,sd);
H20=alpha2thickness(A20);
%% Plotting acceptable alpha values and misfit threshold

```

```

% Finding delta chi-square
nu=numel(xp20)-(numel(A20)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit20(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit20(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A20; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H20=alpha2thickness(A20); %Optimal elastic thickness at misfit minimum
H20min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H20max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

b. Corrected Code

```

load prof20oc.xyz
indexmin20=57;
xp20 = transpose(prof20oc(indexmin20:end,3));
t20 = transpose(prof20oc(indexmin20:end,4));
xpn20=[xp20(1:293-indexmin20),xp20(499-indexmin20:end)];
tn20=[t20(1:293-indexmin20),t20(499-indexmin20:end)];
z=find(round(xpn20) == 600);
z1=find((xpn20(end)) == xpn20);
sd=std(tn20(z:z1));
averagedtopo20=mean(tn20(z:z1));

```



```

newprof20=tn20-averagedtopo20; %Reconstruction of topography without uncertainty

%% Variables
Atest=[10:1:1000]; %Alpha values to be tested
xq=[-800:50:0]; %Load vector
%% Calculation
[A20,Q,M,misfit20,Imin]=InvertElastic(xpn20,newprof20,Atest,xq,0.01,sd);
H20=alpha2thickness(A20);
%% Plotting acceptable alpha values and misfit threshold
% Finding delta chi-square
nu=numel(xpn20)-(numel(A20)+numel(Q)); %Degrees of freedom: Number of data points -
number of parameters being inverted
P=0.683; %Probability value taken from Numerical Recipes 3rd Edition for one sigma value of
delta chi square
deltachisquare=gammaincinv(P,nu/2);
%Calculations of alpha range and misfit threshold
deltachisquare(~isfinite(deltachisquare))=0; %Taking infinity values out
Mmax=max(deltachisquare)+M; %Finding the misfit threshold (Make sure there is no infinite
values)
Amin=Atest(max(find(misfit20(1:Imin)>Mmax))); %Finding minimum alpha value within the
threshold
Amax=Atest(min(find(misfit20(Imin:end)>Mmax))+Imin+1); %Finding maximum alpha value
within the threshold
% Creating the plots
LE=M; %elastic wavelength
AE=A20; % convert to flexural parameter alpha
figure(2); hold on;
plot([Amin,Amin,Amax,Amax],[0,1,1,0]*Mmax,'r--');
plot(AE*[1,1],get(gca,'ylim'),'k');
set(gca,'FontSize',12)
orient portrait
%
print(2,'-dpdf','Mistfit test 1');
figure(1); orient tall;

return
print(1,'-dpdf','Fit test 1');
hold off
%% Finding Range of Acceptable Elastic Thicknesses
Arange=[Amin:Amax];%Acceptable alpha range
H20=alpha2thickness(A20); %Optimal elastic thickness at misfit minimum
H20min=alpha2thickness(Amin); %Minimum acceptable elastic thickness
H20max=alpha2thickness(Amax); %Maximum acceptable elastic thickness

```

Z. Weighted Average Flexural Parameter and Heat Flux Calculation

```

%% Weighted Average Flexural Parameter Calculation

```

%Optimal Flexural Parameter Value for Each Profile in km

A1=45;
A2=122;
A3=74;
A4=28;
A5=181;
A6=432;
A7=86;
A8=128;
A9=1820;
A10=7810;
A11=88;
A12=117;
A13=97;
A14=92;
A15=115;
A16=104;
A17=102;
A18=145;
A19=554;
A20=198;

%Minimum Acceptable Flexural Parameter in km

A1min=A1-35;
A2min=A2-55;
A3min=A3-64;
A4min=A4-18;
A5min=A5-136;
A6min=A6-396;
A7min=A7-inf;
A8min=A8-28;
A9min=A9-1760;
A10min=A10-7700;
A11min=A11-11;
A12min=A12-101;
A13min=A13-81;
A14min=A14-39;
A15min=A15-36;
A16min=A16-22;
A17min=A17-13;
A18min=A18-97;
A19min=A19-inf;
A20min=A20-77;

%Maximum Acceptable Flexural Parameter in km

A1max=A1+72;

```

A2max=A2+122;
A3max=A3+31;
A4max=A4+inf;
A5max=A5+364;
A6max=A6+inf;
A7max=A7+inf;
A8max=A8+42;
A9max=A9+inf;
A10max=A10+inf;
A11max=A11+113;
A12max=A12+inf;
A13max=A13+112;
A14max=A14+40;
A15max=A15+46;
A16max=A16+25;
A17max=A17+15;
A18max=A18+835;
A19max=A19+inf;
A20max=A20+70;

```

```

%emin

```

```

emin1=A1-A1min;
emin2=A2-A2min;
emin3=A3-A3min;
emin4=A4-A4min;
emin5=A5-A5min;
emin6=A6-A6min;
emin7=A7-A7min;
emin8=A8-A8min;
emin9=A9-A9min;
emin10=A10-A10min;
emin11=A11-A11min;
emin12=A12-A12min;
emin13=A13-A13min;
emin14=A14-A14min;
emin15=A15-A15min;
emin16=A16-A16min;
emin17=A17-A17min;
emin18=A18-A18min;
emin19=A19-A19min;
emin20=A20-A20min;

```

```

%emax

```

```

emax1=A1max-A1;
emax2=A2max-A2;
emax3=A3max-A3;

```

```

emax4=A4max-A4;
emax5=A5max-A5;
emax6=A6max-A6;
emax7=A7max-A7;
emax8=A8max-A8;
emax9=A9max-A9;
emax10=A10max-A10;
emax11=A11max-A11;
emax12=A12max-A12;
emax13=A13max-A13;
emax14=A14max-A14;
emax15=A15max-A15;
emax16=A16max-A16;
emax17=A17max-A17;
emax18=A18max-A18;
emax19=A19max-A19;
emax20=A20max-A20;

```

%w

```

w1=(1/emax1)+(1/emin1);
w2=(1/emax2)+(1/emin2);
w3=(1/emax3)+(1/emin3);
w4=(1/emax4)+(1/emin4);
w5=(1/emax5)+(1/emin5);
w6=(1/emax6)+(1/emin6);
w7=(1/emax7)+(1/emin7);
w8=(1/emax8)+(1/emin8);
w9=(1/emax9)+(1/emin9);
w10=(1/emax10)+(1/emin10);
w11=(1/emax11)+(1/emin11);
w12=(1/emax12)+(1/emin12);
w13=(1/emax13)+(1/emin13);
w14=(1/emax14)+(1/emin14);
w15=(1/emax15)+(1/emin15);
w16=(1/emax16)+(1/emin16);
w17=(1/emax17)+(1/emin17);
w18=(1/emax18)+(1/emin18);
w19=(1/emax19)+(1/emin19);
w20=(1/emax20)+(1/emin20);

```

%Weighted average

```

ha1=A1*w1;
ha2=A2*w2;
ha3=A3*w3;
ha4=A4*w4;
ha5=A5*w5;

```

```

ha6=A6*w6;
ha7=A7*w7;
ha8=A8*w8;
ha9=A9*w9;
ha10=A10*w10;
ha11=A11*w11;
ha12=A12*w12;
ha13=A13*w13;
ha14=A14*w14;
ha15=A15*w15;
ha16=A16*w16;
ha17=A17*w17;
ha18=A18*w18;
ha19=A19*w19;
ha20=A20*w20;
h=[ha1 ha2 ha3 ha4 ha5 ha6 ha7 ha8 ha9 ha10 ha11 ha12 ha13 ha14 ha15 ha16 ha17 ha18 ha19
ha20];
havg=sum(h);
w=[w1 w2 w3 w4 w5 w6 w7 w8 w9 w10 w11 w12 w13 w14 w15 w16 w17 w18 w19 w20];
havg=sum(h)/sum(w);
disp(sprintf('Weight Average Flexural Parameter = %g km',havg))

he=alpha2thickness(havg);
disp(sprintf('Weighted Average Elastic Thickness = %g km',he))

```

AA. Enceladus, Europa, Ganymede, and Pluto Temperature Gradient Calculation

```

%% Enceladus
%D,pm,pc,g,E,h,g,v
E=1e9; %Young's modulus in GPa
dp=920; %Density in kg/m^3
g=0.113; %Gravity in m/s^2
v=0.33; %Poisson's ratio
hen=300; %Effective elastic thickness in m
D=(E*hen^3)/(12*(1-(v^2))); %Flexural rigidity
aen=((4*D)/(dp*g))^(1/4)/1000; %Flexural parameter in km

disp(sprintf('Flexural parameter = %g km',aen))

%% Europa
%D,pm,pc,g,E,h,g,v
E=1e9; %Young's modulus in GPa
pm=900; %Density in kg/m^3
g=1.3; %Gravity in m/s^2
v=0.3; %Poisson's ratio

```

```

he=6000;
D=(E*he^3)/(12*(1-(v^2))); %Flexural rigidity
ae=((4*D)/(pm*g))^(1/4)/1000; %Flexural parameter

disp(sprintf('Flexural parameter = %g km',ae))

%% Ganymede
%D,pm,pc,g,E,h,g,v
E=1e9; %Young's modulus in GPa
pm=1000; %Density in kg/m^3
g=1.4; %Gravity in m/s^2
v=0.33; %Poisson's ratio
hg=1000; %Effective elastic thickness in m
D=(E*hg^3)/(12*(1-(v^2))); %Flexural rigidity
a=((4*D)/(pm*g))^(1/4)/1000; %Flexural parameter

disp(sprintf('Flexural parameter = %g km',a))

%% Converting to Elastic Thickness with Pluto's Parameters
hpen=alpha2thickness(aen);
hpe=alpha2thickness(ae);
hpa=alpha2thickness(a);

%% Temperature Gradients (Using Parameters from Papers)
Tr=150; %For ice in K

%Enceladus
Tsen=65; %Surface Temperature in K...in paper they use 85k
Tgen=(Tr-Tsen)/hen;

%Europa
Tse=105; %Surface Temperature in K
Tge=(Tr-Tse)/he;

%Ganymede
Tsg=120; %Surface Temperature in K
Tgg=(Tr-Tsg)/hg;

%Pluto
Tsp=33; %Surface Temperature in K
Tgp=(Tr-Tsp)/30000;

disp(sprintf('Enceladus Temperature Gradient = %g K/m',Tgen))
disp(sprintf('Europa Temperature Gradient = %g K/m',Tge))

```

```
disp(sprintf('Ganymede Temperature Gradient = %g K/m',Tgg))  
disp(sprintf('Pluto Temperature Gradient = %g K/m',Tgp))
```

III. Table Appendix

a. Table 1

| Physical Parameters | | |
|---------------------|--|-------------------------------------|
| Symbol | Name | Value/Unit |
| ρ_m | Density of underlying layer of ice shell | 1030 $\frac{\text{kg}}{\text{m}^3}$ |
| g | Gravity | 0.62 $\frac{\text{m}}{\text{s}^2}$ |
| E | Young's modulus | 9 GPa |
| ν | Poisson's ratio | 0.3 |

Table 1. Rheological parameters used for both the one-dimensional and two-dimensional elastic flexure models. (Sources for parameters are from Johnson et al., 2016 and Nimmo et al., 2016.)

b. Table 2

| Uncorrected Profiles | Zero Offset (km) | Background Elevation (m) | Topography Variance (m) | χ^2 Minimum | χ^2 Threshold | α (km) | Elastic Thickness (km) |
|----------------------|------------------|--------------------------|-------------------------|------------------|--------------------|----------------------------|-------------------------------|
| Profile 1 | 32 | 599.1 | 466.0 | 651.3 | 927.9 | 56 $^{+69}_{-46}$ | 12.4 $^{+23.8}_{-11.2}$ |
| Profile 2 | 39 | -246.8 | 748.5 | 390.1 | 673.2 | 115 $^{+184}_{-103}$ | 32.4 $^{+83.3}_{-30.8}$ |
| Profile 3 | 24 | 229.6 | 683.8 | 584.7 | 865.3 | 17 $^{+\infty}_{-7}$ | 2.5 $^{+\infty}_{-\infty}$ |
| Profile 4 | 54 | 119.5 | 442.4 | 909.0 | 1174 | 75 $^{+49}_{-65}$ | 18.3 $^{+17.5}_{-17.1}$ |
| Profile 5 | 36 | 64.7 | 662.6 | 542.4 | 816.9 | 164 $^{+\infty}_{-122}$ | 51.5 $^{+\infty}_{-43.3}$ |
| Profile 6 | 8 | 1918 | 848.2 | 497.3 | 793.1 | 53 $^{+\infty}_{-\infty}$ | 11.5 $^{+\infty}_{-\infty}$ |
| Profile 7 | 46 | 278.2 | 1170 | 259.3 | 535.8 | 149 $^{+\infty}_{-118}$ | 45.7 $^{+\infty}_{-40.1}$ |
| Profile 8 | 45 | 1164 | 761.9 | 280.5 | 555.5 | 86 $^{+\infty}_{-\infty}$ | 22.0 $^{+\infty}_{-\infty}$ |
| Profile 9 | 183 | 1767 | 833.0 | 162.7 | 343.7 | 1820 $^{+\infty}_{-1760}$ | 1285.7 $^{+\infty}_{-1272.1}$ |
| Profile 10 | 223 | -806.6 | 980.3 | 240.5 | 428.1 | 7810 $^{+\infty}_{-7700}$ | 8966 $^{+\infty}_{-8893.5}$ |
| Profile 11 | 409 | 896.6 | 540.1 | 160.9 | 245.4 | 87 $^{+\infty}_{-27}$ | 22.3 $^{+\infty}_{-8.7}$ |
| Profile 12 | 21 | 1830 | 787.8 | 452.5 | 741.7 | 346 $^{+\infty}_{-330}$ | 140.5 $^{+\infty}_{-138.2}$ |
| Profile 13 | 32 | 817.3 | 830.8 | 792.5 | 1076 | 28 $^{+\infty}_{-\infty}$ | 4.9 $^{+\infty}_{-\infty}$ |
| Profile 14 | 34 | 728.2 | 328.5 | 601.5 | 871.9 | 92 $^{+59}_{-76}$ | 24.0 $^{+22.5}_{-21.7}$ |
| Profile 15 | 24 | 339.1 | 298.5 | 1186 | 1473 | 113 $^{+42}_{-35}$ | 31.6 $^{+16.6}_{-12.3}$ |
| Profile 16 | 29 | 795.6 | 236.6 | 1519 | 1804 | 94 $^{+27}_{-22}$ | 24.7 $^{+9.9}_{-7.4}$ |
| Profile 17 | 21 | 739.5 | 125.2 | 2974 | 3263 | 104 $^{+16}_{-14}$ | 28.3 $^{+5.9}_{-5.0}$ |
| Profile 18 | 10 | 406.2 | 529.6 | 315.2 | 610.0 | 148 $^{+\infty}_{-108}$ | 45.3 $^{+\infty}_{-37.4}$ |
| Profile 19 | 3 | -146.9 | 584.9 | 604.0 | 902.3 | 554 $^{+\infty}_{-\infty}$ | 263.3 $^{+\infty}_{-\infty}$ |
| Profile 20 | 57 | -547.9 | 498.7 | 1979 | 2300 | 219 $^{+84}_{-50}$ | 76.4 $^{+41.4}_{-22.3}$ |

Table 2. Uncorrected profiles summary of the values for the offset on the beginning of the tracks to begin at the edge of Sputnik Planitia, the background topography removed, the topography variance, χ^2

minimum, the χ^2 threshold, the range of acceptable flexural parameters, and acceptable range of elastic thickness.

c. Table 3

| Corrected Profiles | Zero Offset (km) | Background Topography (m) | Topography Variance (m) | χ^2 Minimum | χ^2 Threshold | α (km) | Elastic Thickness (km) |
|--------------------|------------------|---------------------------|-------------------------|------------------|--------------------|---------------------------|------------------------------|
| Profile 1 | 32 | 742.1 | 394.8 | 360.8 | 570.8 | 45^{+72}_{-35} | $9.3^{+23.8}_{-8.1}$ |
| Profile 2 | 39 | 82.20 | 405.5 | 359.0 | 573.6 | 122^{+64}_{-55} | $35.0^{+26.4}_{-19.3}$ |
| Profile 3 | 24 | 670.6 | 233.8 | 460.0 | 740.6 | 74^{+31}_{-64} | $18.0^{+10.7}_{-16.8}$ |
| Profile 4 | 54 | 1085.6 | 345.8 | 141.0 | 406.4 | $28^{+\infty}_{-18}$ | $4.9^{+\infty}_{-3.7}$ |
| Profile 5 | 36 | 279.7 | 541.2 | 386.9 | 668.5 | 181^{+364}_{-136} | $59.2^{+198.4}_{-49.9}$ |
| Profile 6 | 8 | 2294 | 581.8 | 545.1 | 774.9 | $432^{+\infty}_{-396}$ | $189.0^{+\infty}_{-182.1}$ |
| Profile 7 | 46 | 988.1 | 991.0 | 174.8 | 451.4 | $86^{+\infty}_{-\infty}$ | $22.0^{+\infty}_{-\infty}$ |
| Profile 8 | 45 | 726.8 | 190.1 | 1040 | 1315 | 128^{+42}_{-28} | $37.3^{+17.2}_{-10.4}$ |
| Profile 9* | 183 | 1767 | 833.0 | 162.7 | 343.7 | $1820^{+\infty}_{-1760}$ | $1285.7^{+\infty}_{-1272.1}$ |
| Profile 10* | 223 | -806.6 | 980.3 | 240.5 | 428.1 | $7810^{+\infty}_{-7700}$ | $8966^{+\infty}_{-8893.5}$ |
| Profile 11 | 409 | 1404 | 300.9 | 206.5 | 291.0 | 88^{+113}_{-11} | $22.6^{+45.5}_{-3.6}$ |
| Profile 12 | 21 | 1830 | 787.8 | 325.7 | 615.0 | $117^{+\infty}_{-101}$ | $33.1^{+\infty}_{-30.8}$ |
| Profile 13 | 32 | 1088 | 521.3 | 442.6 | 660.7 | 97^{+112}_{-81} | $25.8^{+46}_{-23.5}$ |
| Profile 14 | 34 | 747.6 | 286.7 | 503.0 | 738.4 | 92^{+40}_{-39} | $24.0^{+14.9}_{-12.5}$ |
| Profile 15 | 24 | 364.5 | 284.9 | 624.3 | 912.0 | 115^{+46}_{-36} | $32.4^{+18.3}_{-12.8}$ |
| Profile 16 | 29 | 836.8 | 186.2 | 1160 | 1391 | 104^{+25}_{-22} | $29.3^{+9.4}_{-7.7}$ |
| Profile 17 | 21 | 734.5 | 126.7 | 2225 | 2379 | 102^{+15}_{-13} | $27.6^{+5.5}_{-4.6}$ |
| Profile 18 | 10 | 480.8 | 490.7 | 267.9 | 536.8 | 145^{+835}_{-97} | $44.1^{+519.1}_{-34.0}$ |
| Profile 19* | 3 | -146.9 | 584.9 | 604.0 | 902.3 | $554^{+\infty}_{-\infty}$ | $263.3^{+\infty}_{-\infty}$ |
| Profile 20 | 57 | -547.9 | 498.7 | 453.6 | 671.3 | 198^{+70}_{-77} | $66.8^{+33.2}_{-32.2}$ |

Table 3. Corrected profiles summary of the values for the offset on the beginning of the tracks to begin at the edge of Sputnik Planitia, the background topography removed, the topography variance, χ^2 minimum, the χ^2 threshold, the range of acceptable flexural parameters, and acceptable range of elastic thickness. * means no removal of craters and pits were done.

IV. Figure Appendix
a. Figure 1

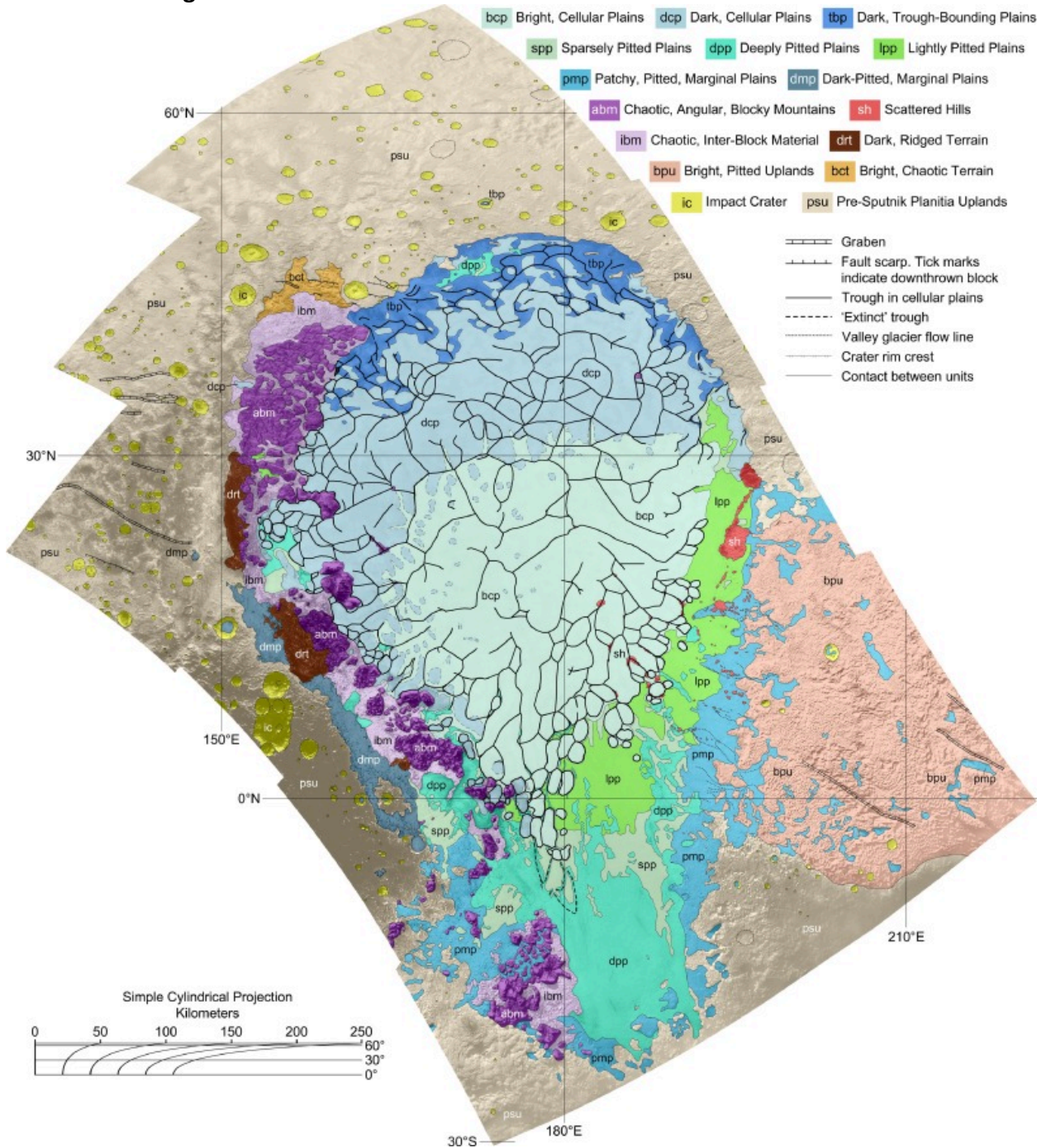


Fig. 1. Geological map of Sputnik Planitia displaying the different geological units imaged by the New Horizons spacecraft with a key of the different units and features (White et al., 2017).

b. Figure 2

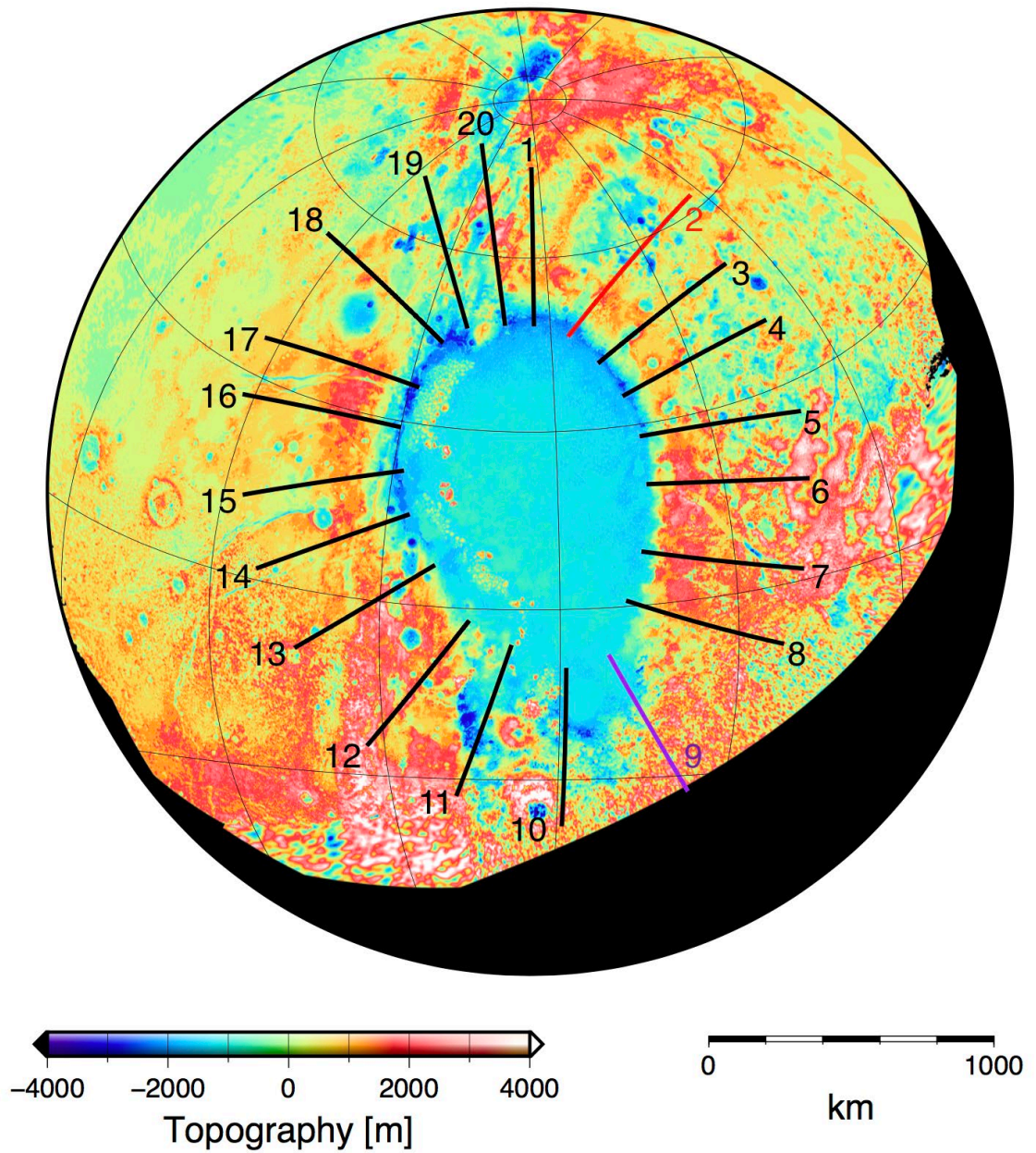


Fig. 2. Topography map of Pluto centered at 175°/20°, using the Lambert Equal Area projection. The map also includes 20 randomly selected tracks on the edge of Sputnik Planitia, highlighting two specific tracks that will be mentioned further in this paper. The black region is an area of no data.

c. Figure 3

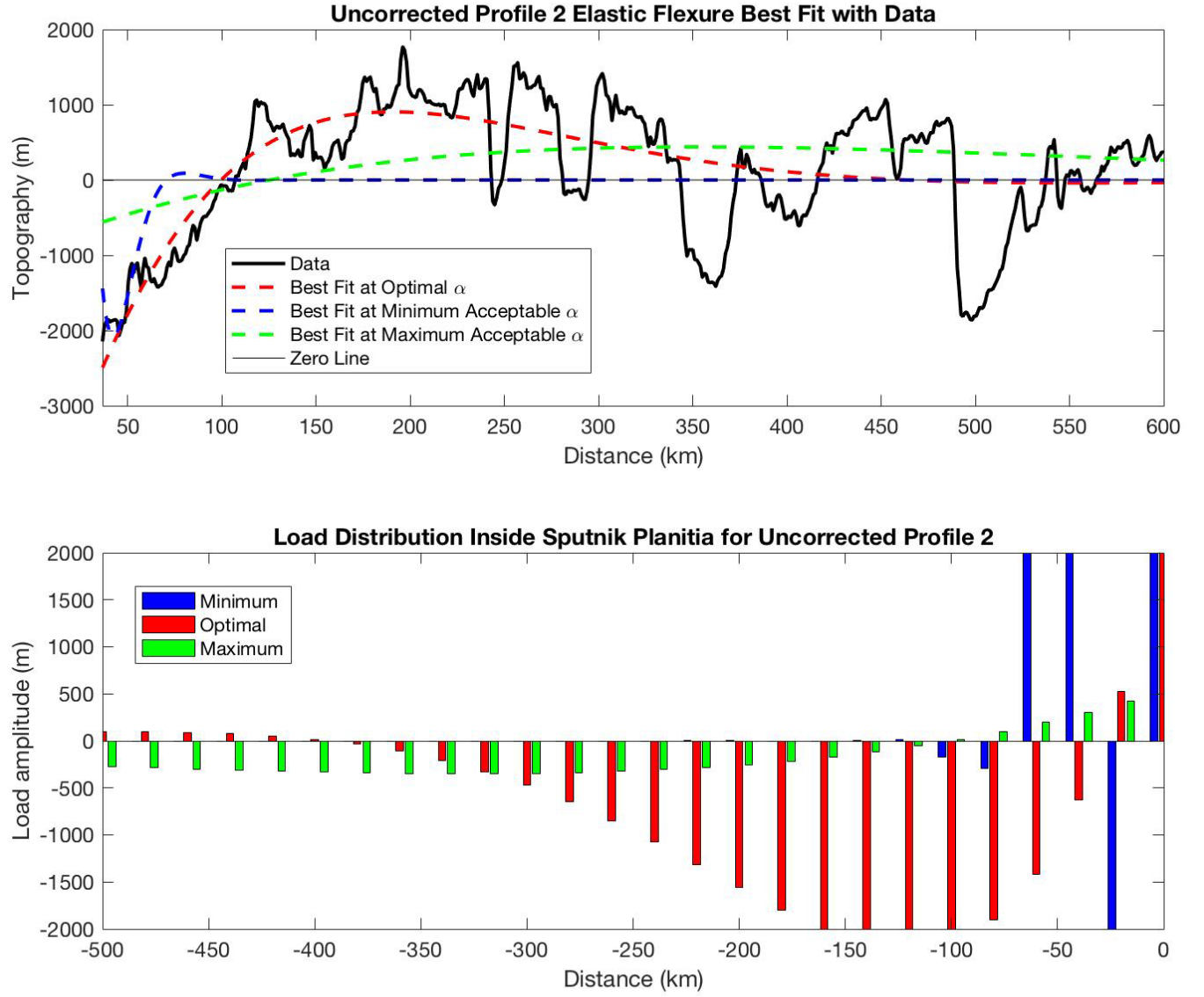


Fig. 3. Top panel: Track 2's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot still has craters present in the data at distance of 240 to 255, 275 to 304, 333 to 375, and 486 to 541. Lower panel: Profile 2's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

d. Figure 4

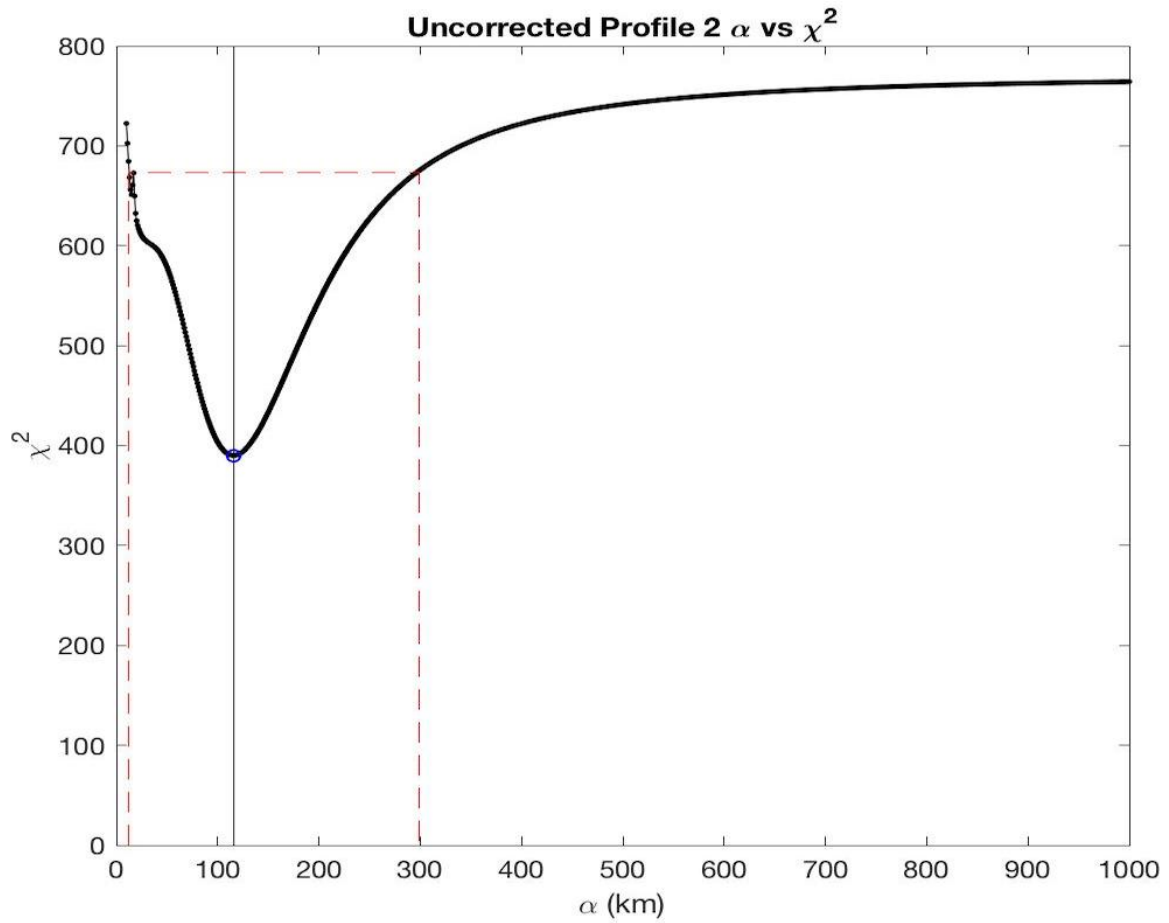


Fig.4. χ^2 versus flexural parameter plot of track 2 with craters where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

e. Figure 5

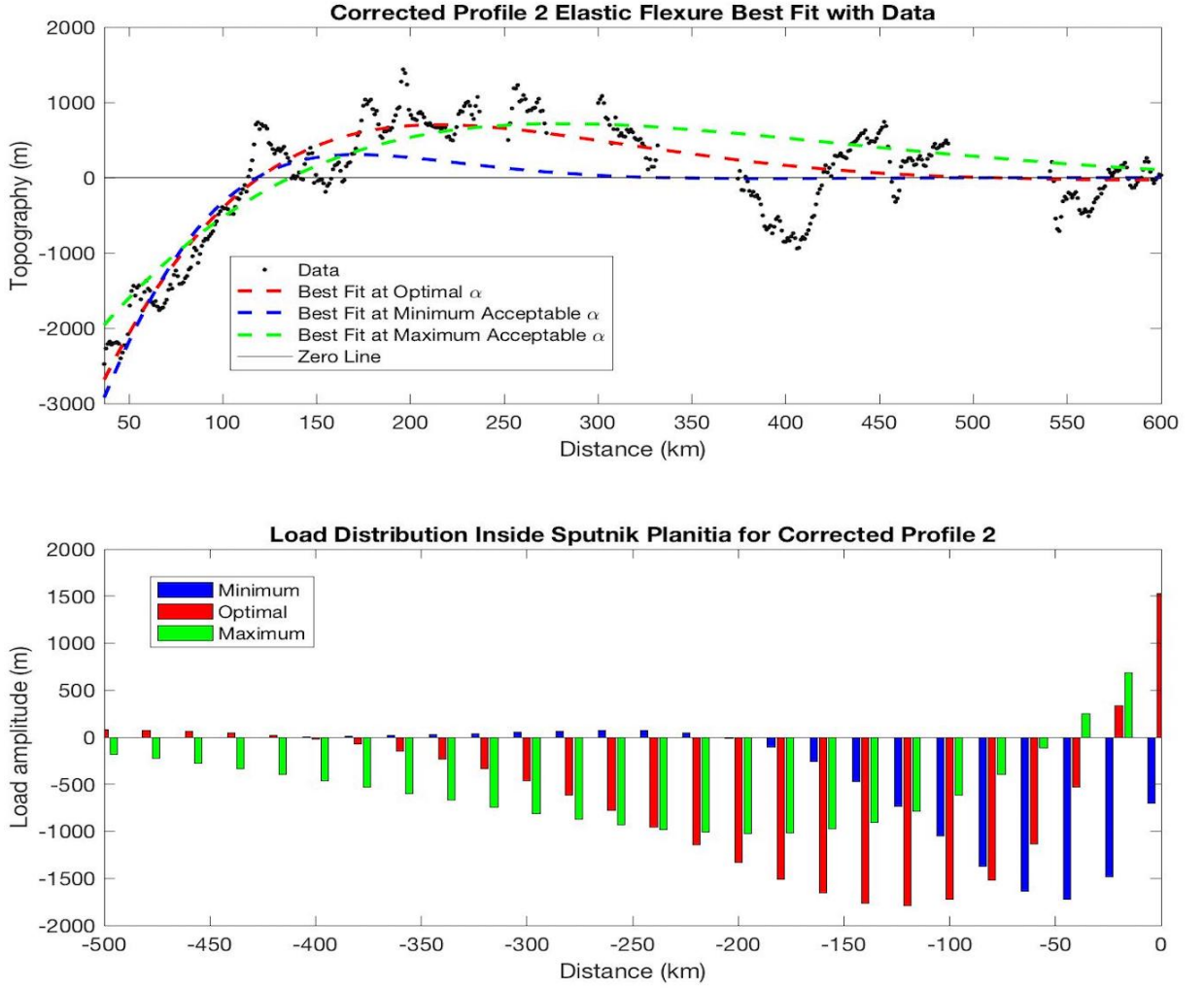


Fig. 5. Top panel: Track 2's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data at distance of 240 to 255, 275 to 304, 333 to 375, and 486 to 541. Lower panel: Profile 2's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

f. Figure 6

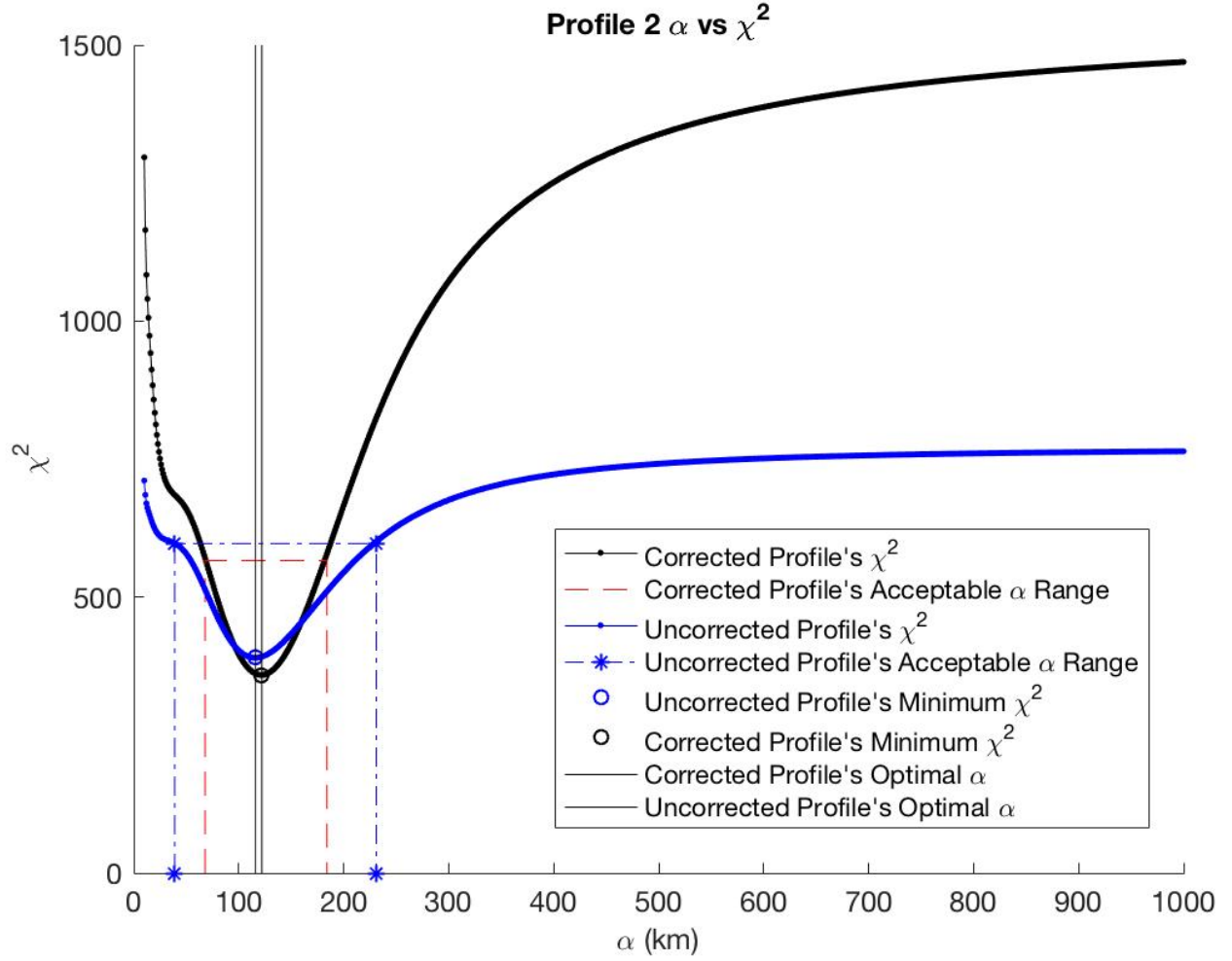


Fig.6. χ^2 versus flexural parameter plot of track 2 without craters where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized. Also on this figure is the acceptable flexural parameters for the uncorrected profile 2 to show that the range became narrower with the removal of craters along with its χ^2 curve.

g. Figure 7

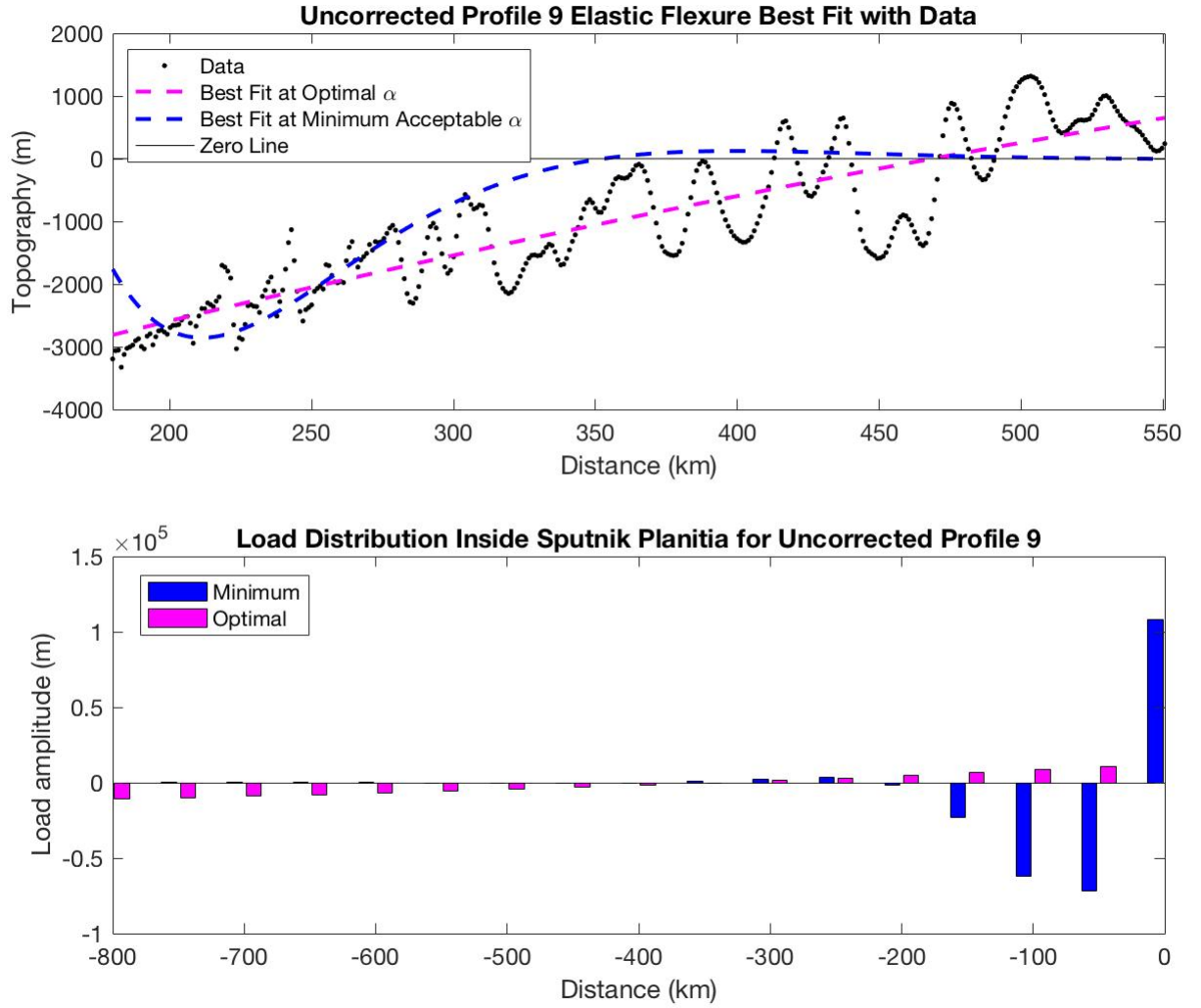


Fig.7. Top panel: Track 9's profile with the best fits from the acceptable range of α where the magenta dashed line is the best with the optimal α where χ^2 is minimized, and the blue dashed line is the best fit with the minimum acceptable α . Lower panel: Profile 9's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

h. Figure 8

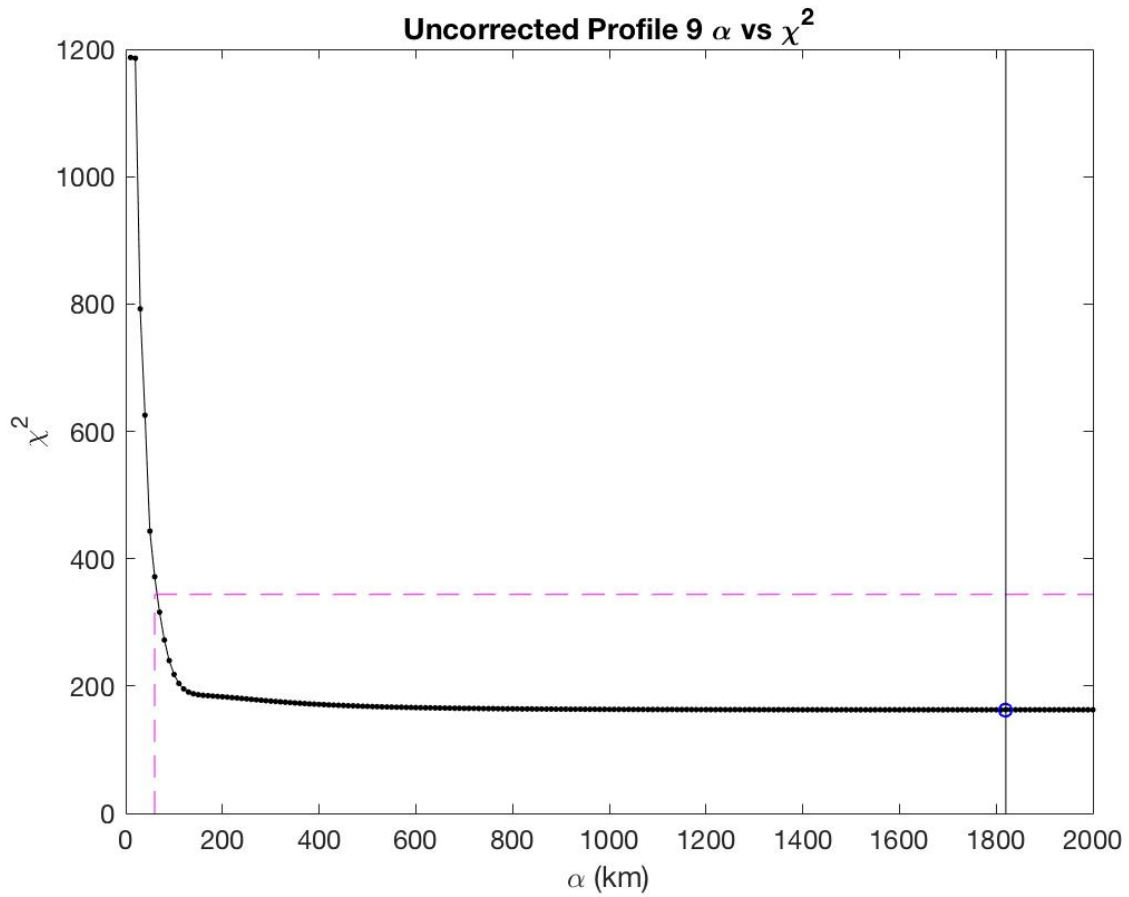


Fig. 8. χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

i. Figure 9

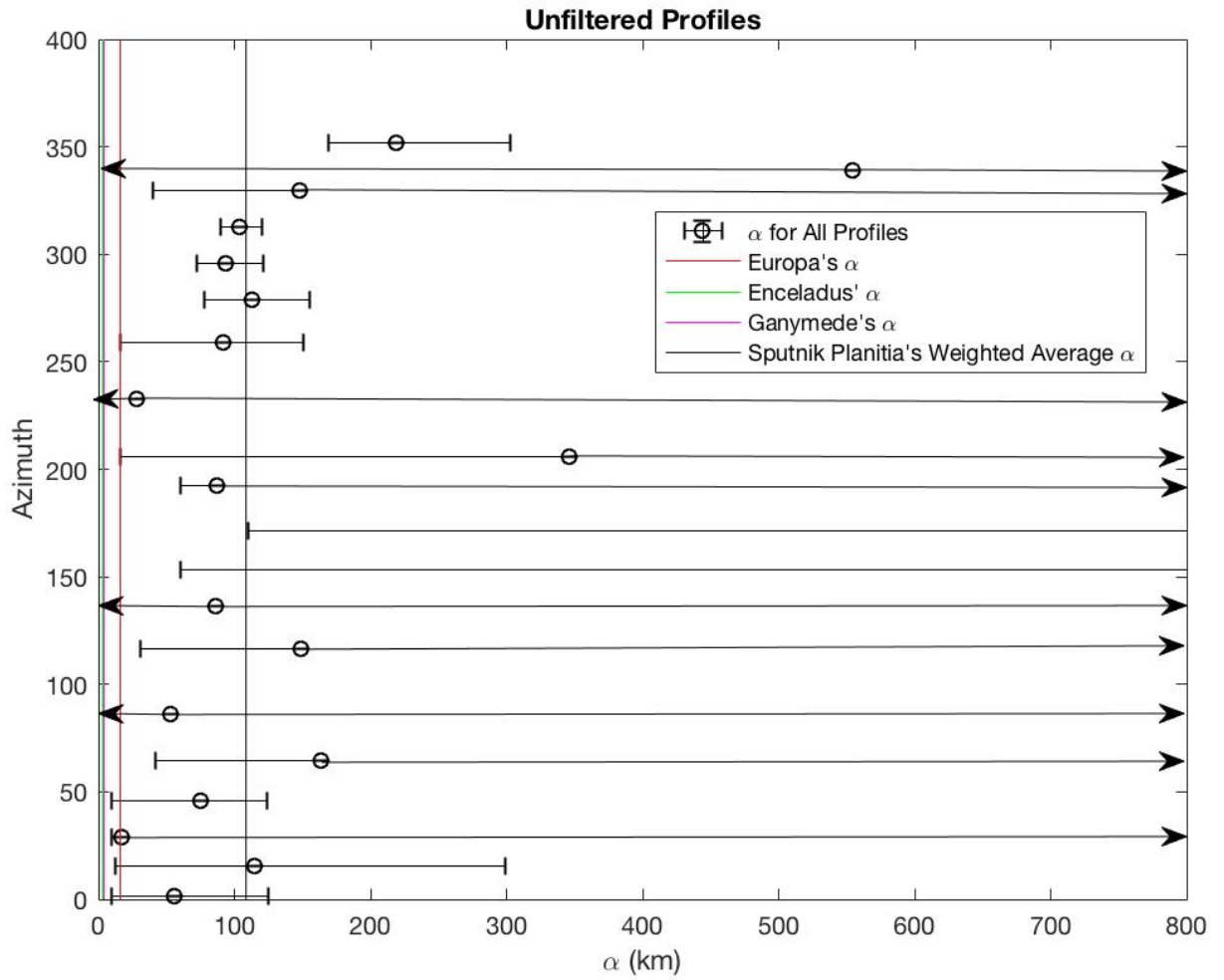


Figure 9. Azimuth versus α plot for profiles with craters and pits, showing that the flexural parameter range for most profiles are between 17 and 554 with two outlier values for Profiles 9 and 10 which are past the bounds of 800 km. The arrows indicate infinite values that are past the bounds of 0 to 800 km. The α values for Europa, Enceladus, and Ganymede are plotted in red, green, and magenta respectively. Sputnik Planitia's weighted average α is in black.

j. Figure 10

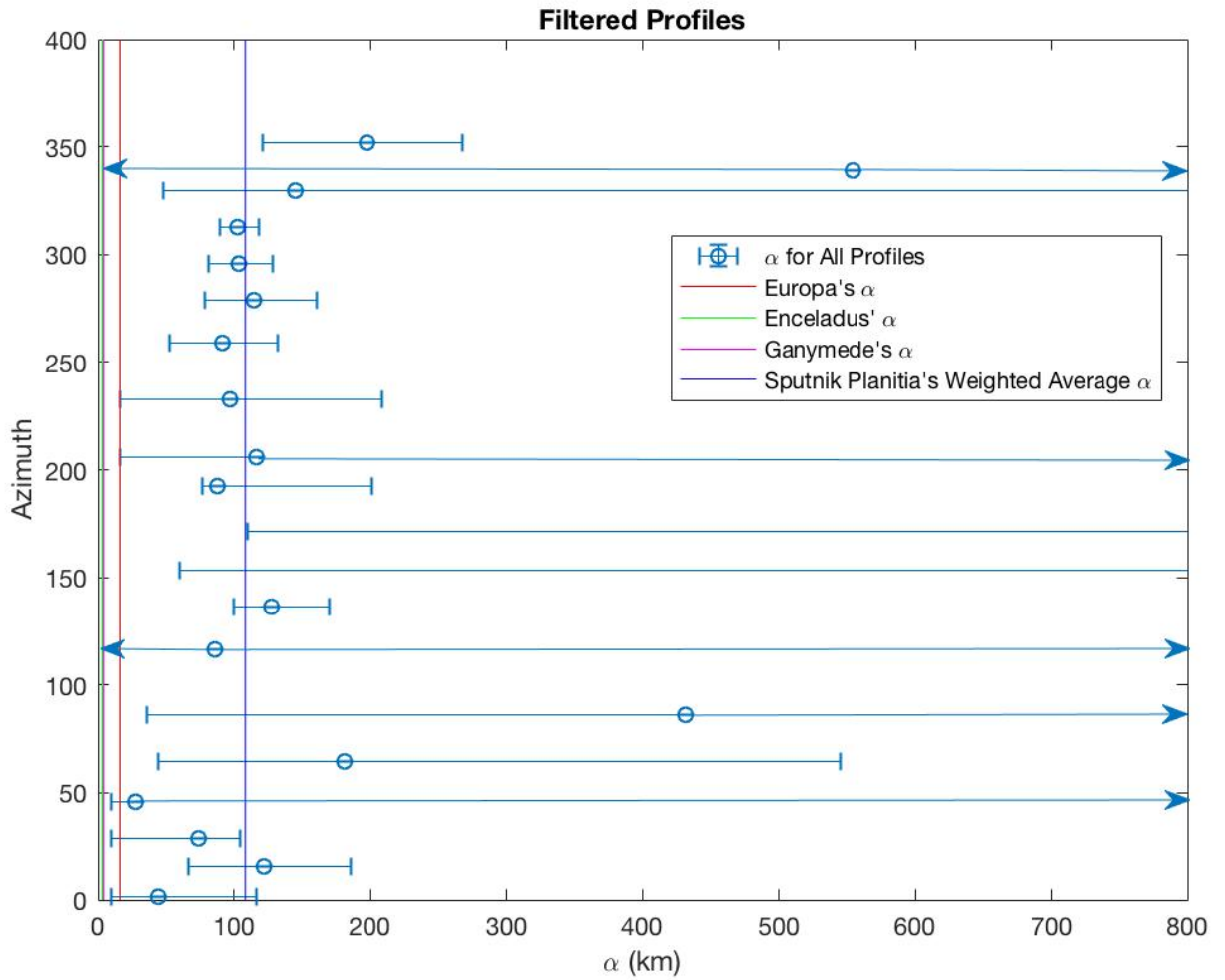
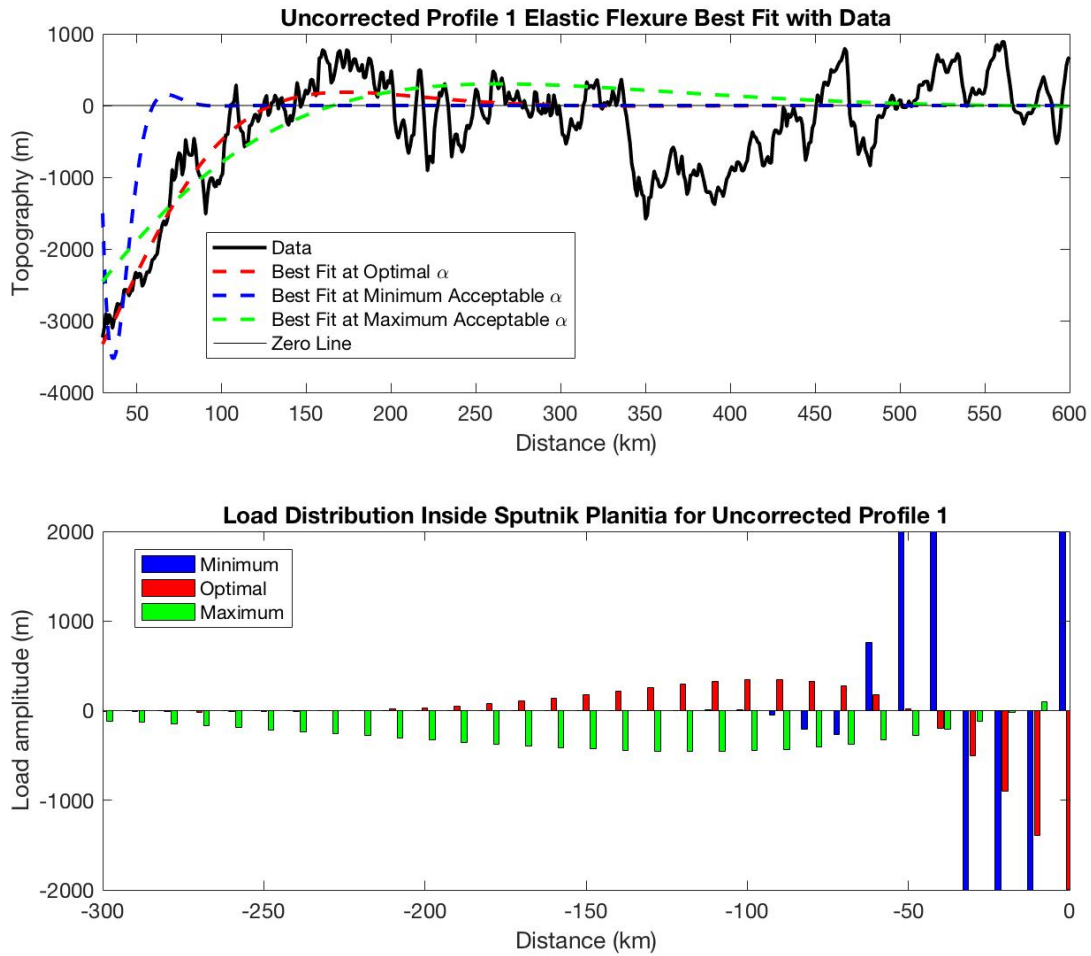
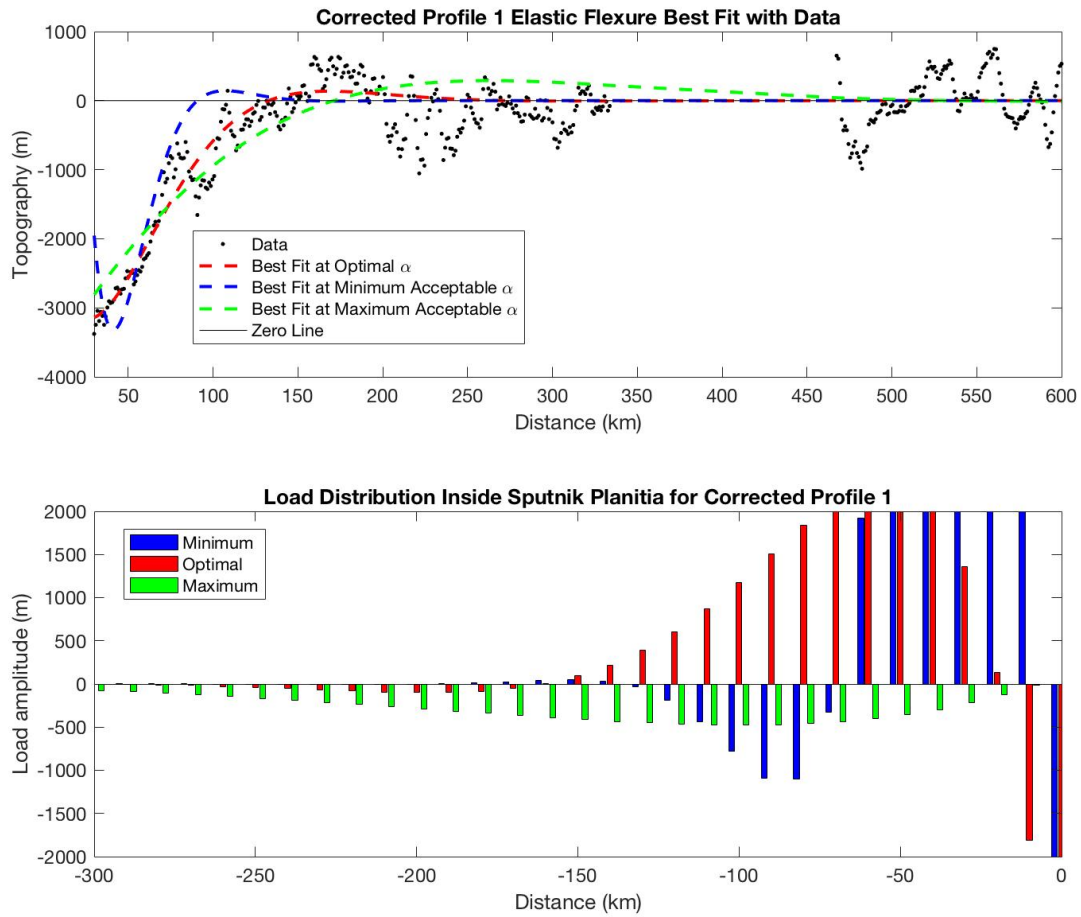


Figure 10. Azimuth versus α plot for profiles with craters and pits, showing that the flexural parameter range for most profiles are between 28 and 554 with two outlier values for Profiles 9 and 10 which are past the bounds of 800 km. The arrows indicate infinite values that are past the bounds of 0 to 800 km. The α values for Europa, Enceladus, and Ganymede are plotted in red, green, and magenta respectively. Sputnik Planitia's weighted average α is in black.

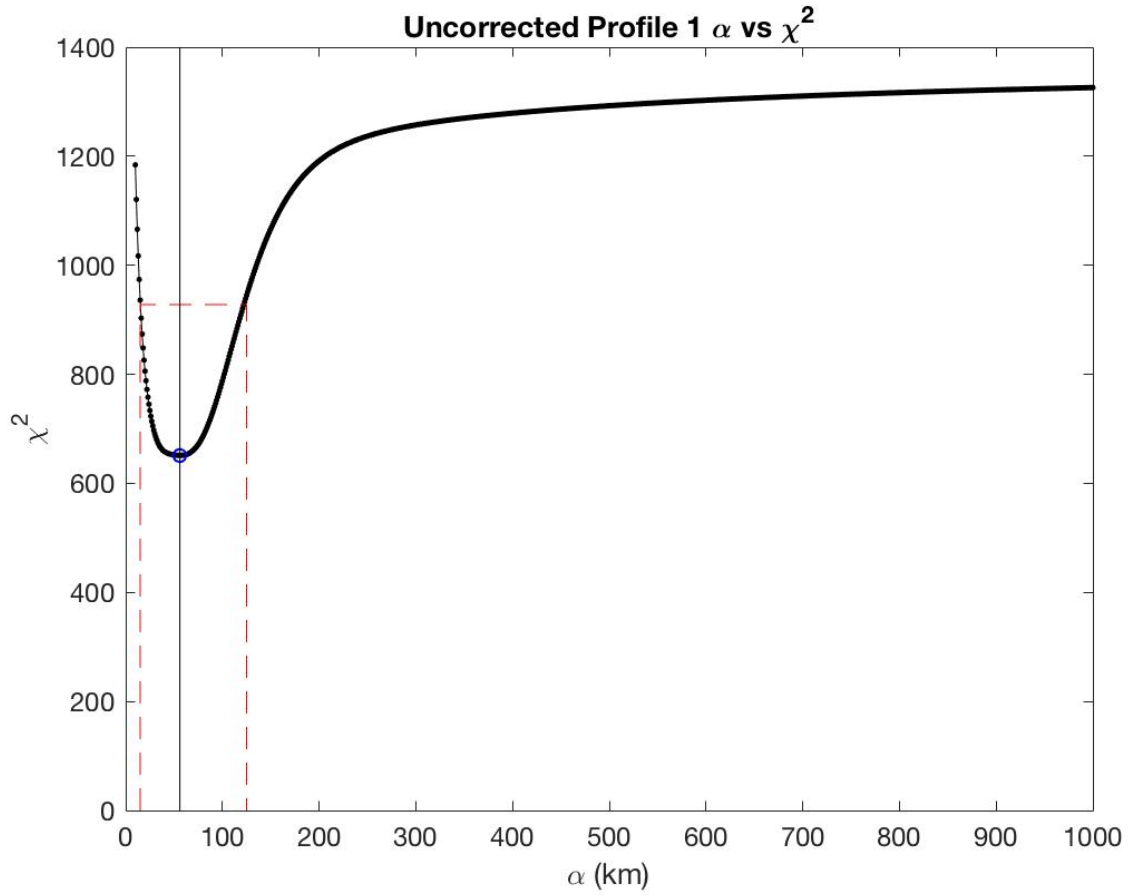
k. Profile 1's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



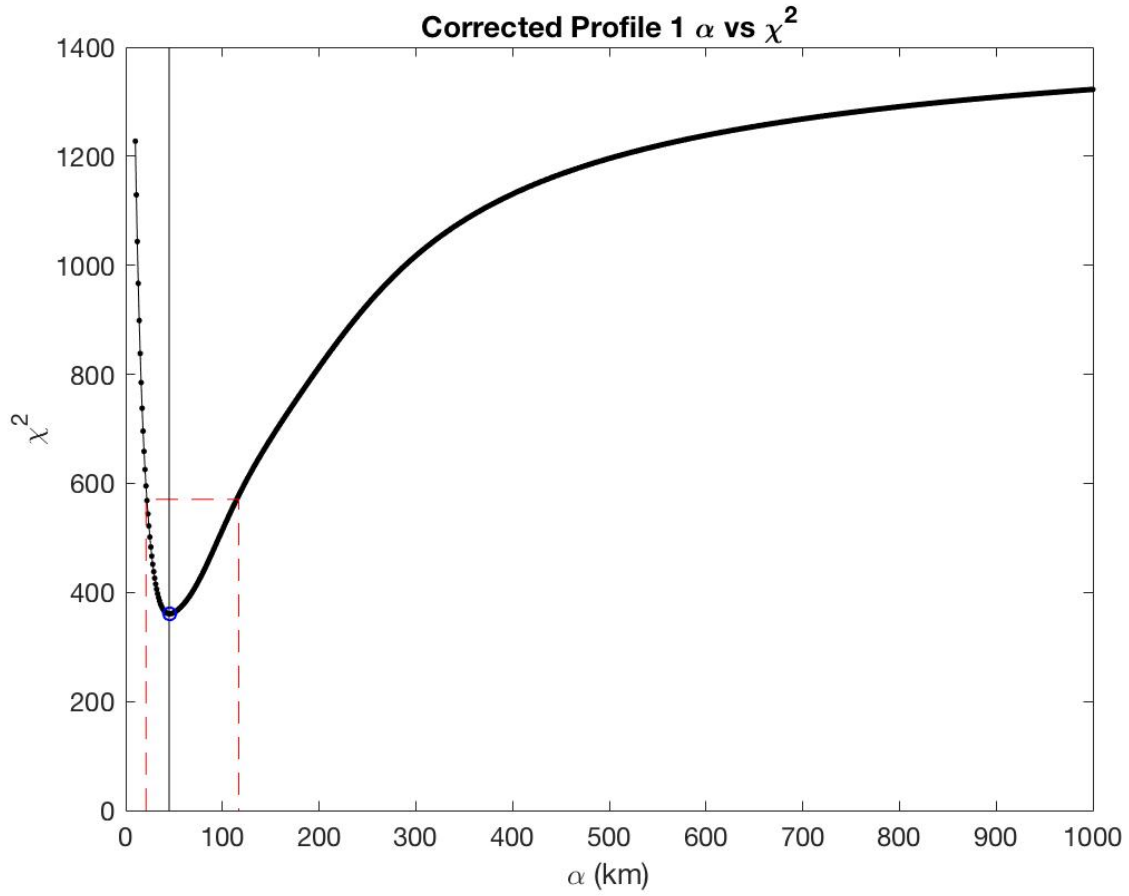
Track 1's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot still has craters present in the data. Lower panel: Profile 1's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .



Track 1's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data. Lower panel: Profile 1's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

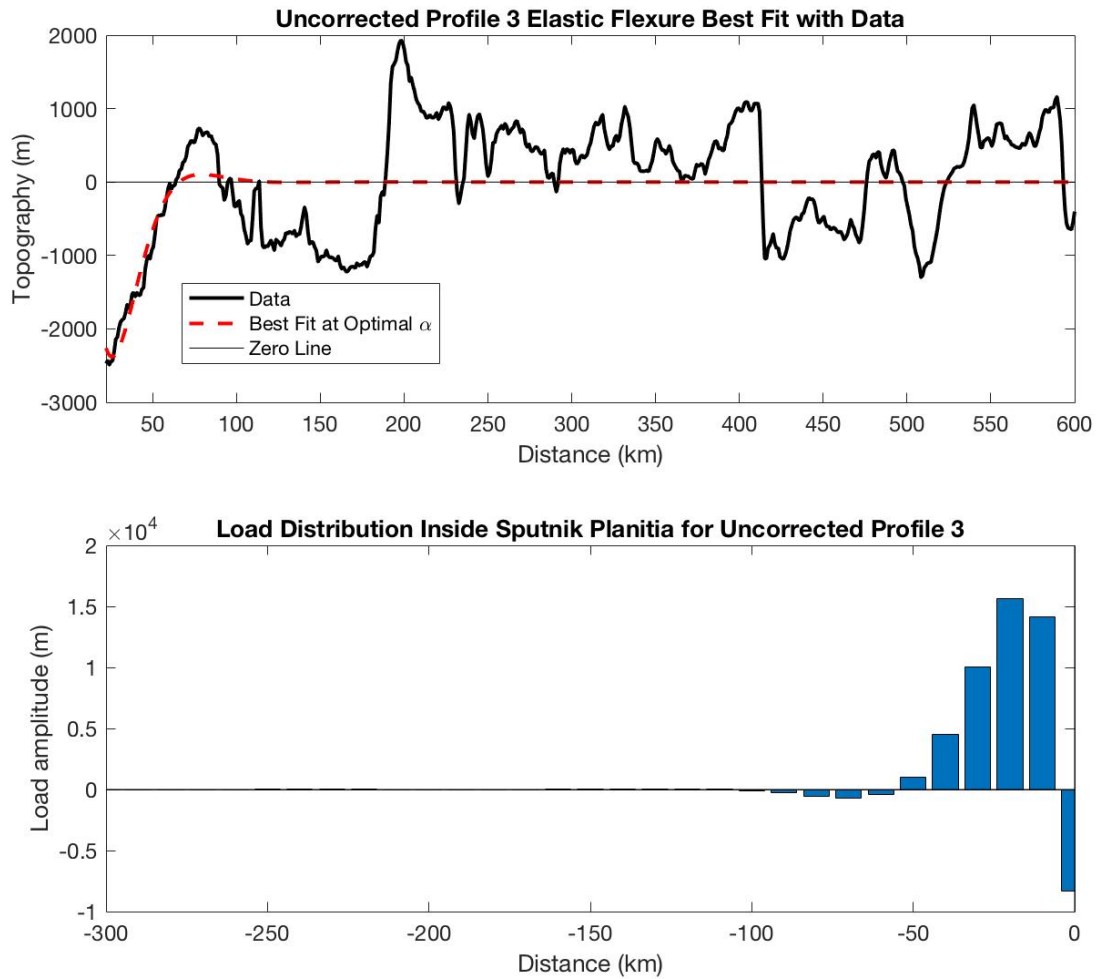


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

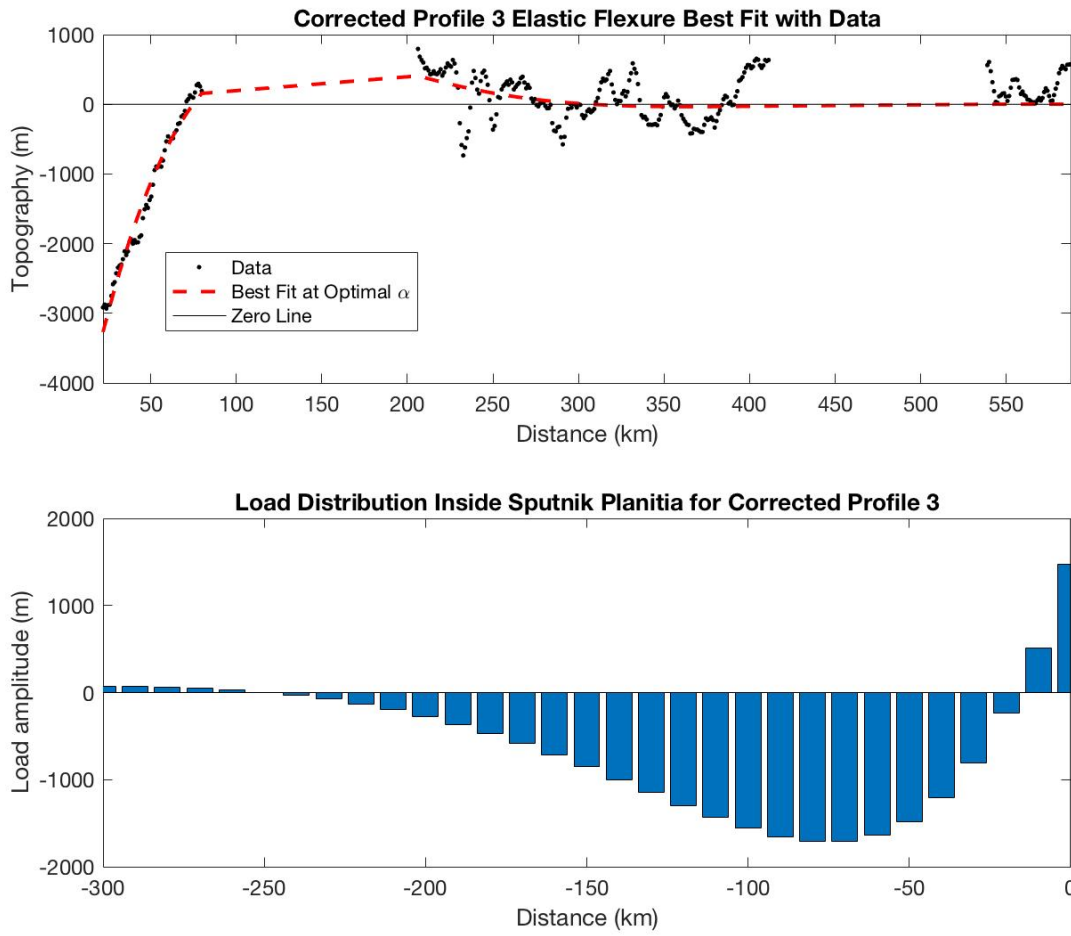


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

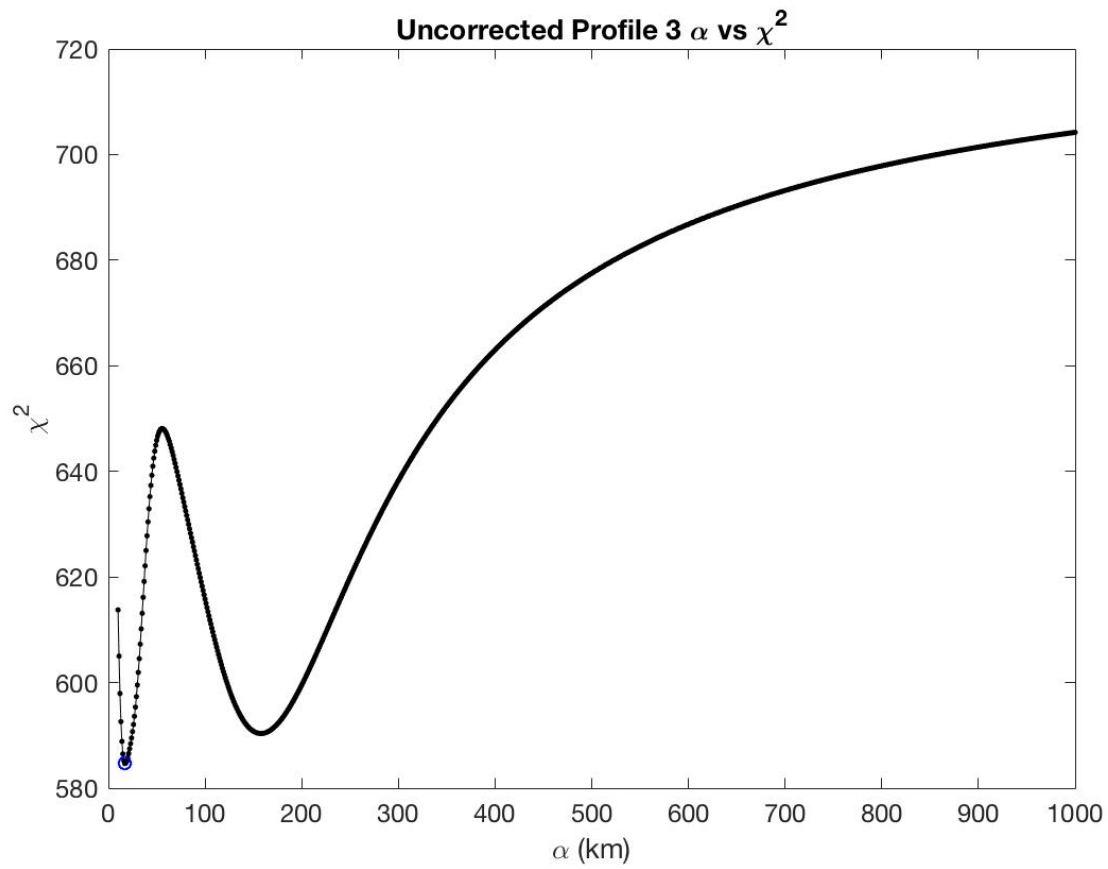
I. Profile 3's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



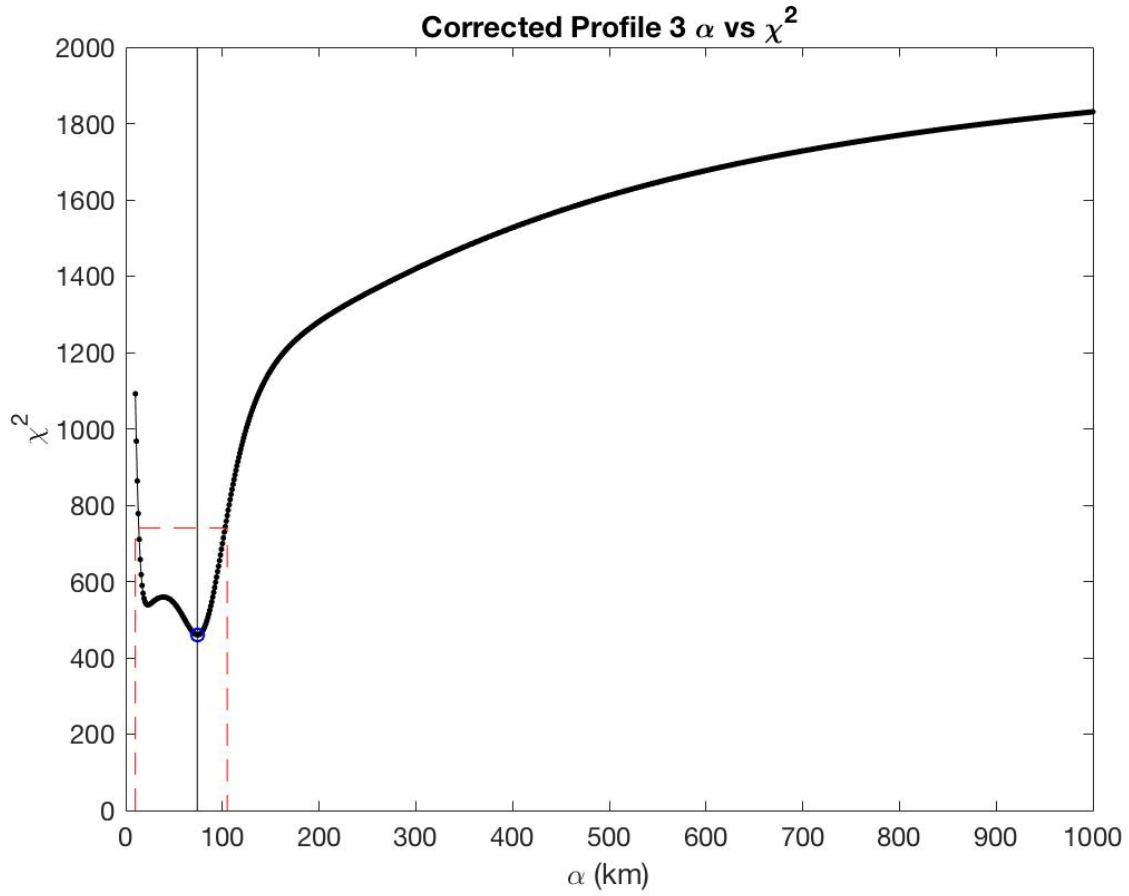
Track 3's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized. This plot still has craters present in the data. Lower panel: Profile 3's corresponding load amplitudes of the best fit that corresponds to the acceptable range of α .



Track 3's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized. This plot has craters removed in the data. Lower panel: Profile 3's corresponding load amplitudes of the best fit that corresponds to the acceptable range of α .

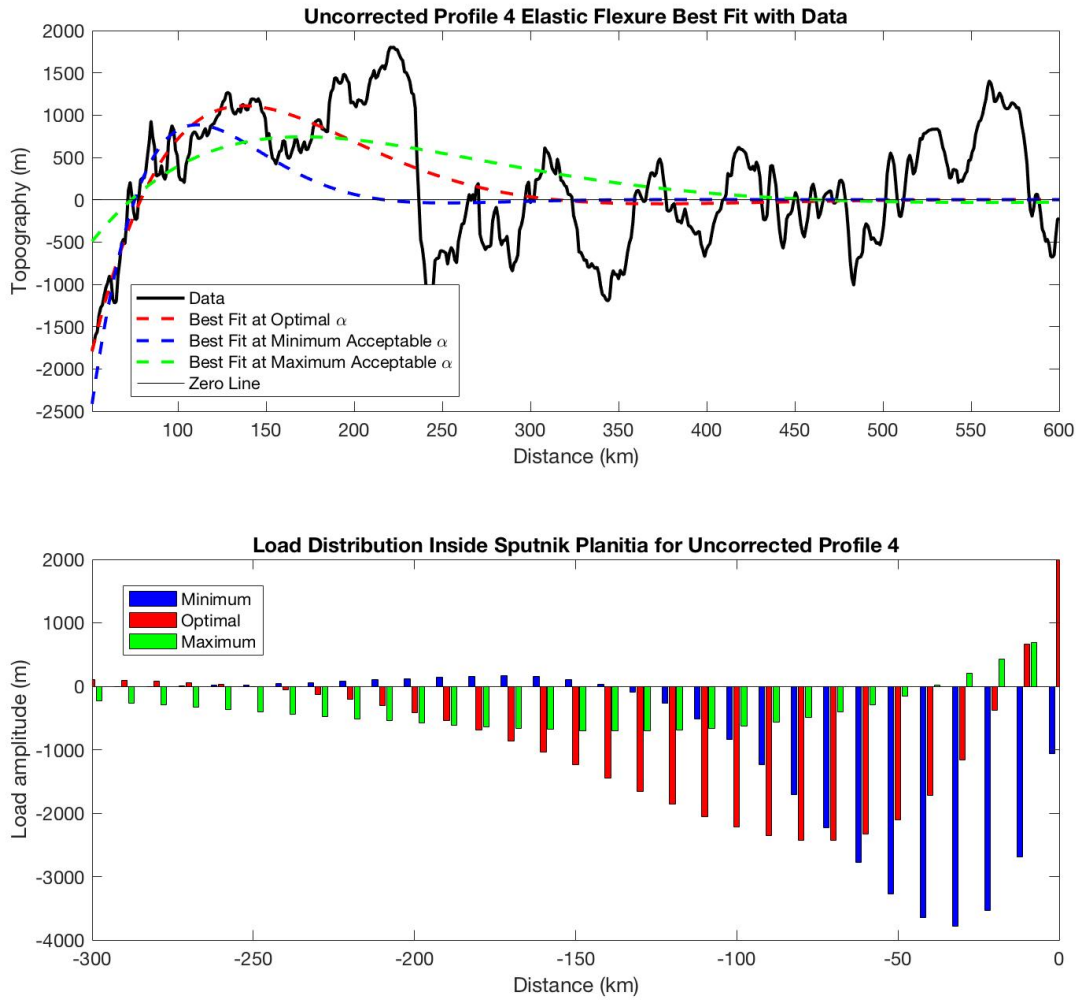


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

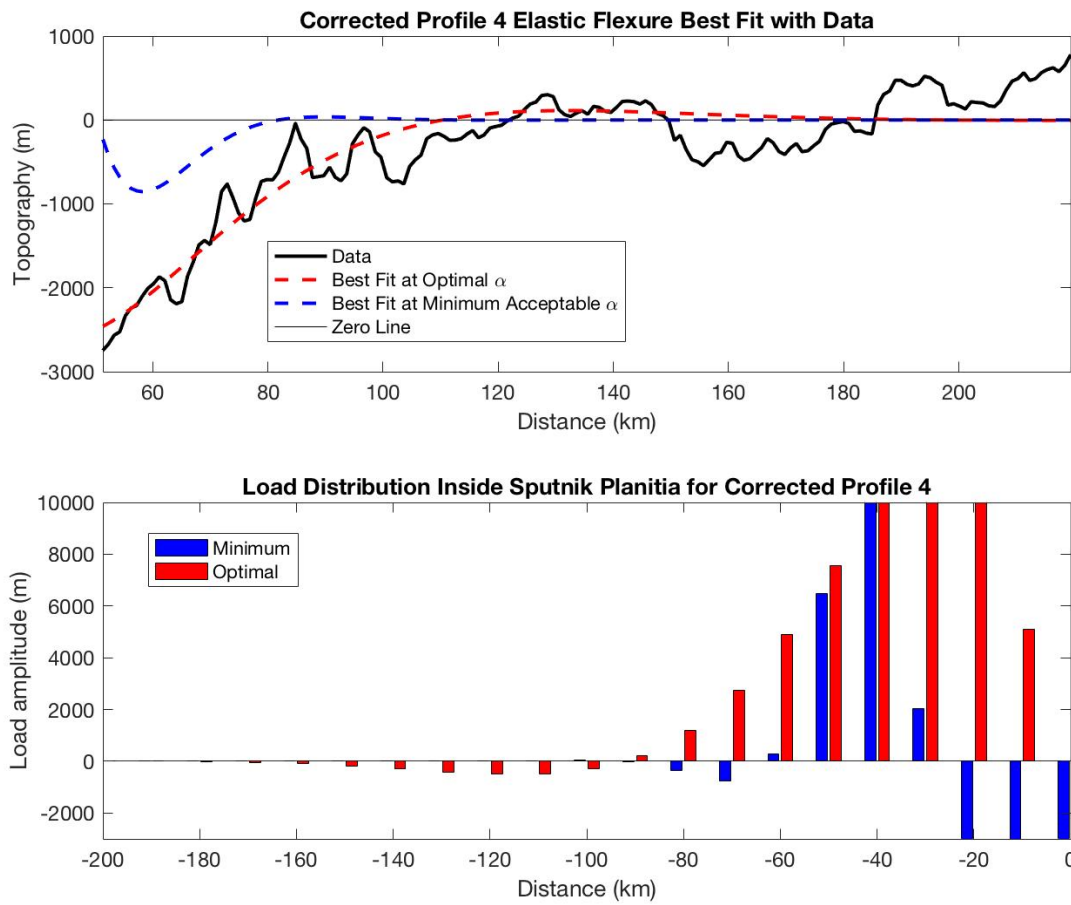


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

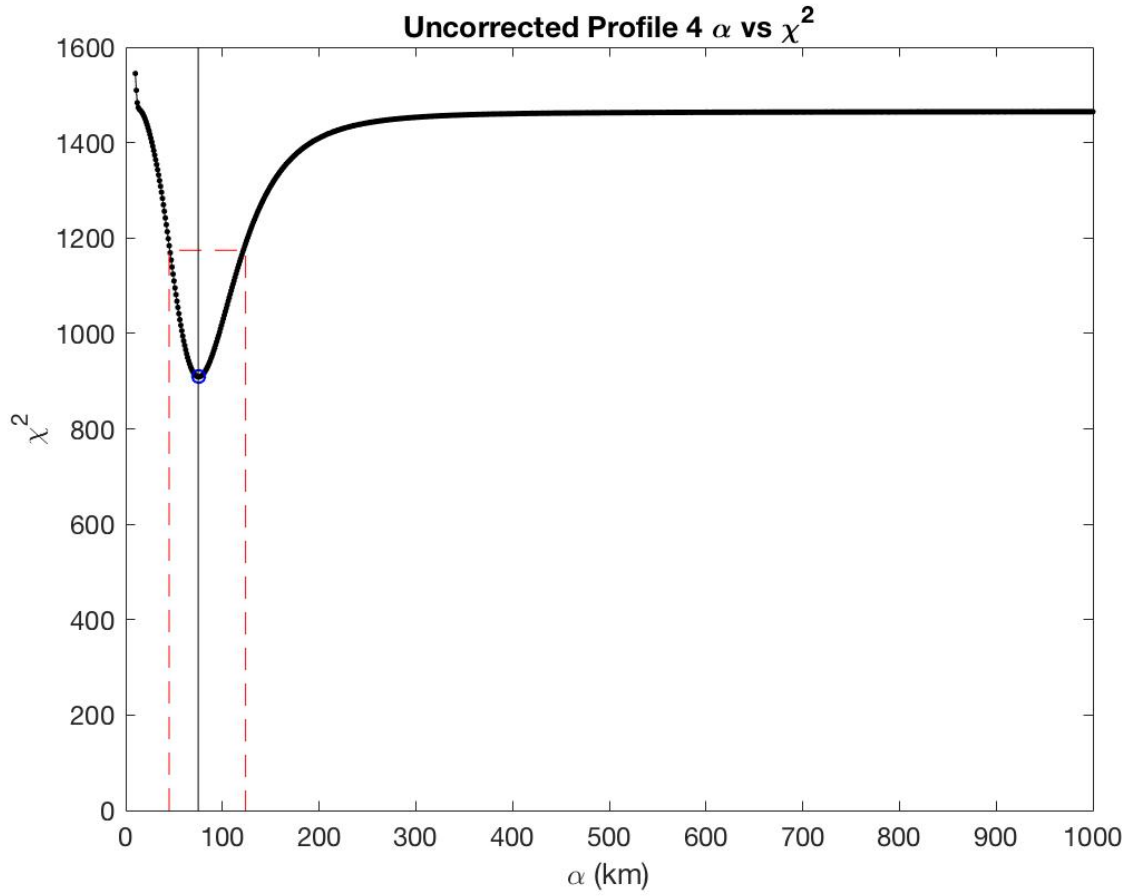
m. Profile 4's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



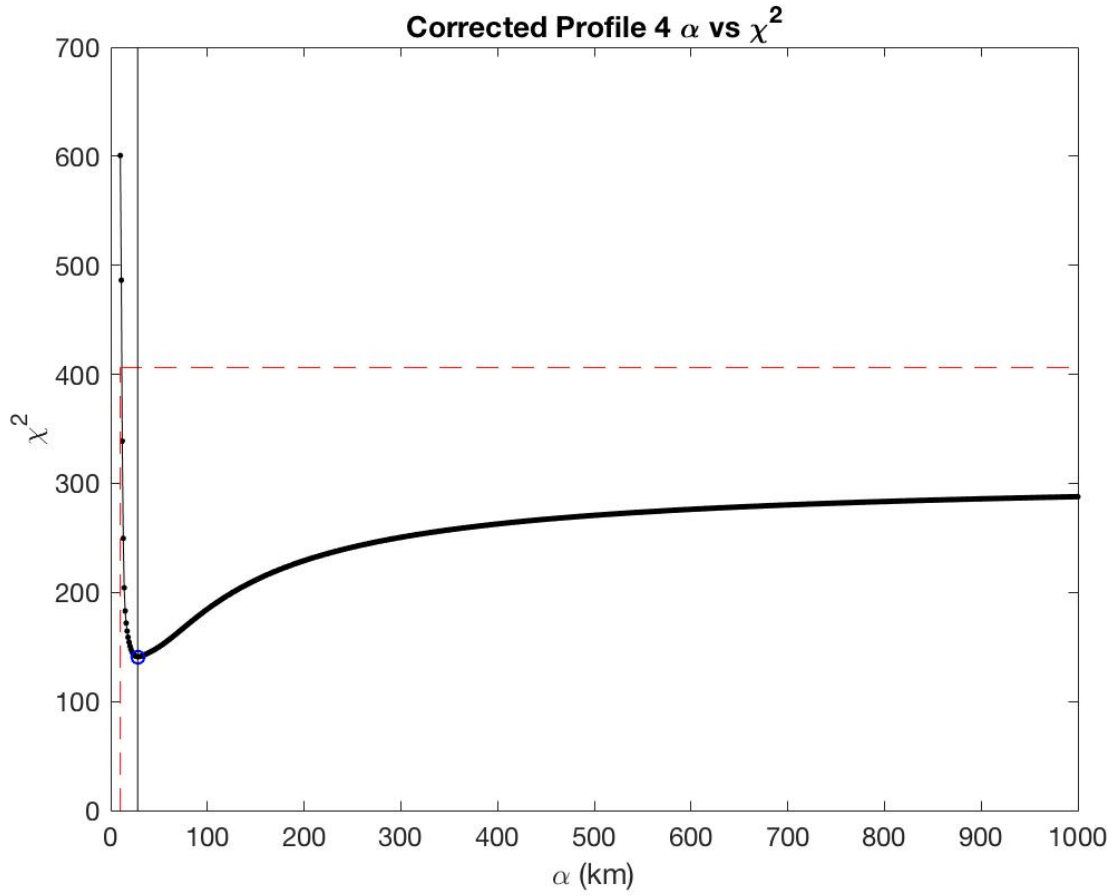
Track 4's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters still in the data. Lower panel: Profile 4's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .



Track 4's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized, and the blue dashed line is the best fit with the minimum acceptable. This plot has craters removed in the data. Lower panel: Profile 4's corresponding load amplitudes of the two best fits that correspond to the acceptable range of α .

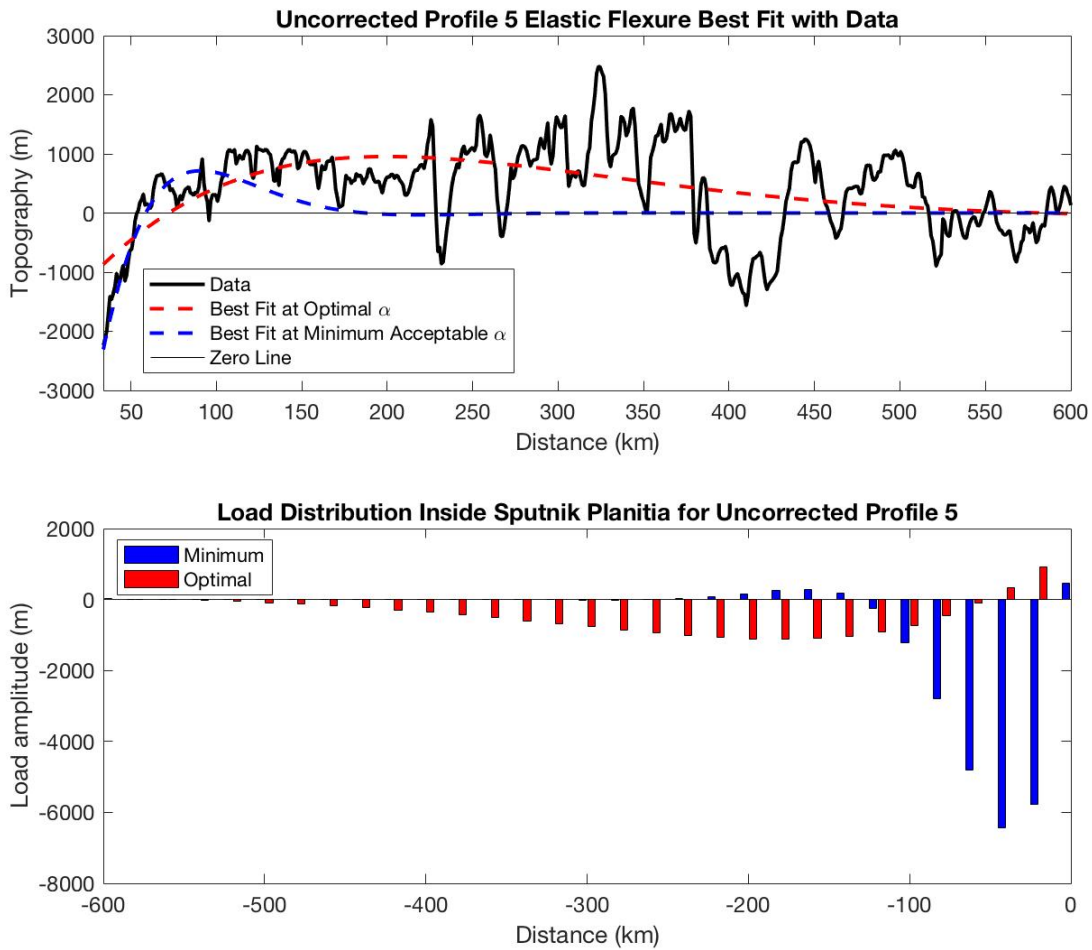


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

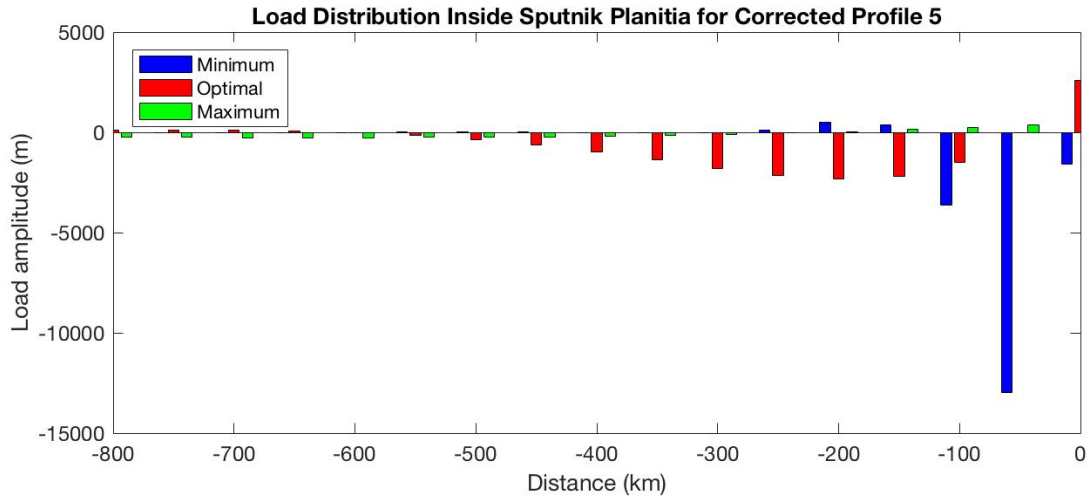
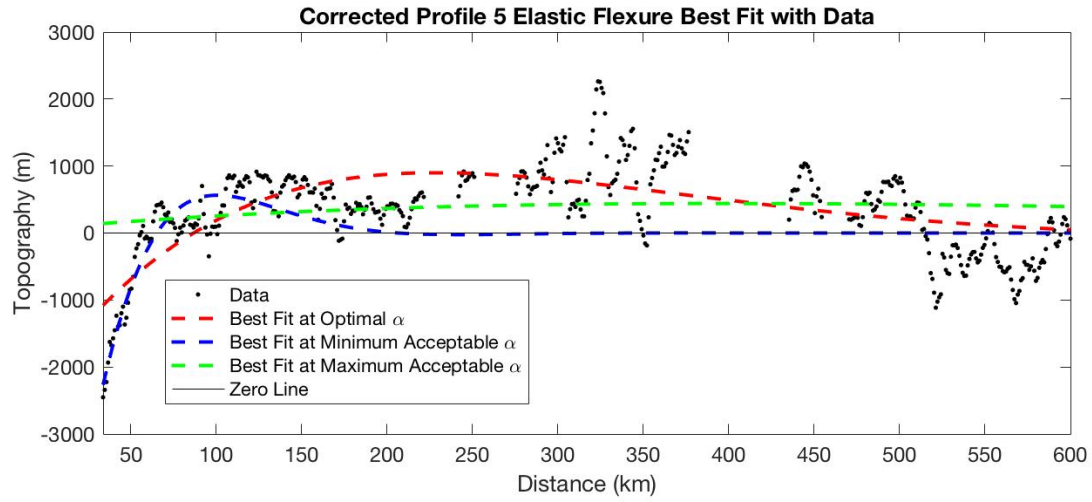


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

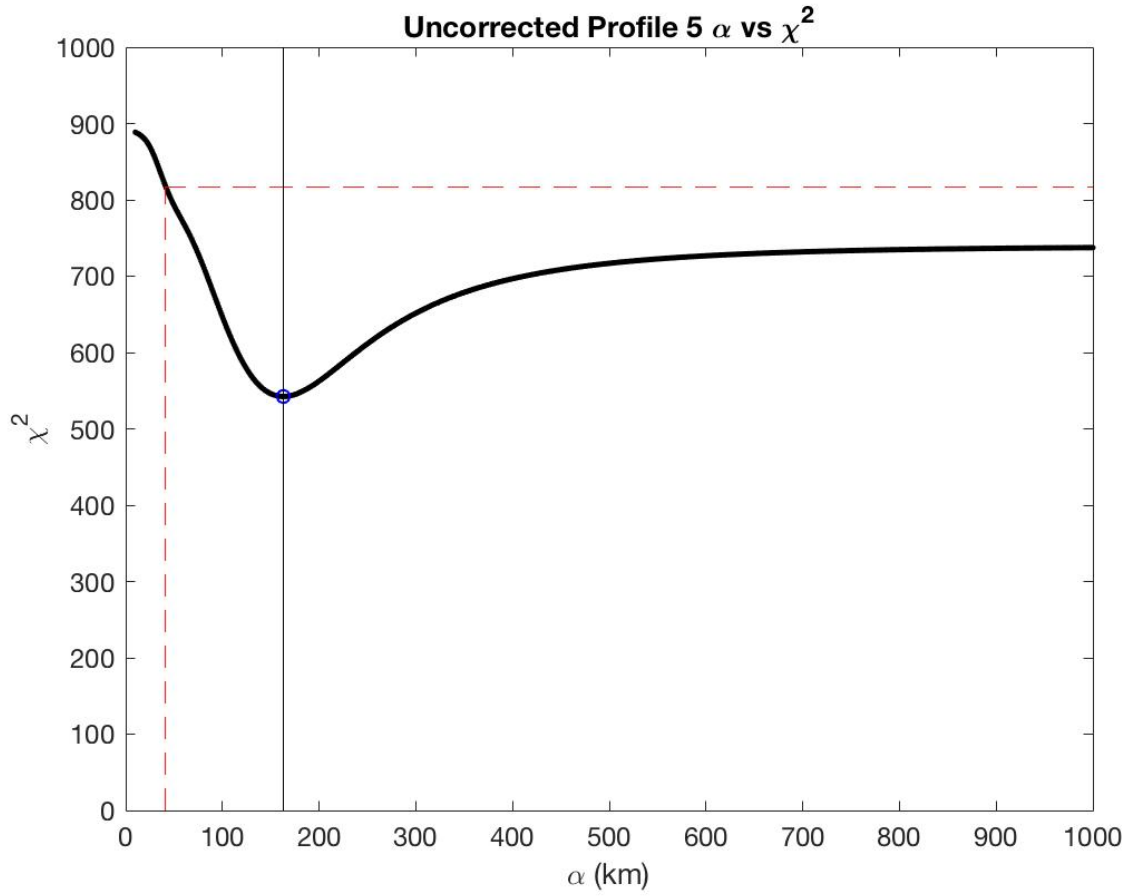
n. Profile 5's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



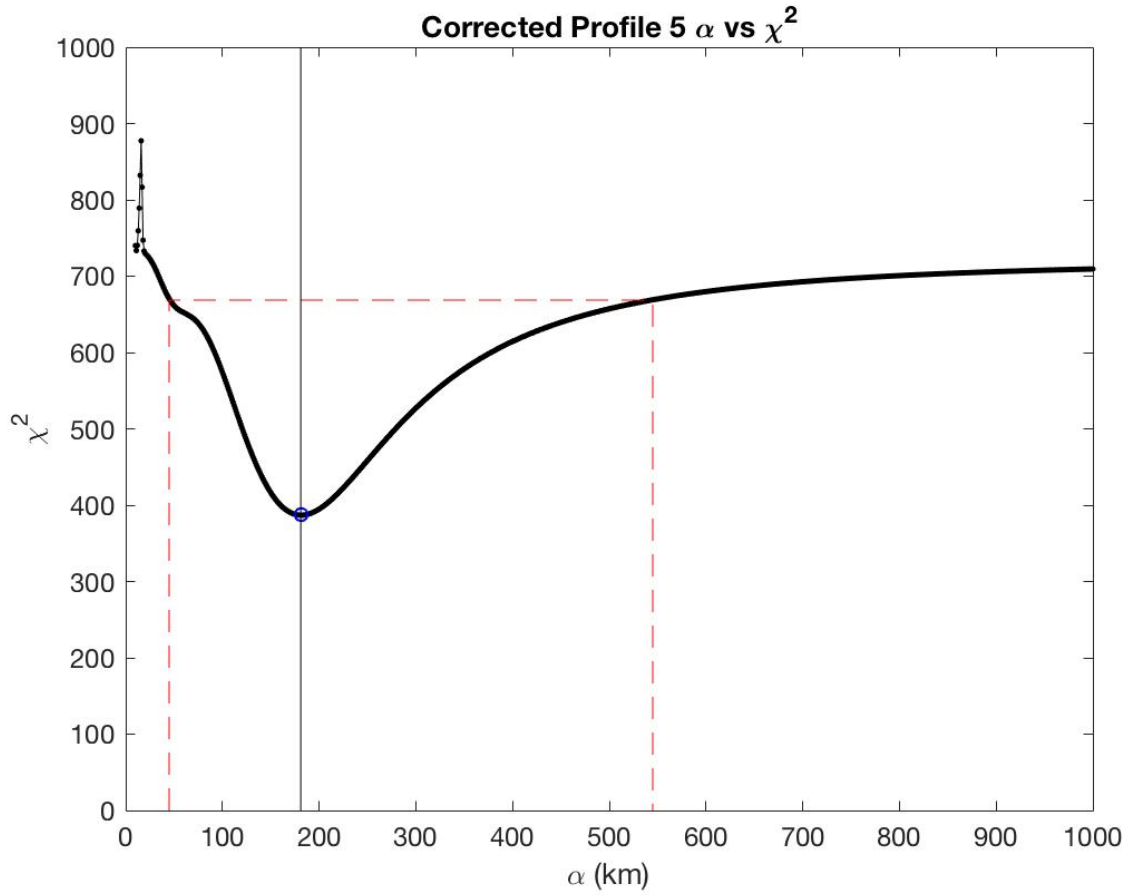
Track 5's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized, and the blue dashed line is the best fit with the minimum acceptable α . This plot has craters present in the data. Lower panel: Profile 5's corresponding load amplitudes of the two best fits that correspond to the acceptable range of α .



Track 5's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data. Lower panel: Profile 5's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

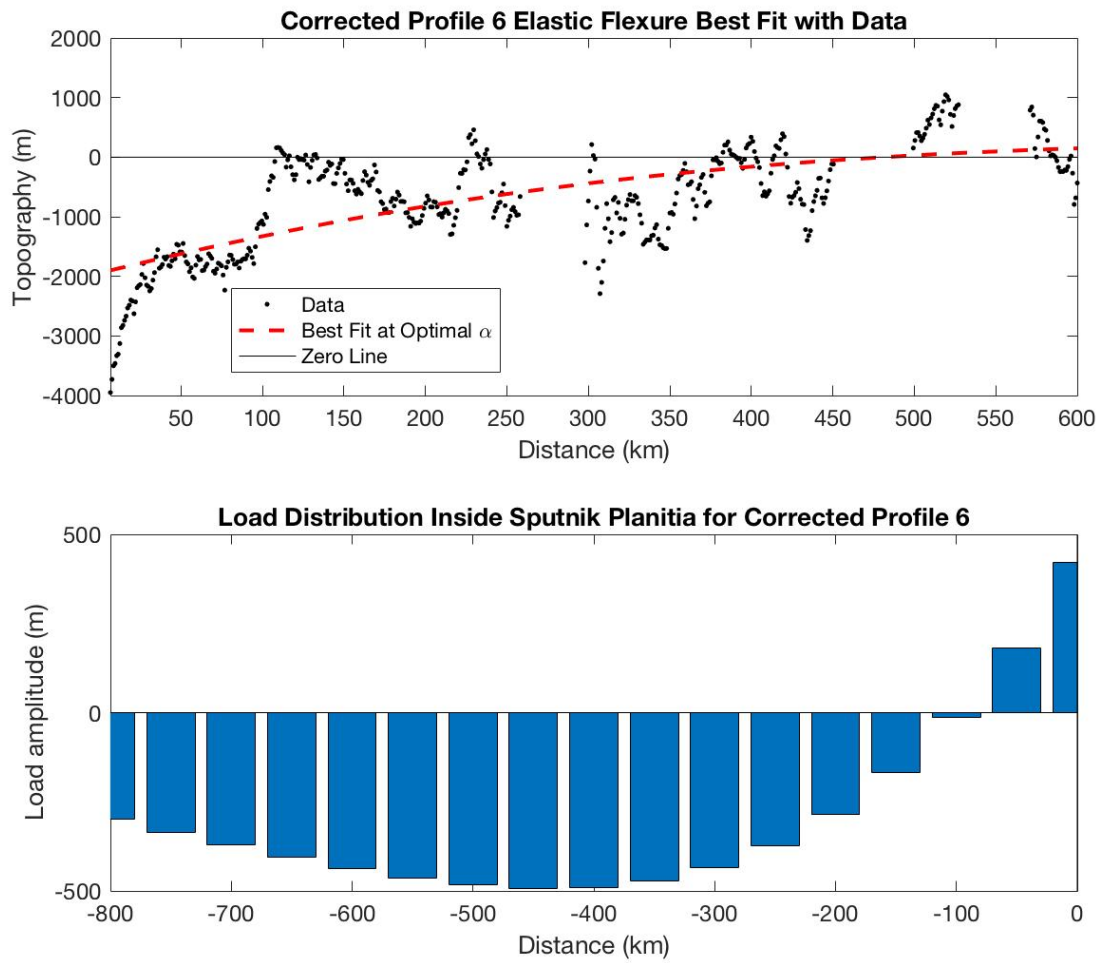


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

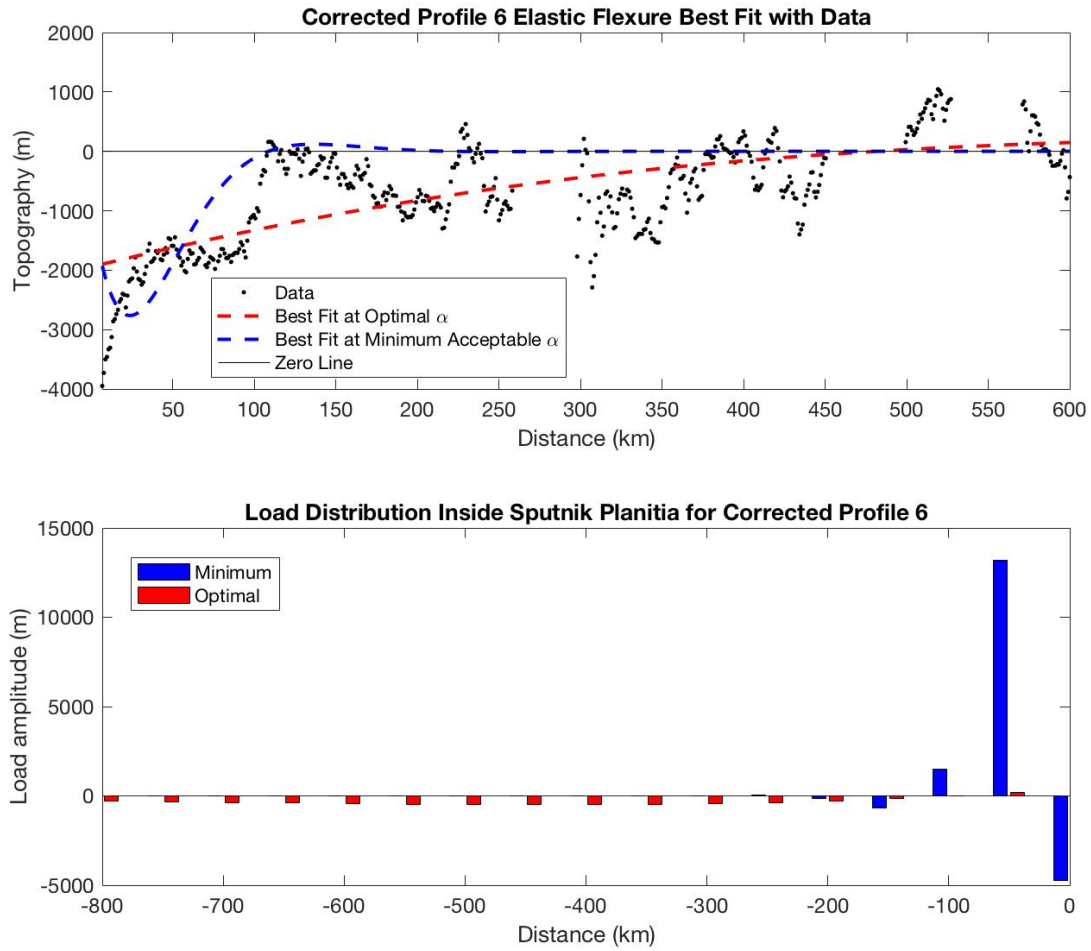


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

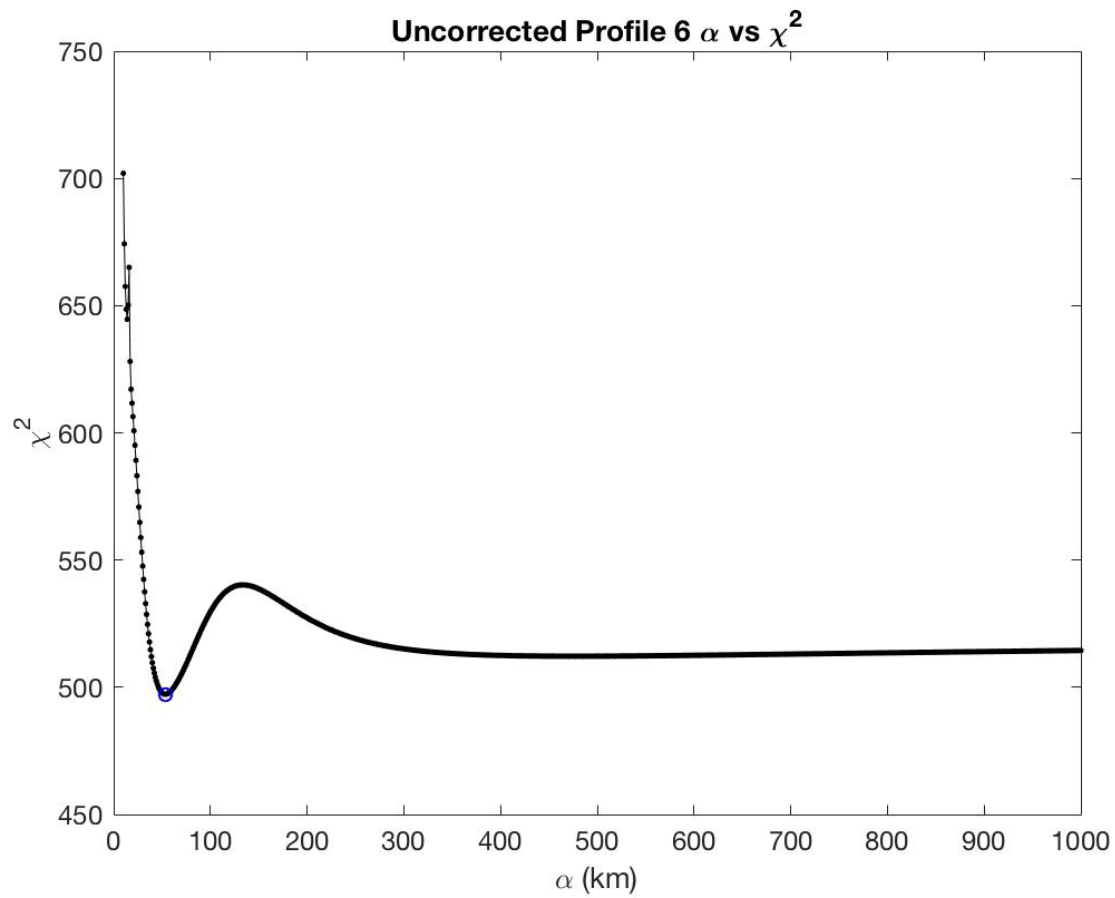
o. Profile 6's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



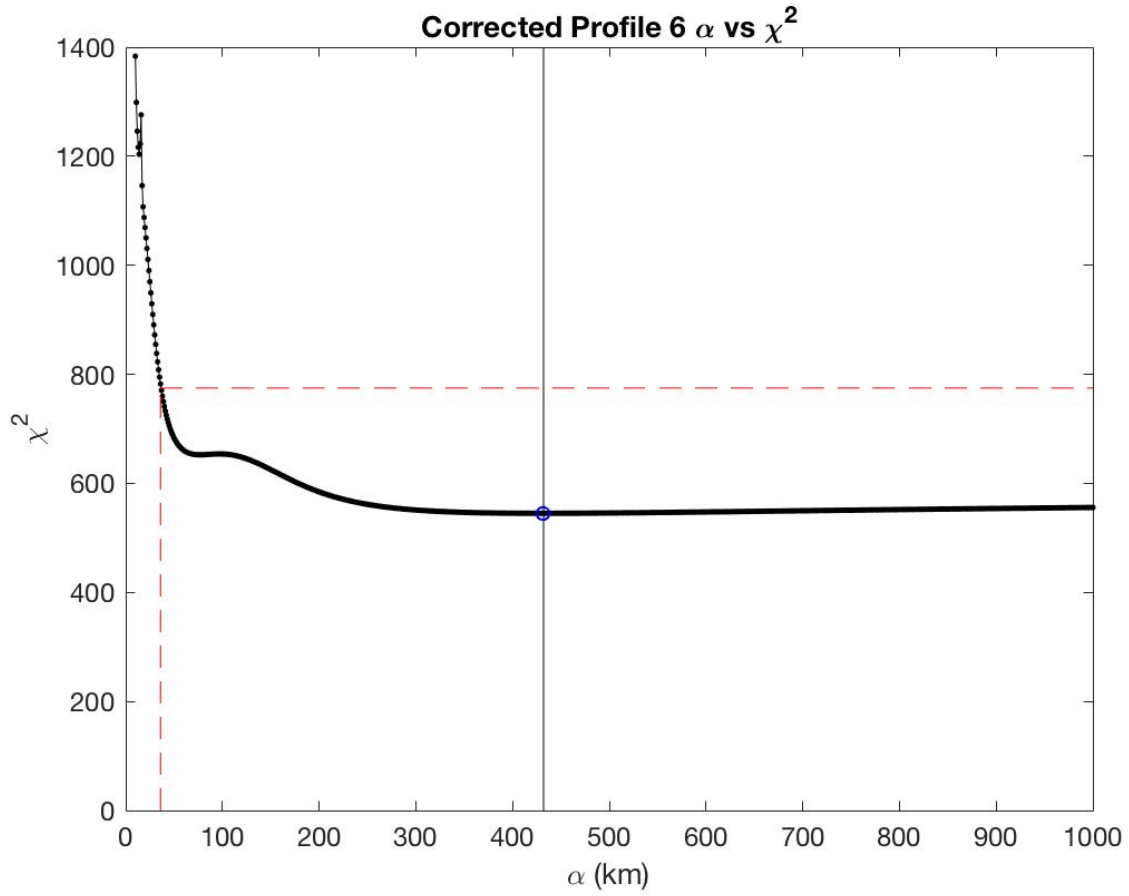
Track 6's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized. Lower panel: Profile 6's corresponding load amplitudes of the best fit that corresponds to the acceptable range of α .



Track 6's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized, and the blue dashed line is the best fit with the minimum acceptable α . This plot has craters removed in the data. Lower panel: Profile 6's corresponding load amplitudes of the two best fits that correspond to the acceptable range of α .

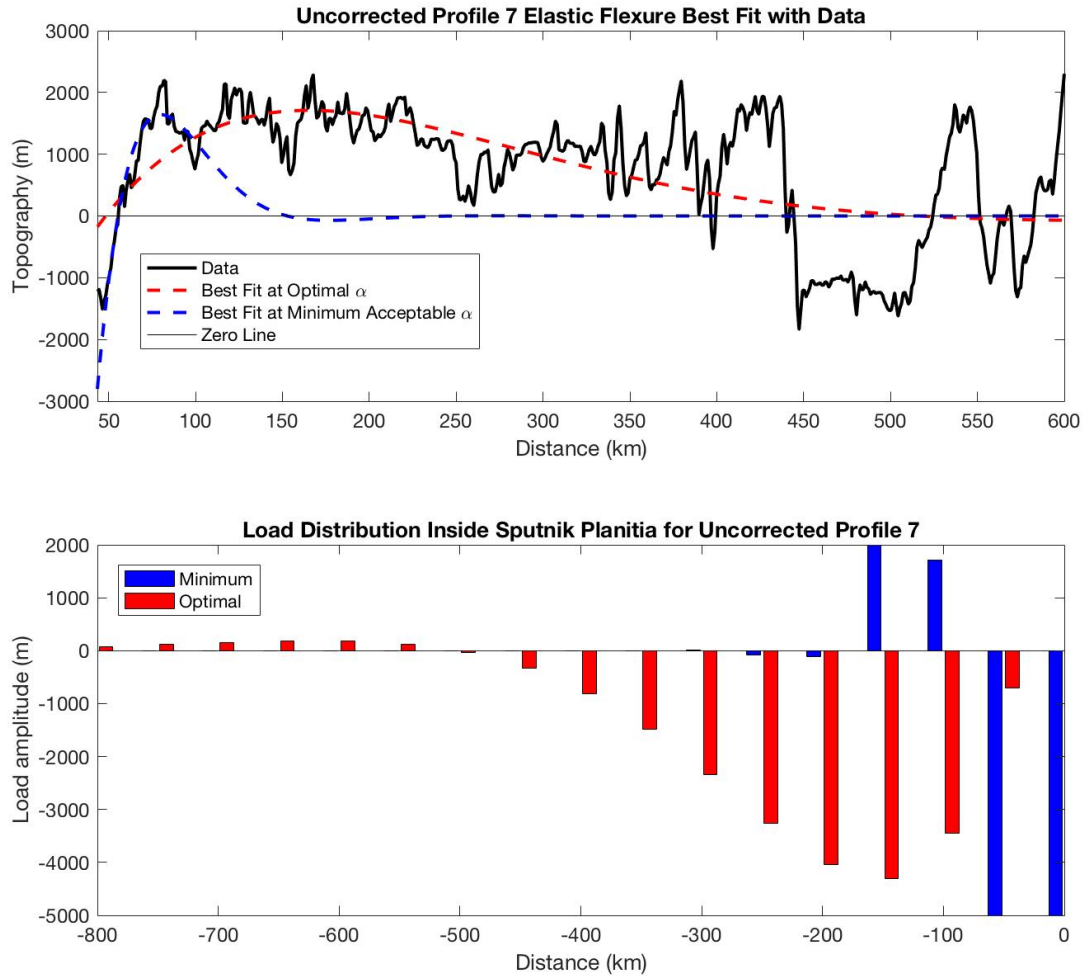


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

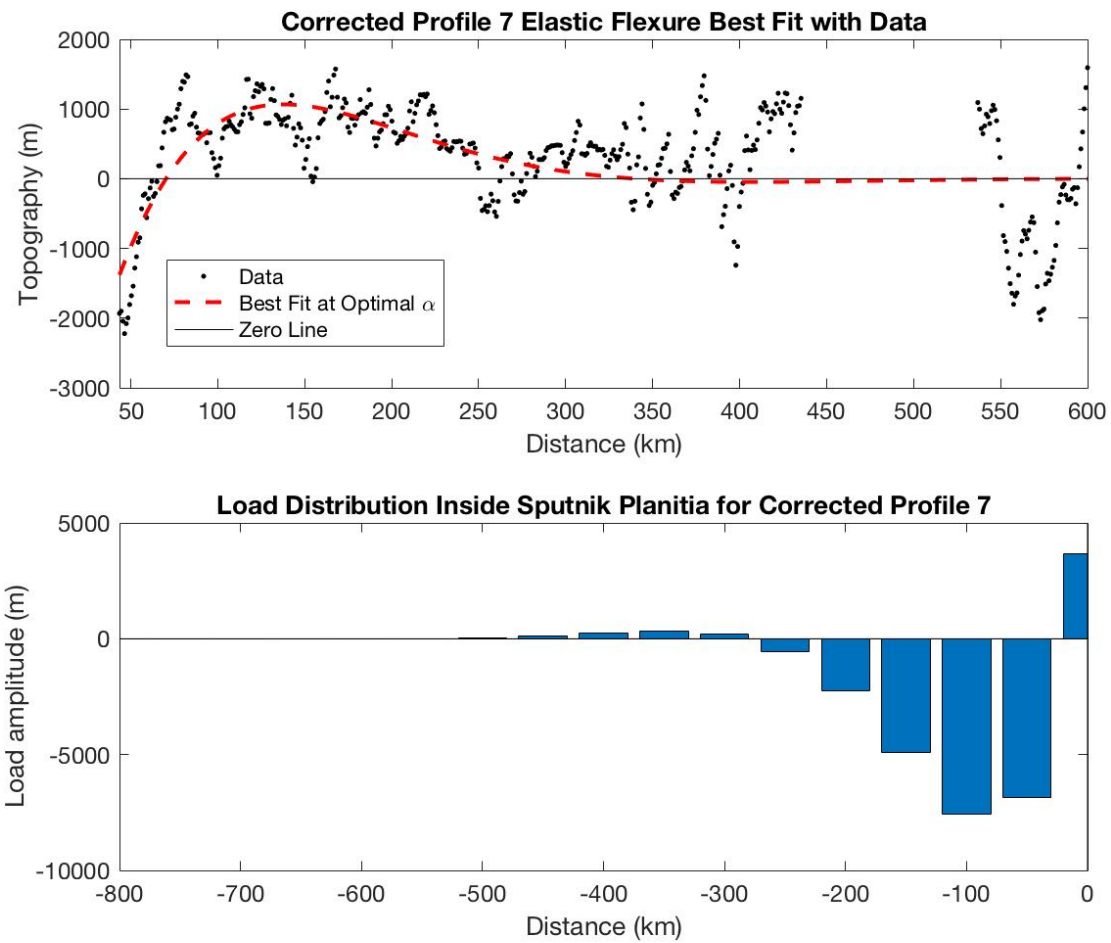


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

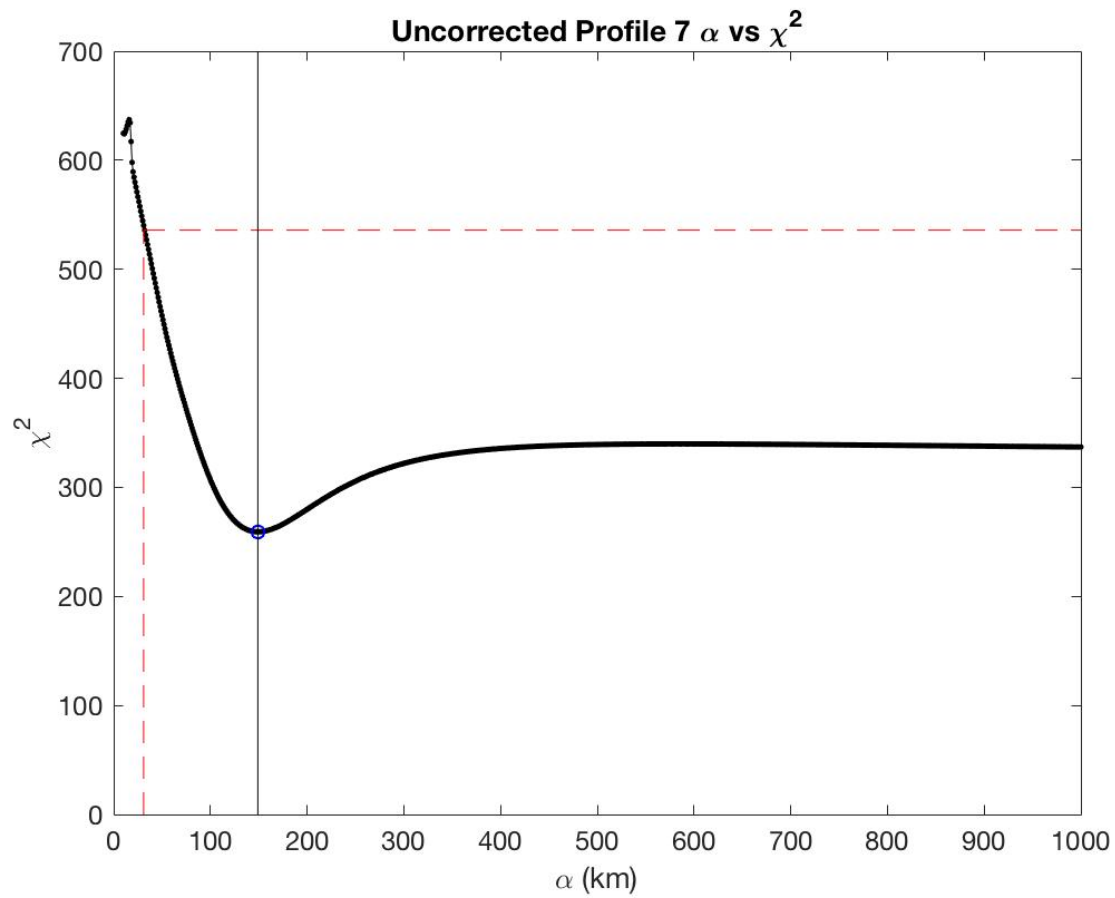
p. Profile 7's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



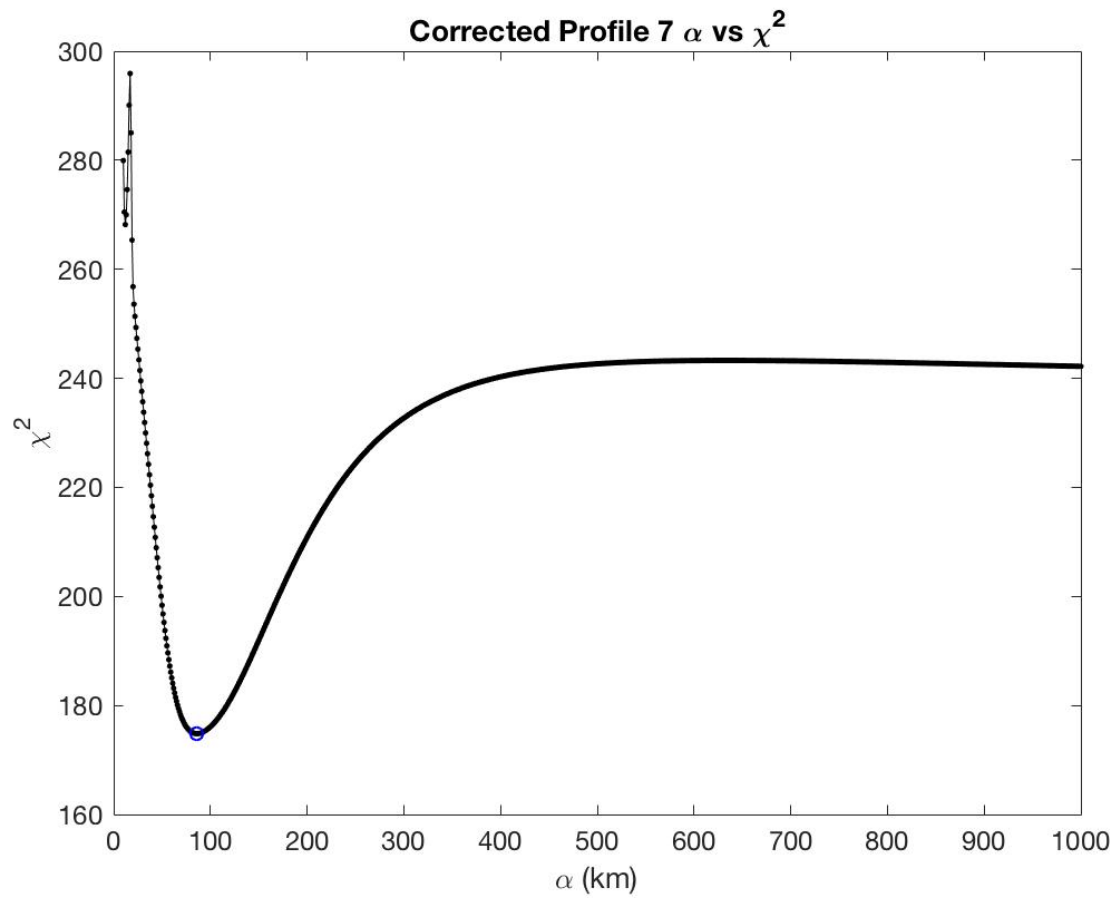
Track 7's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized, and the blue dashed line is the best fit with the minimum acceptable α . This plot has craters still in the data. Lower panel: Profile 7's corresponding load amplitudes of the two best fits that correspond to the acceptable range of α .



Track 7's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized. This plot has craters removed in the data. Lower panel: Profile 7's corresponding load amplitudes of the best fit that corresponds to the acceptable range of α .

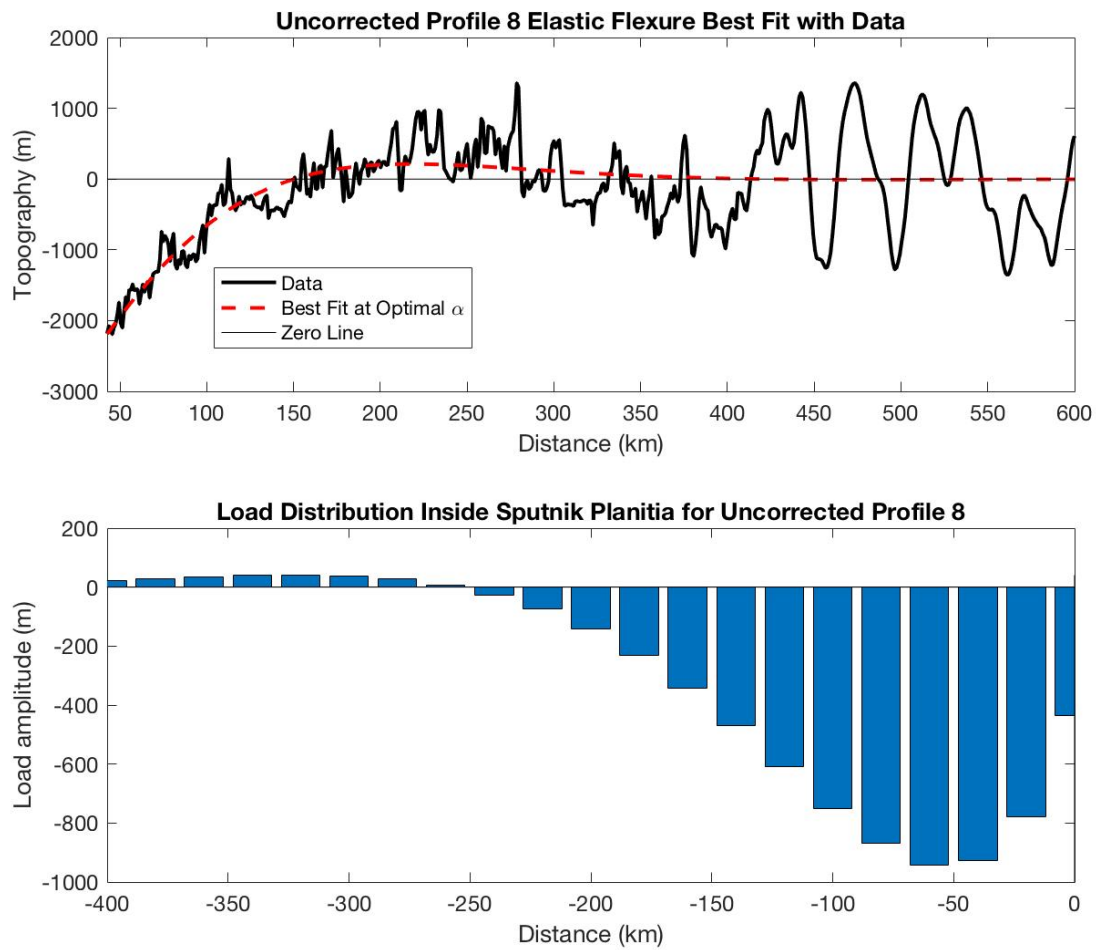


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

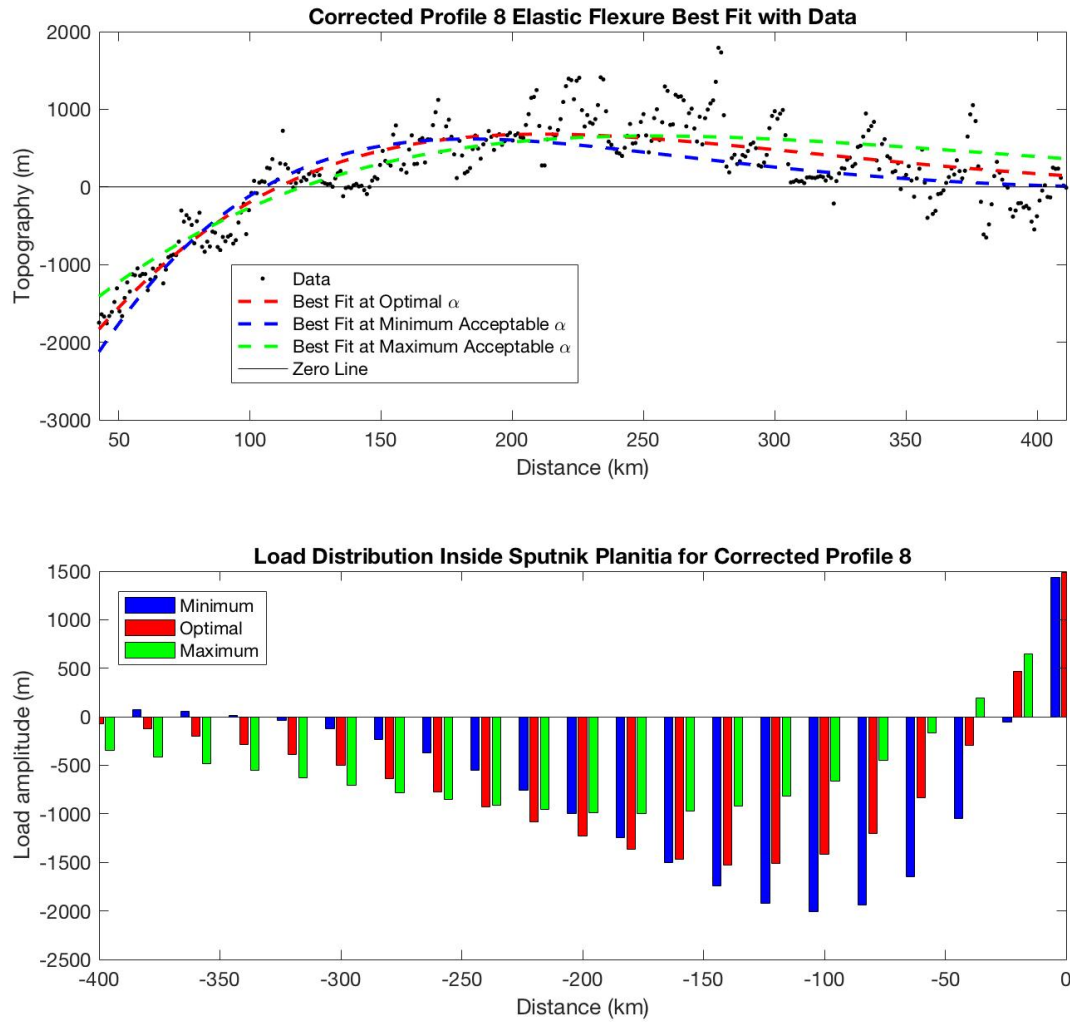


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

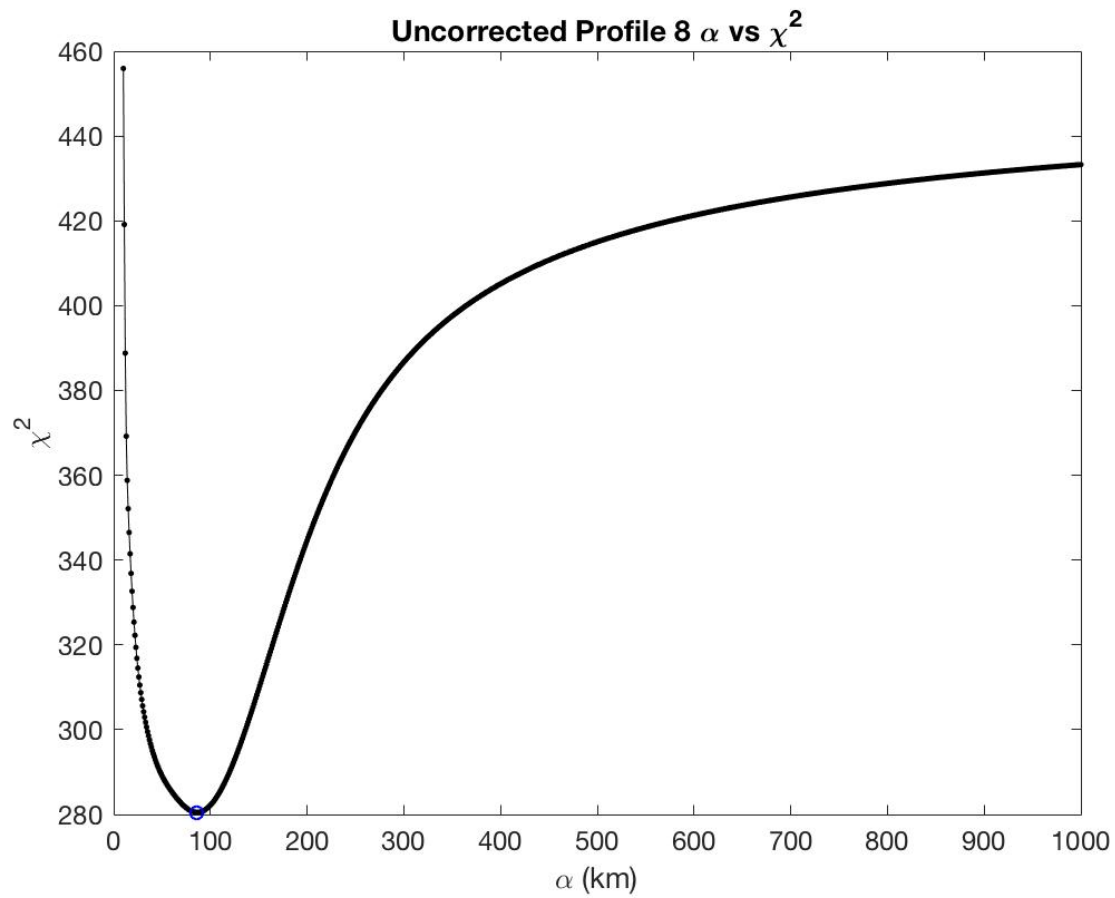
q. Profile 8's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



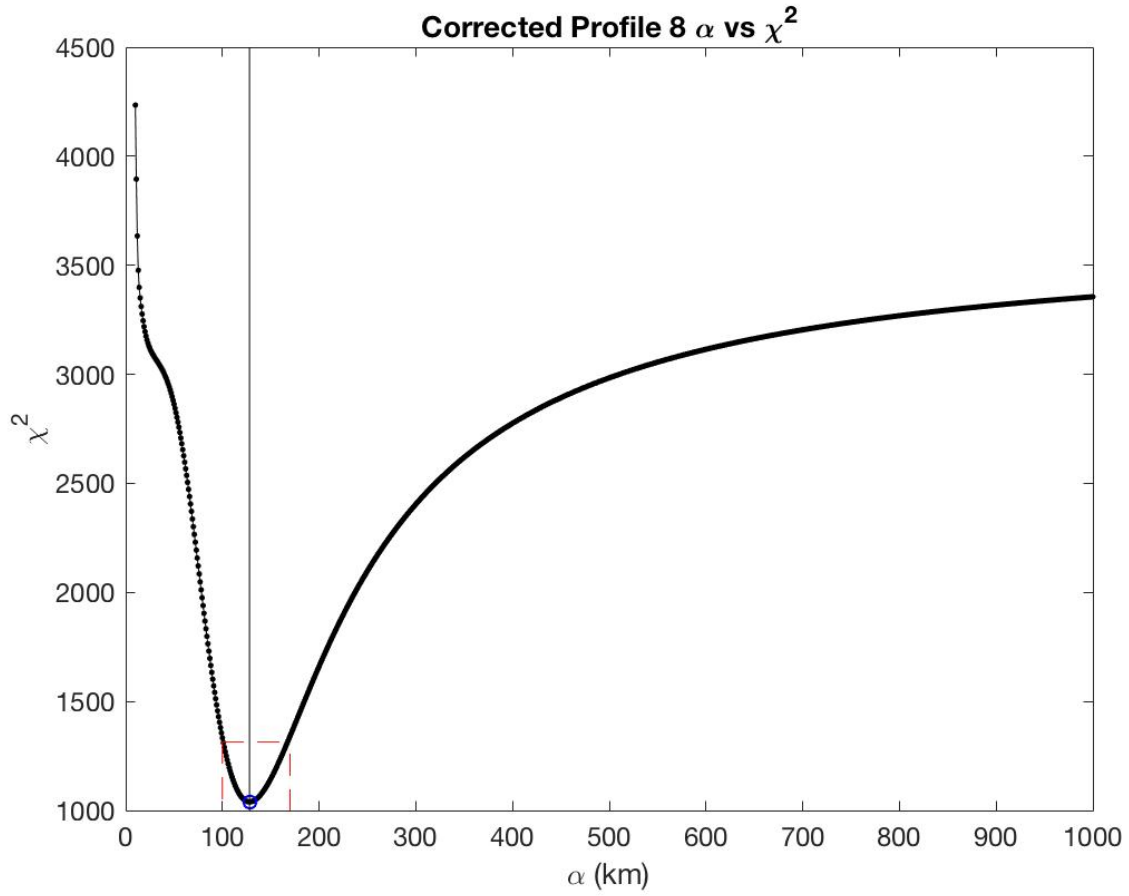
Track 8's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized. This plot has craters still in the data. Lower panel: Profile 8's corresponding load amplitudes of the best fit that corresponds to the acceptable range of α .



Track 8's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data. Lower panel: Profile 8's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

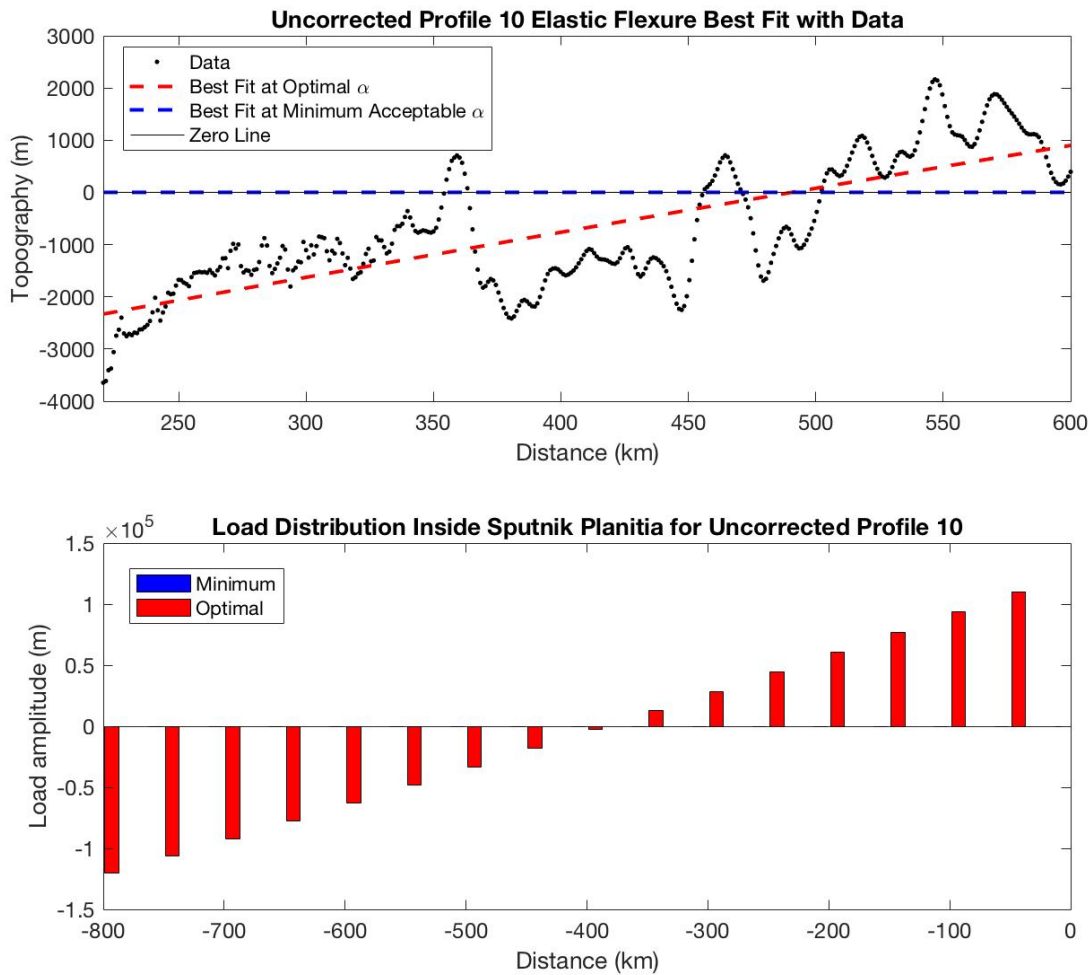


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

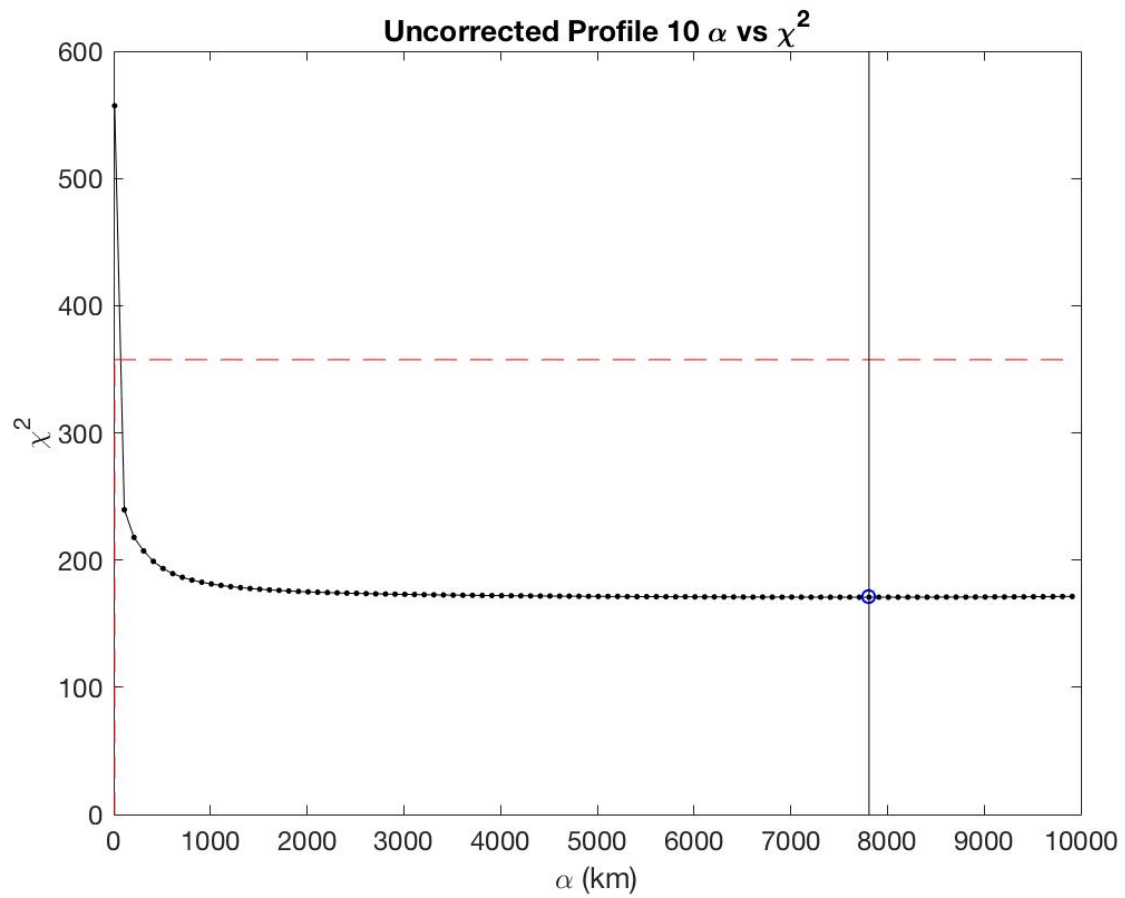


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

r. **Profile 10's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot**

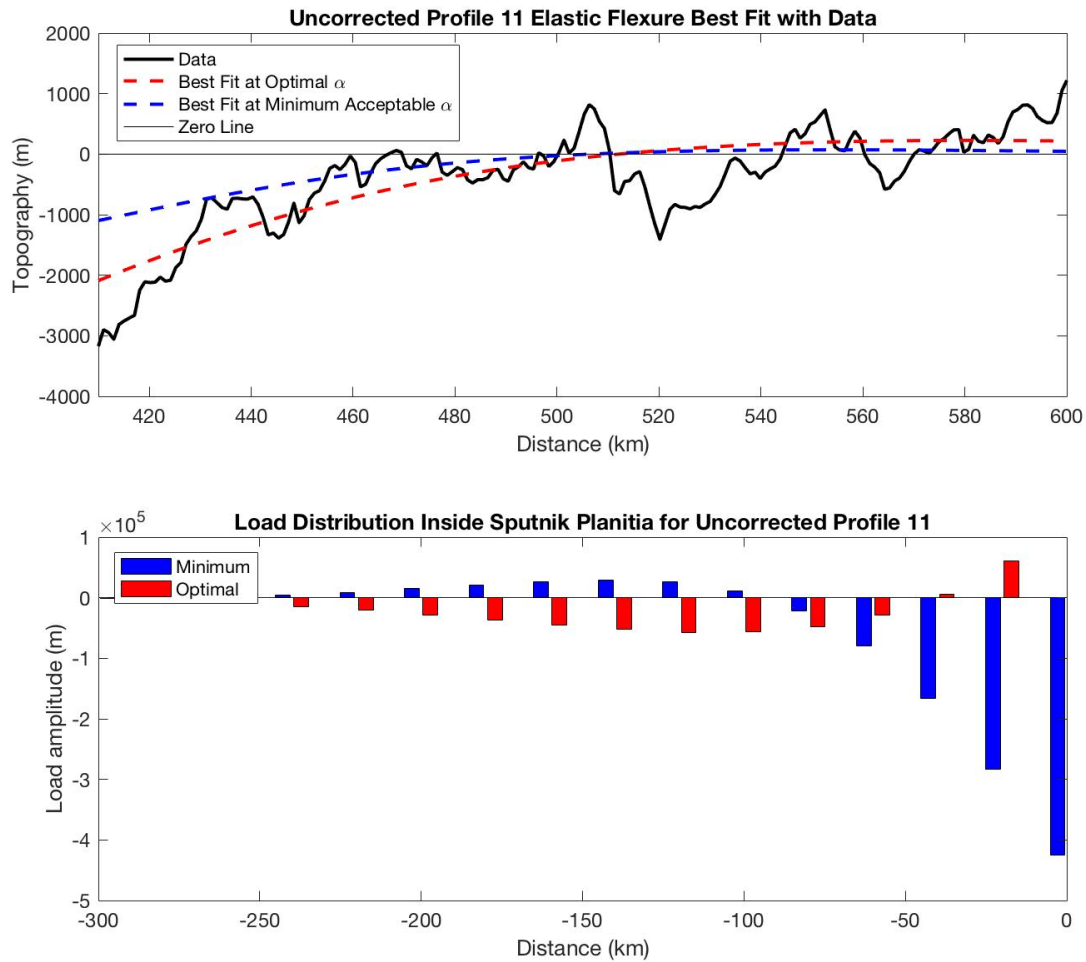


Track 10's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized, and the blue dashed line is the best fit with the minimum acceptable α . This plot has craters present in the data. Lower panel: Profile 10's corresponding load amplitudes of the two best fits that correspond to the acceptable range of α .

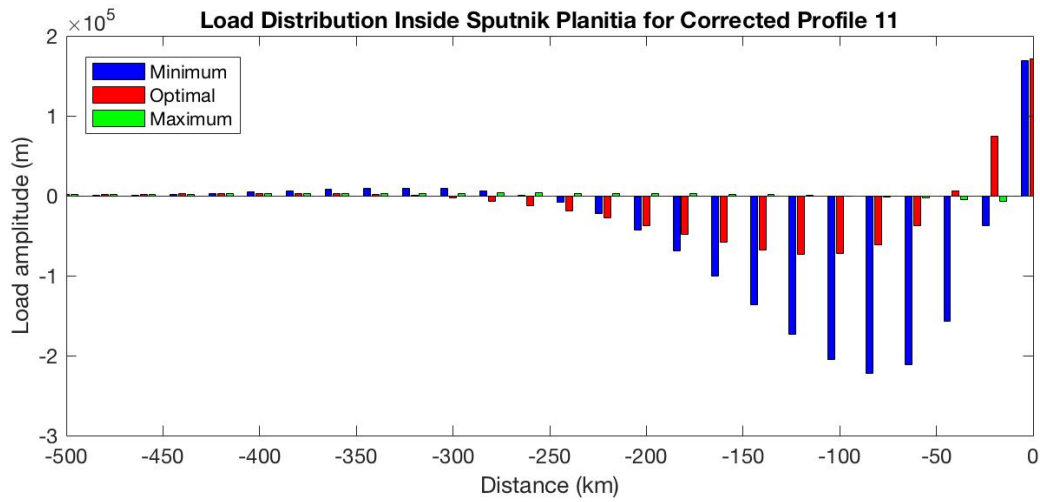
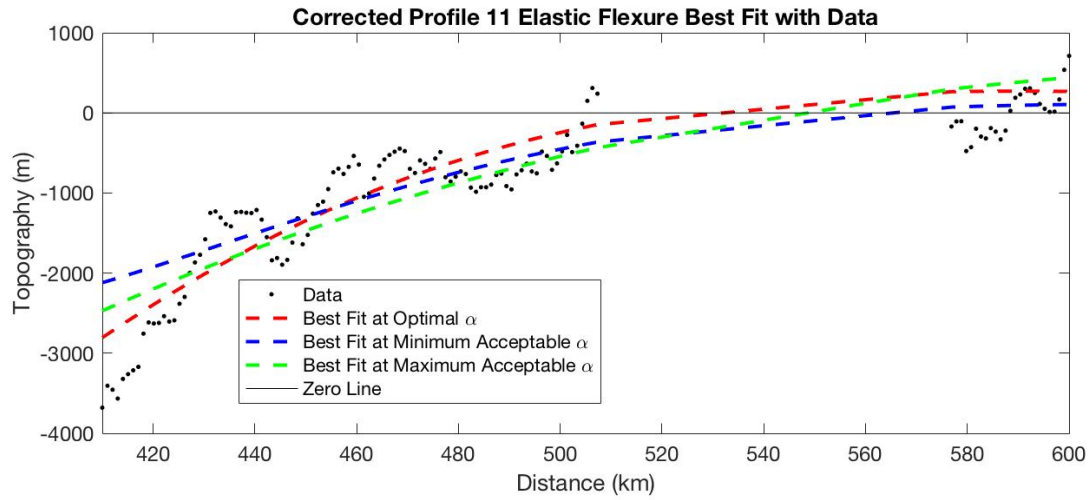


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

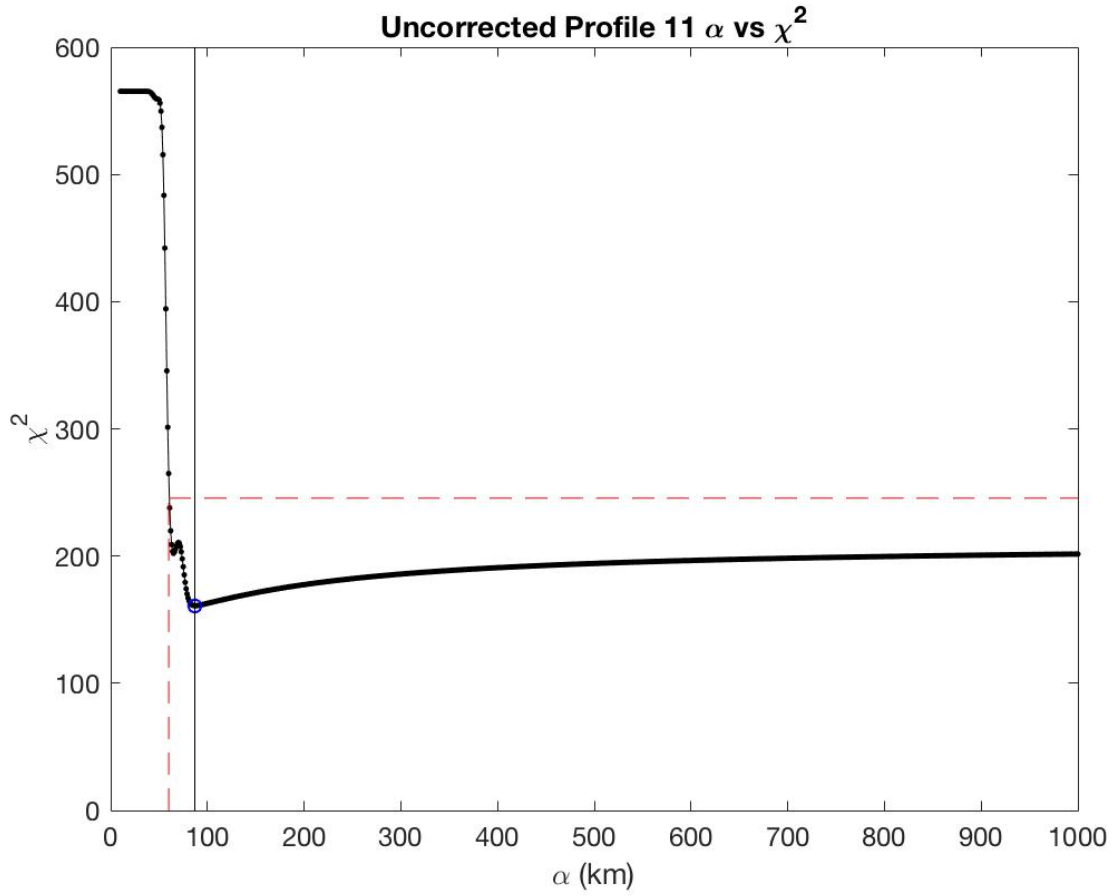
s. **Profile 11's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot**



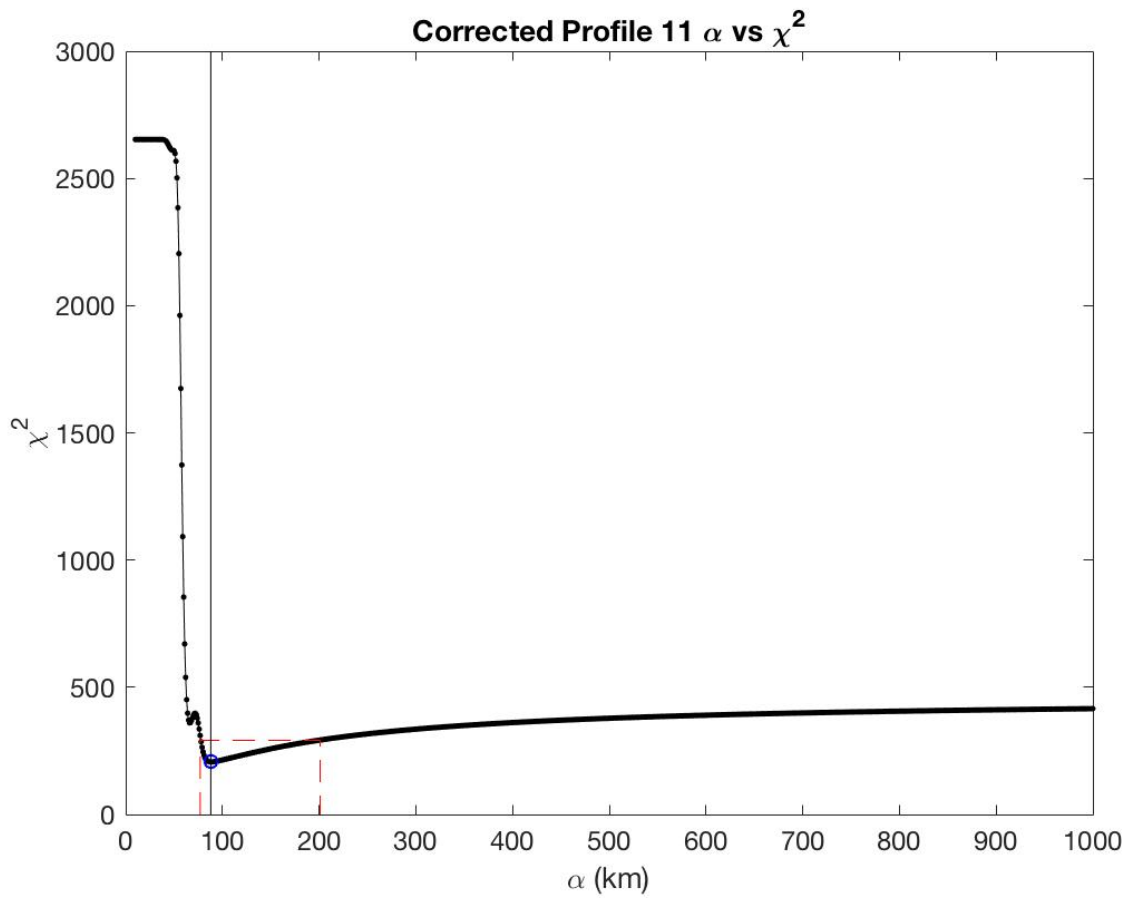
Track 11's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized, and the blue dashed line is the best fit with the minimum acceptable α . This plot has craters present in the data. Lower panel: Profile 11's corresponding load amplitudes of the two best fits that correspond to the acceptable range of α .



Track 11's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data. Lower panel: Profile 11's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

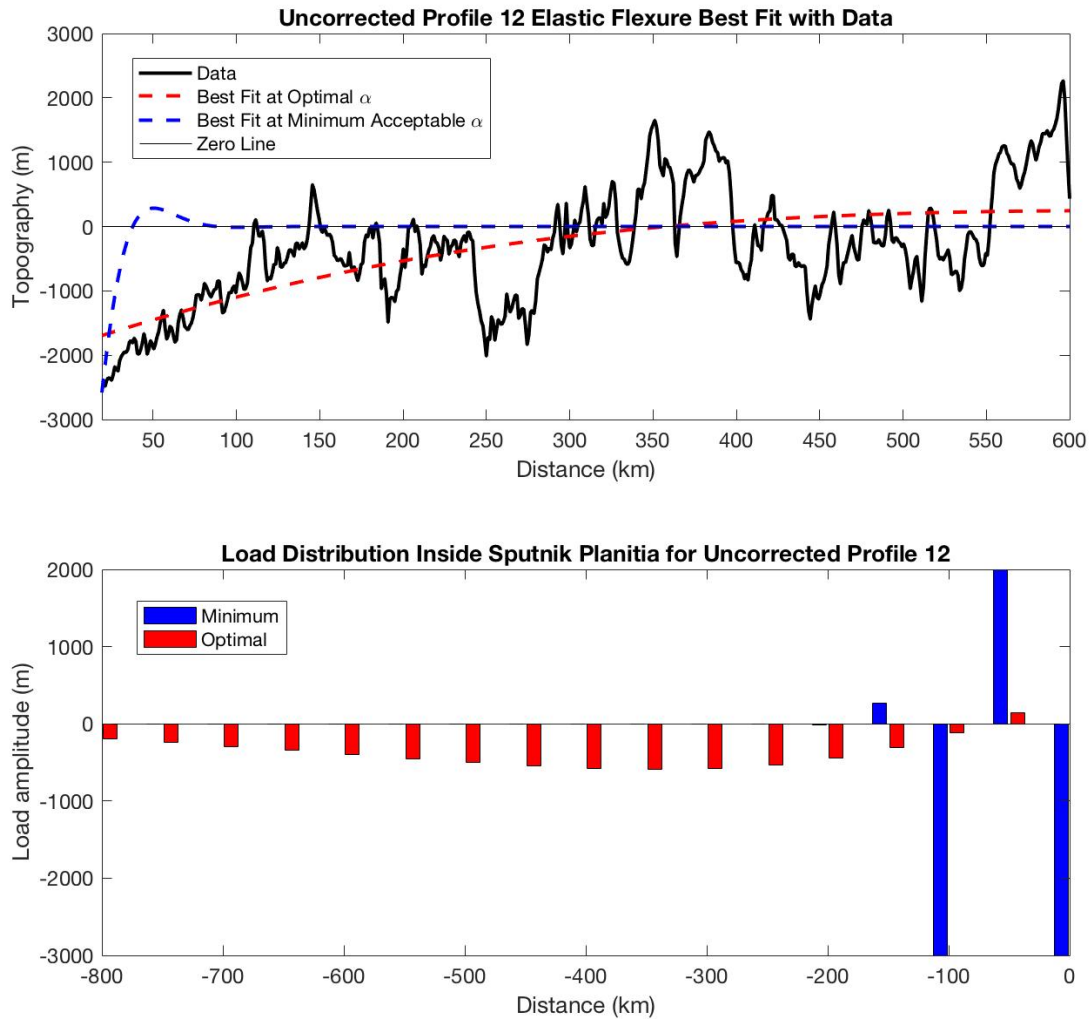


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

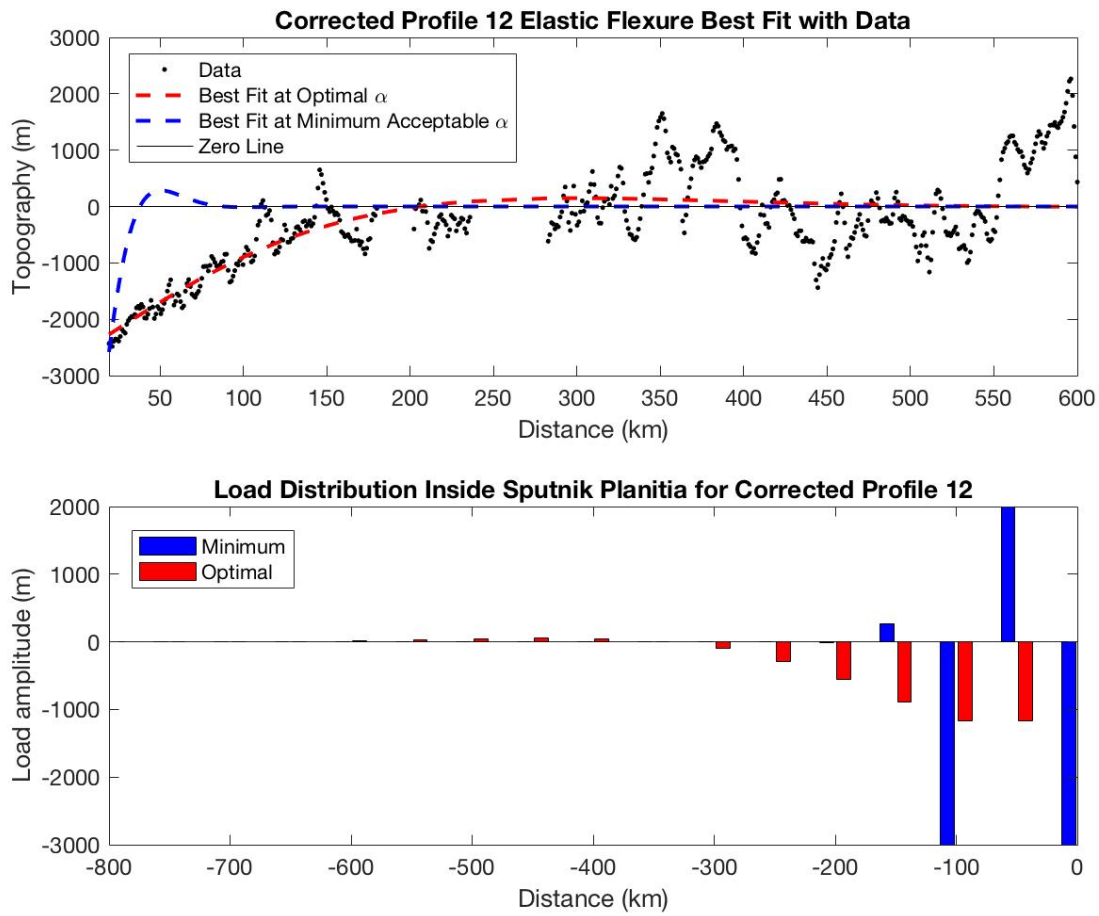


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

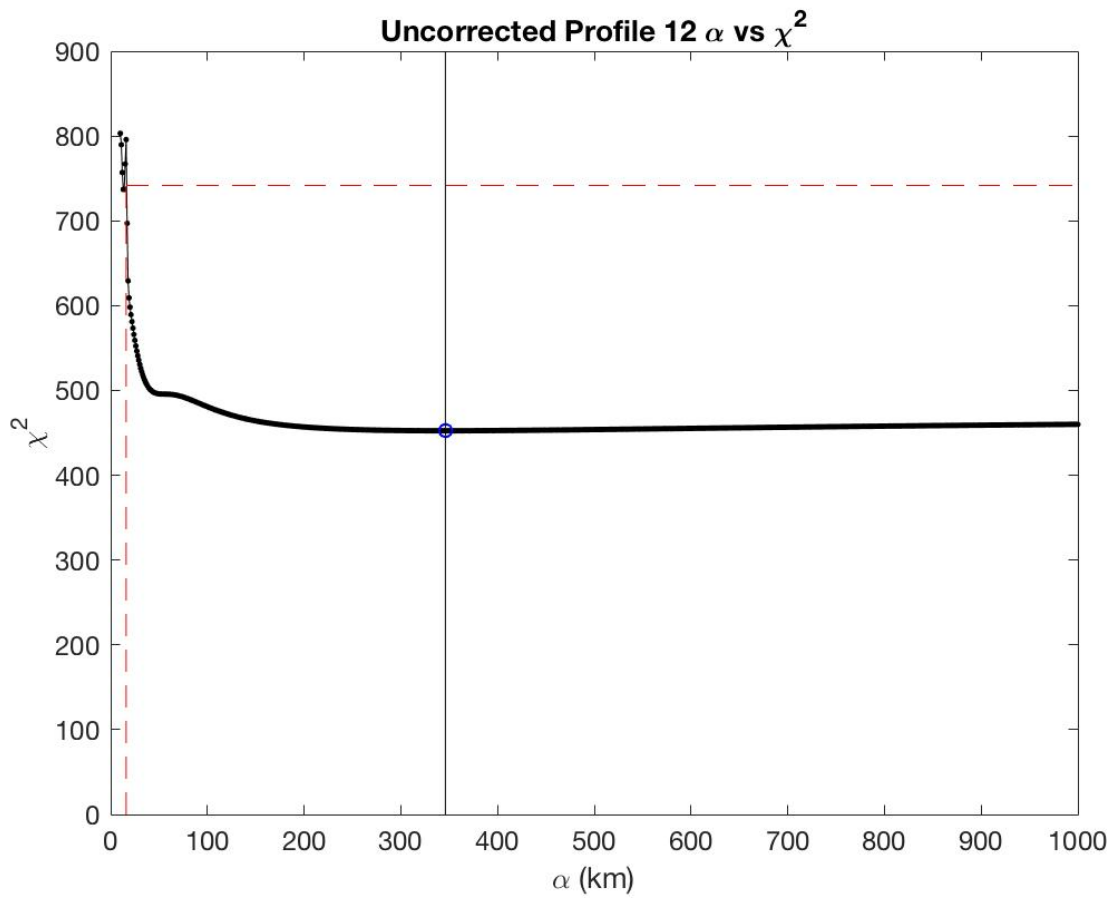
t. **Profile 12's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot**



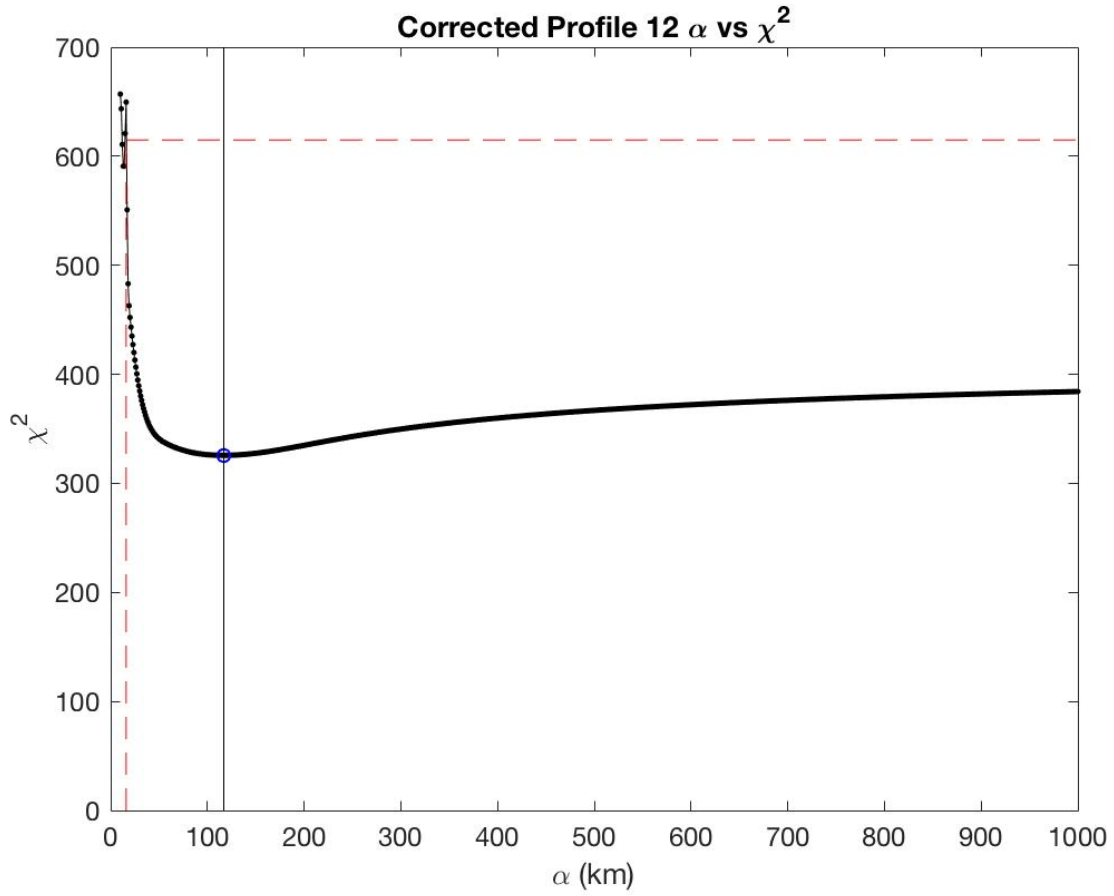
Track 12's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized, and the blue dashed line is the best fit with the minimum acceptable α . This plot has craters present in the data. Lower panel: Profile 12's corresponding load amplitudes of the two best fits that correspond to the acceptable range of α .



Track 12's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized, and the blue dashed line is the best fit with the minimum acceptable α . The plot has craters removed in the data. Lower panel: Profile 12's corresponding load amplitudes of the two best fits that correspond to the acceptable range of α .

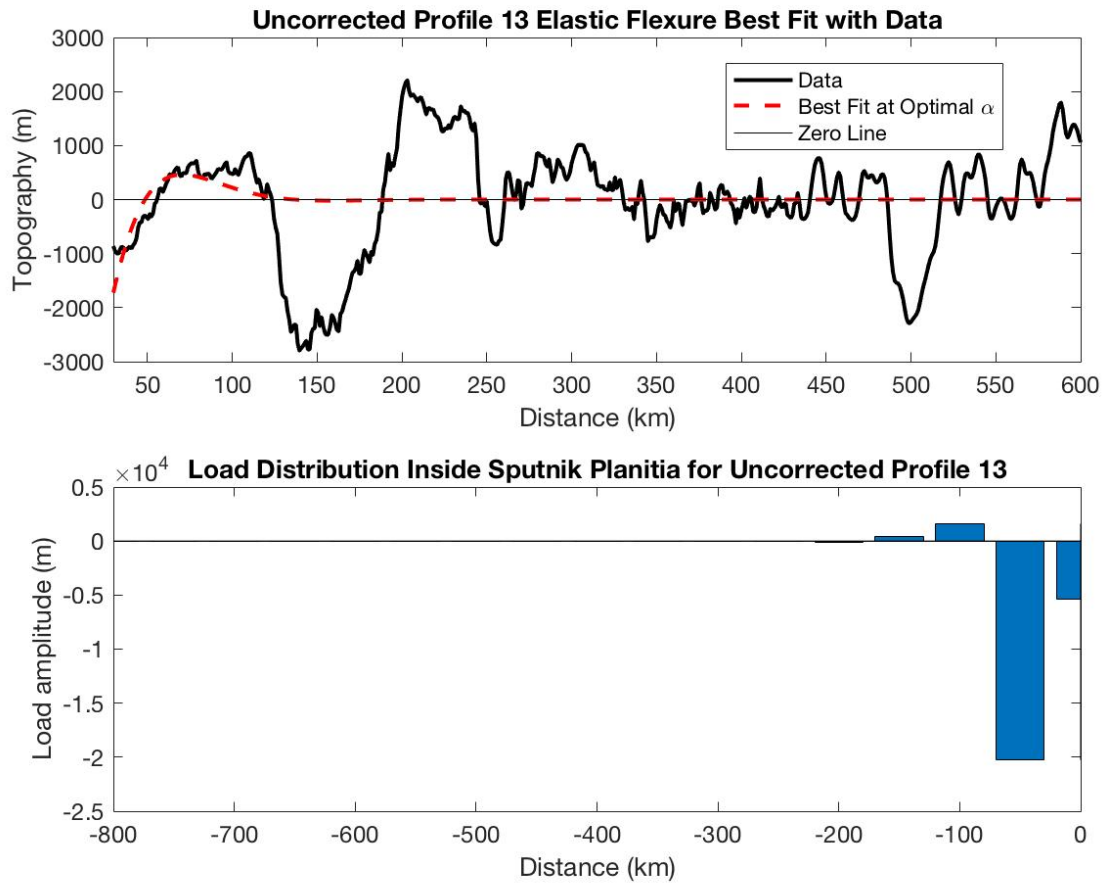


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

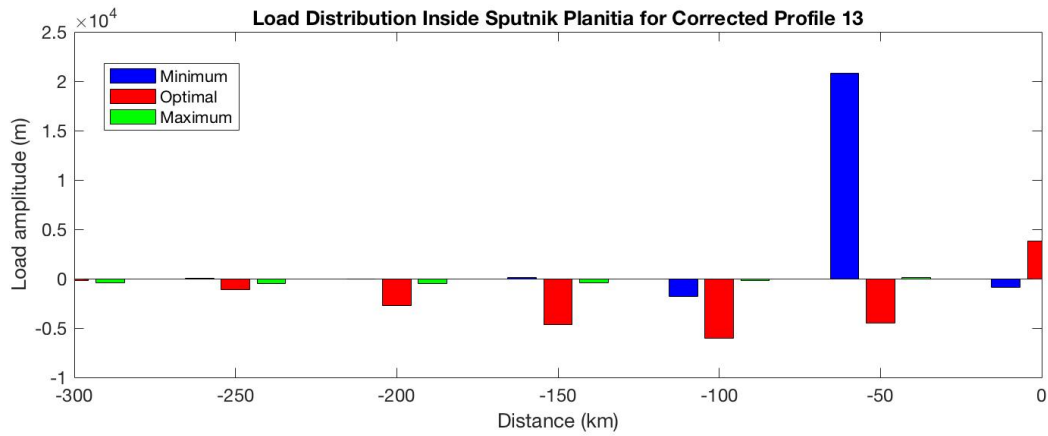
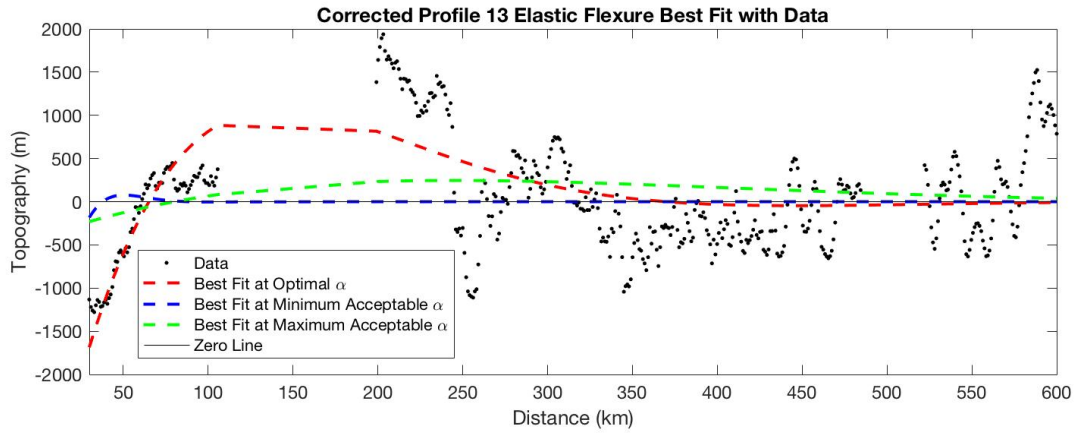


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

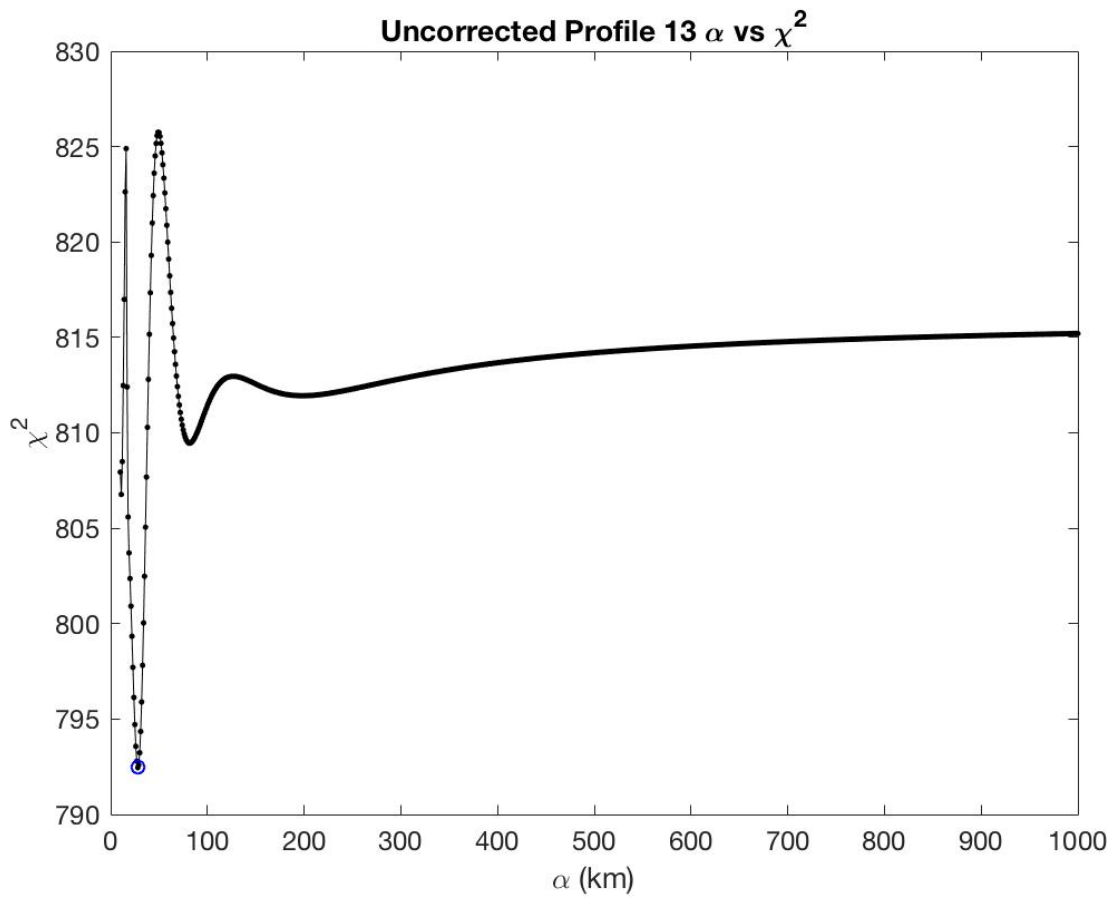
u. **Profile 13's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot**



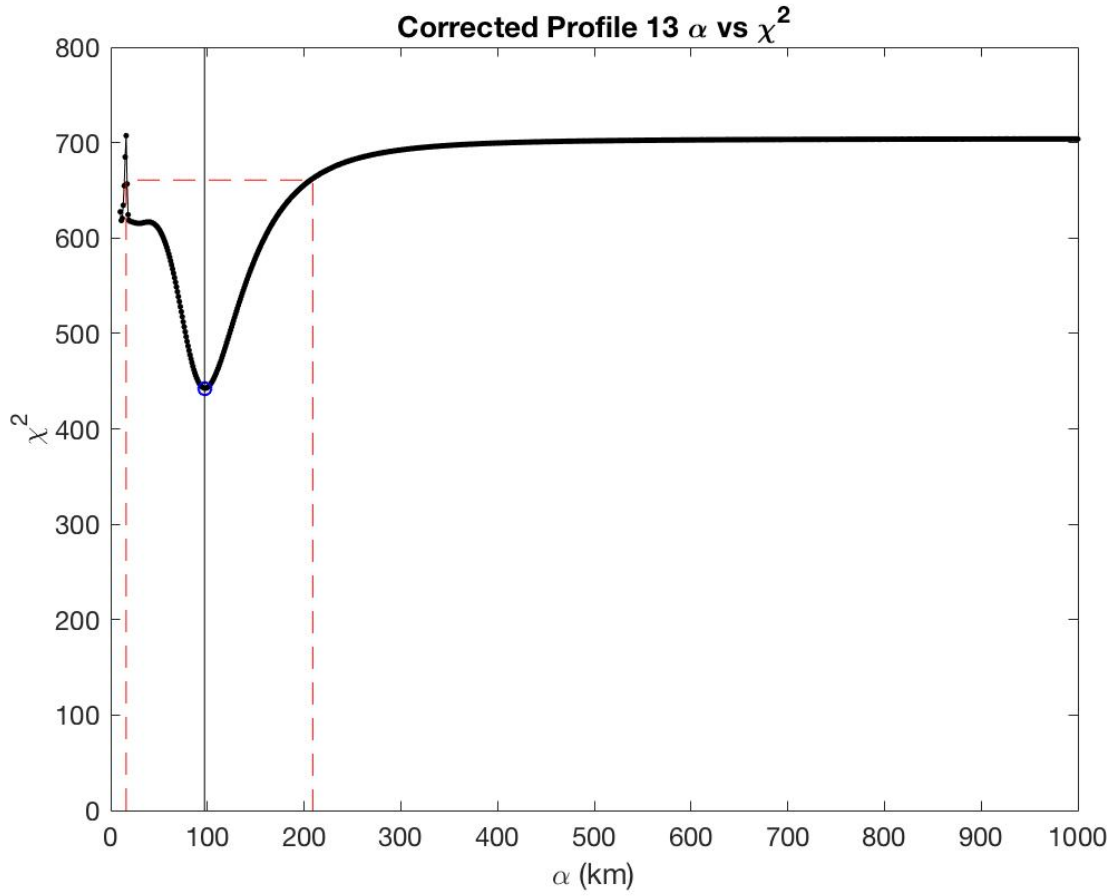
Track 13's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized. This plot has craters present in the data. Lower panel: Profile 13's corresponding load amplitudes of the best fit that corresponds to the acceptable range of α .



Track 13's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data. Lower panel: Profile 13's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

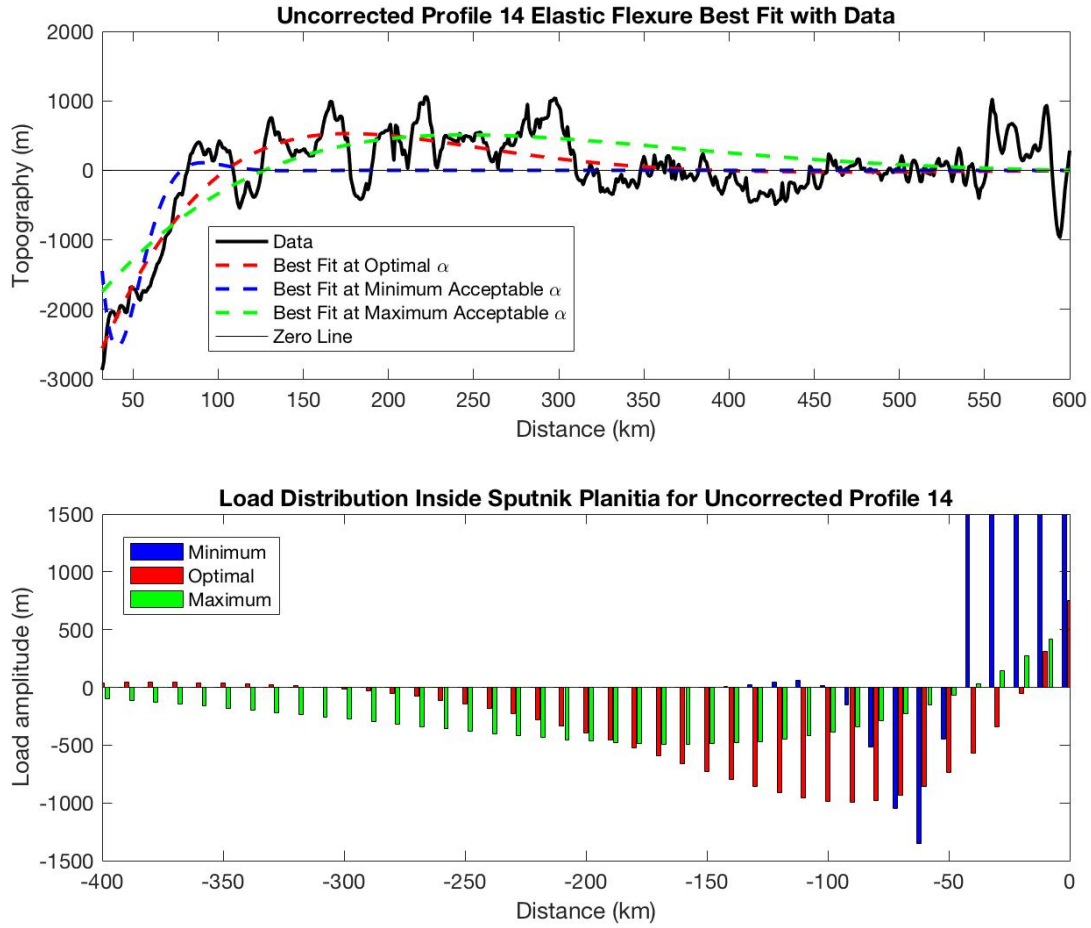


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

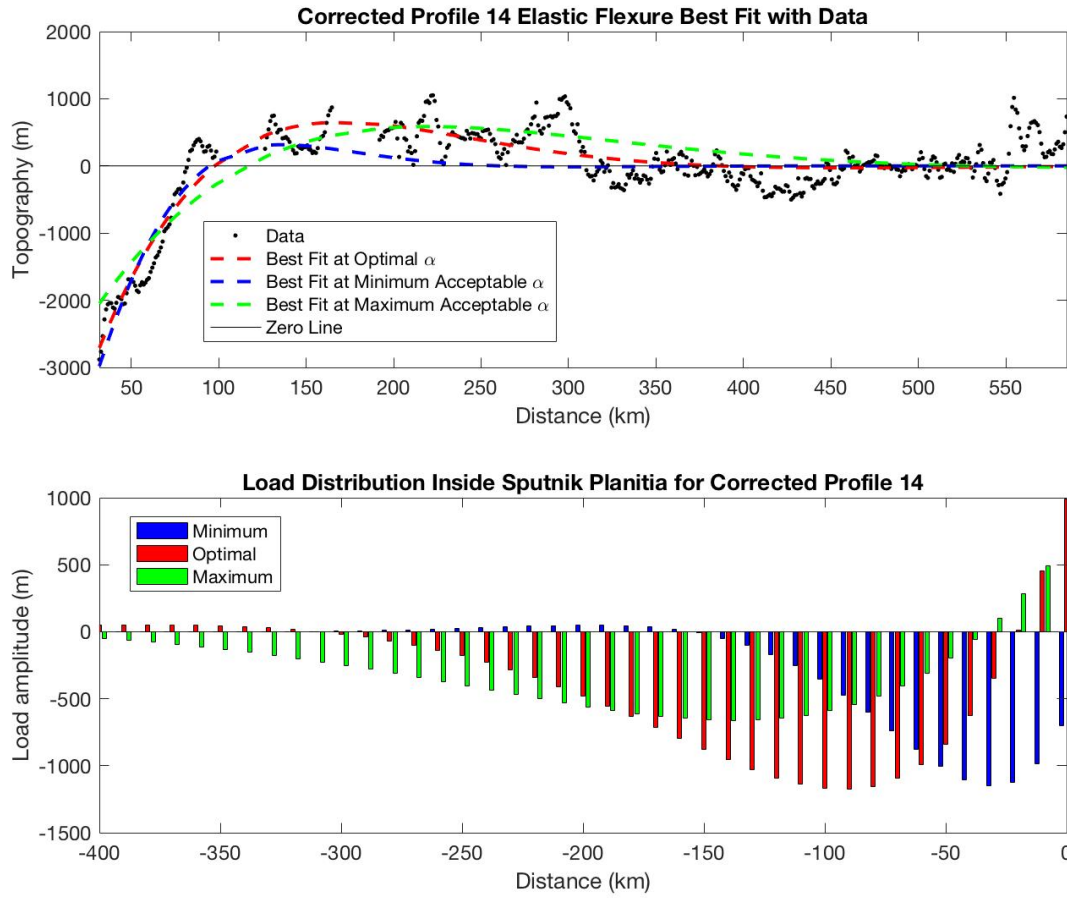


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

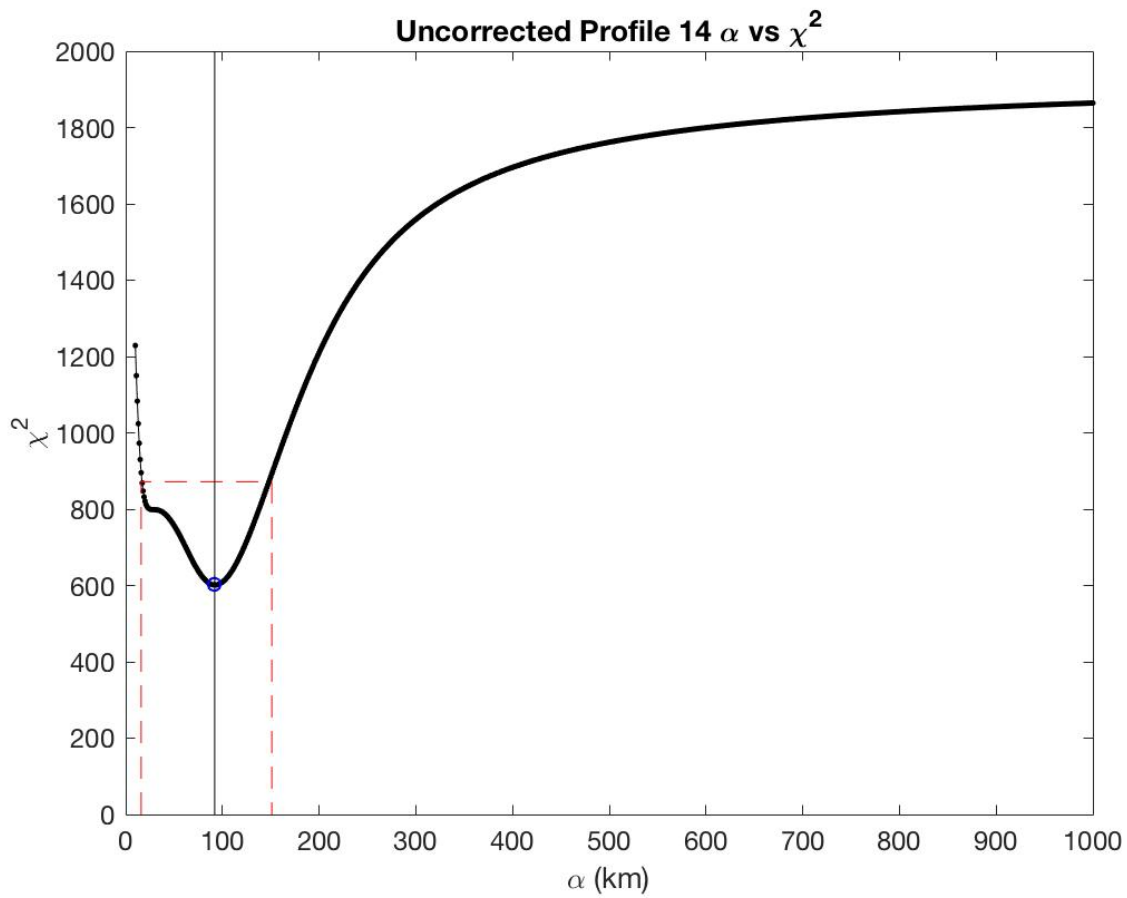
v. Profile 14's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



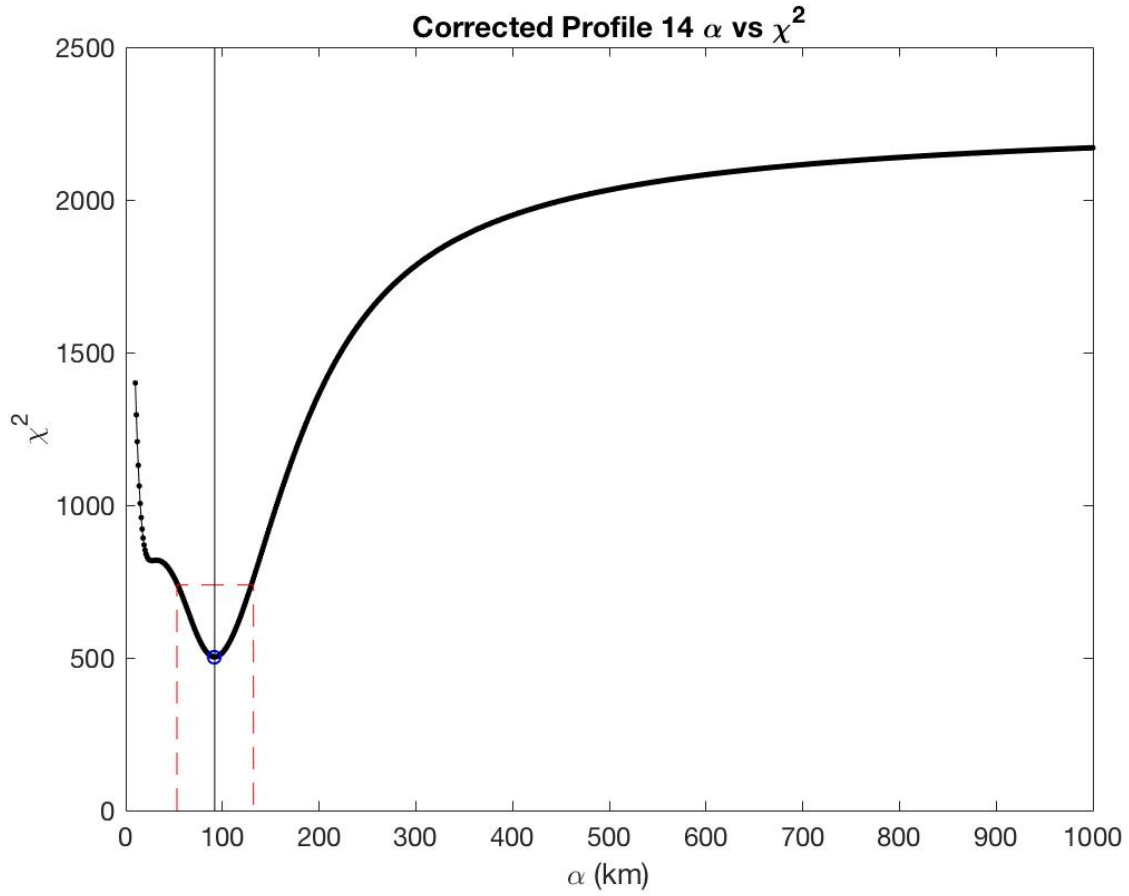
Track 14's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters present in the data. Lower panel: Profile 14's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .



Track 14's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data. Lower panel: Profile 14's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

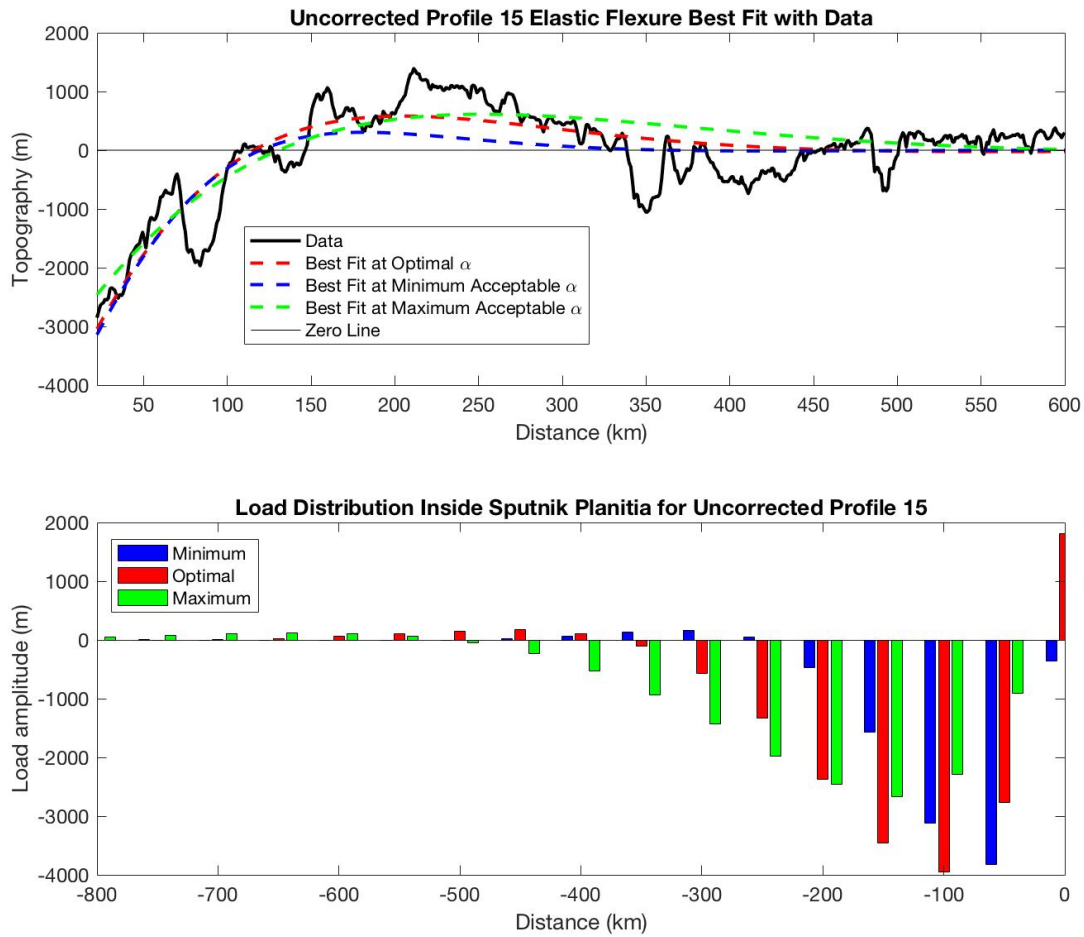


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

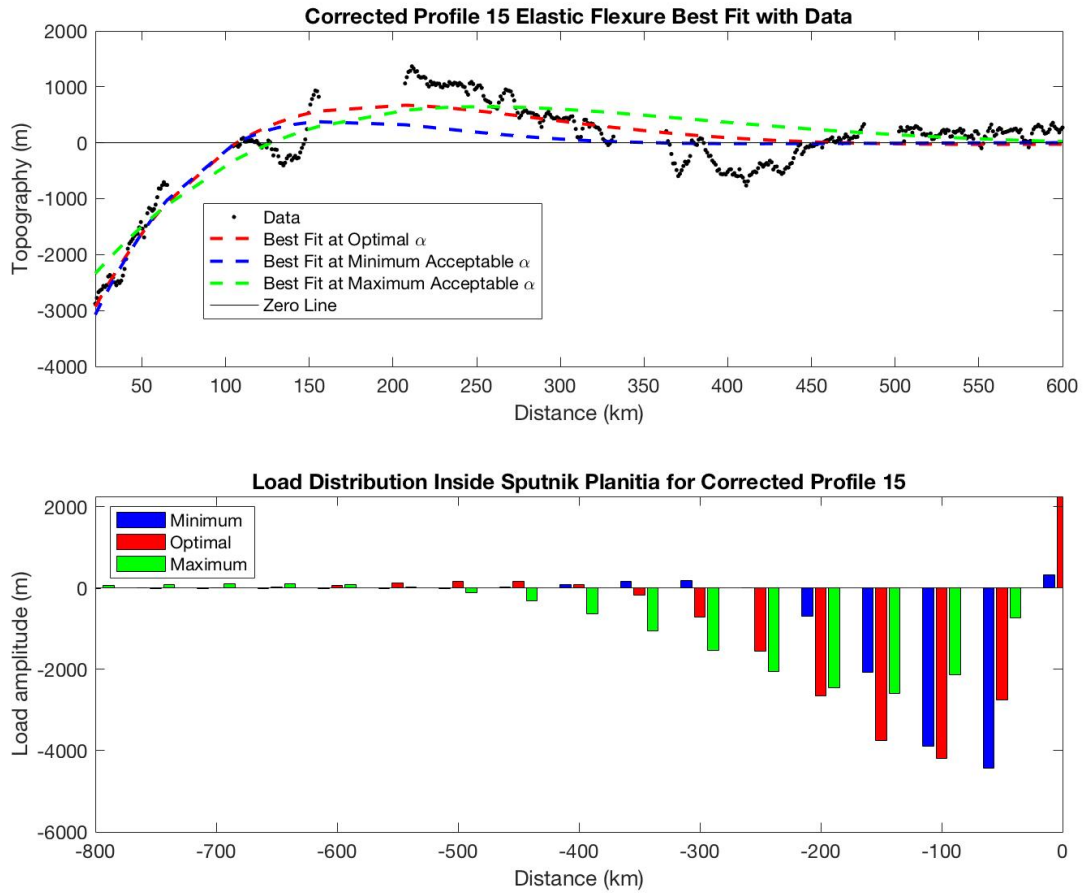


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

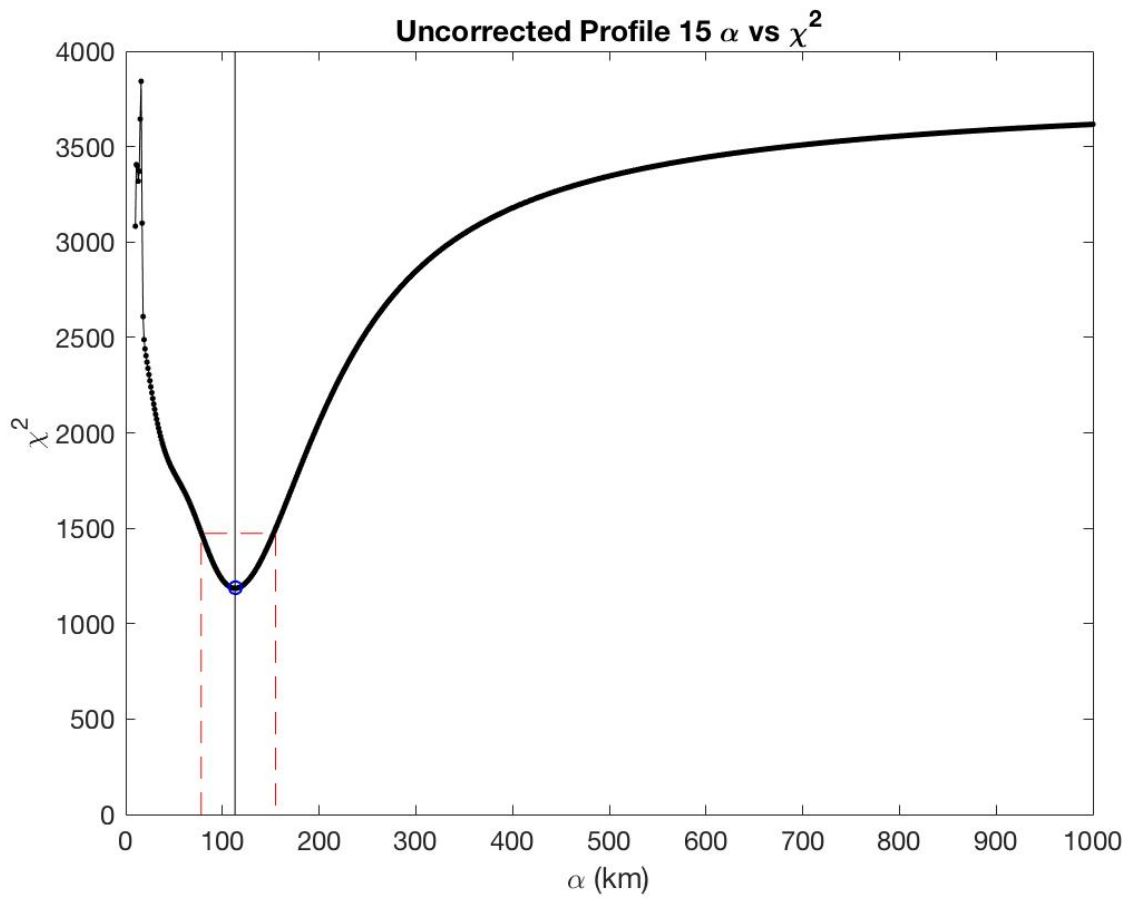
w. Profile 15's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



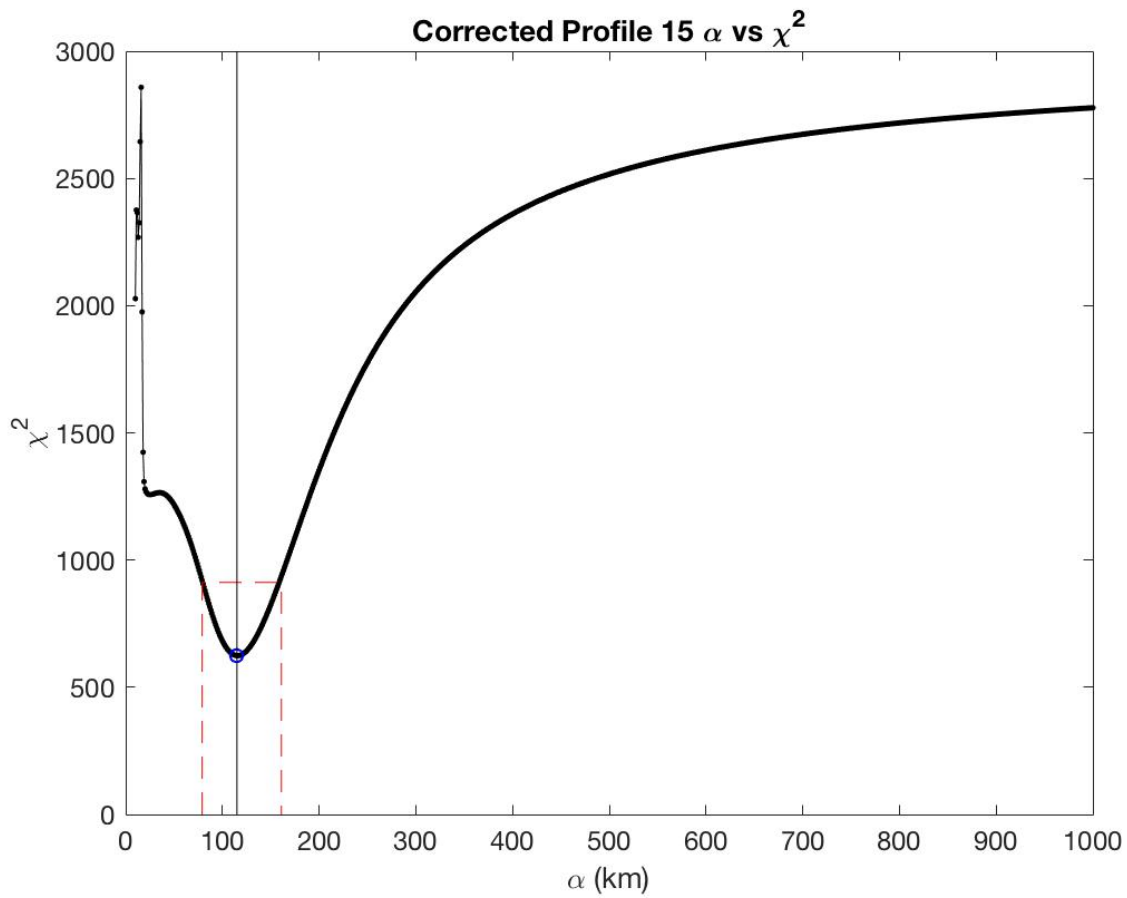
Track 15's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters present in the data. Lower panel: Profile 15's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .



Track 15's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data. Lower panel: Profile 15's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

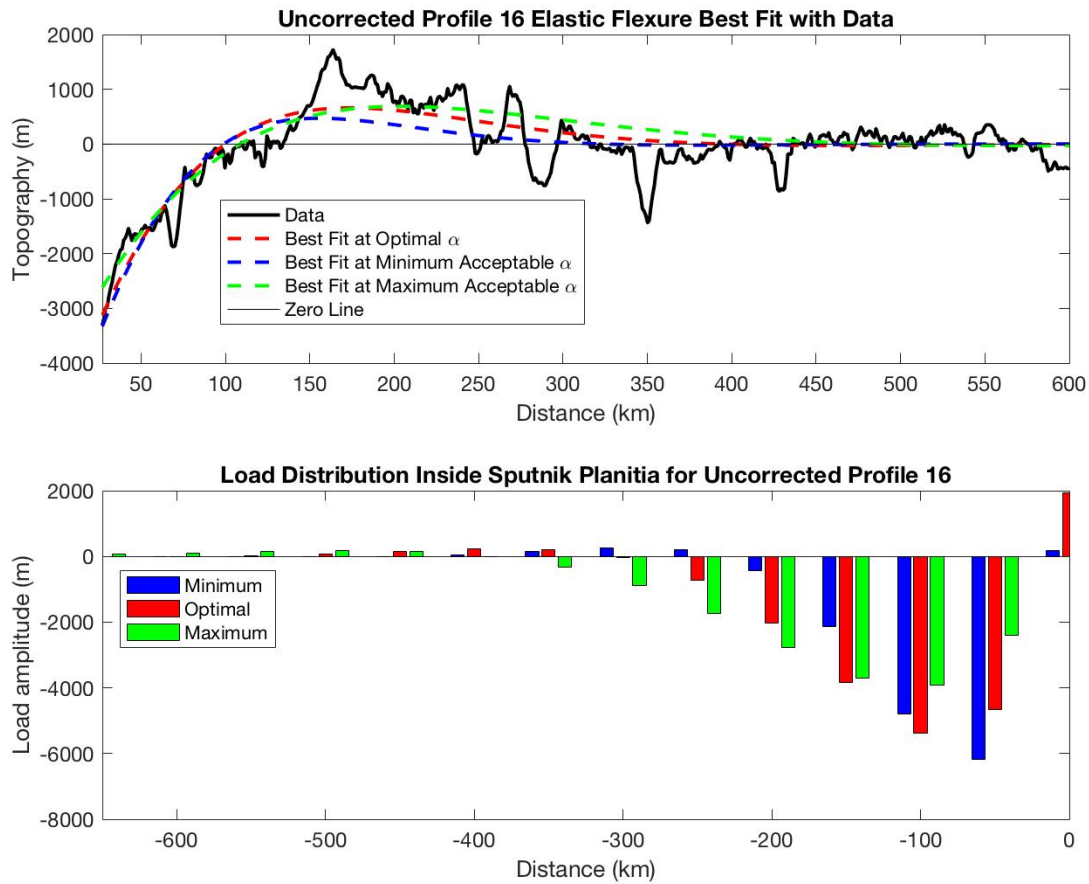


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

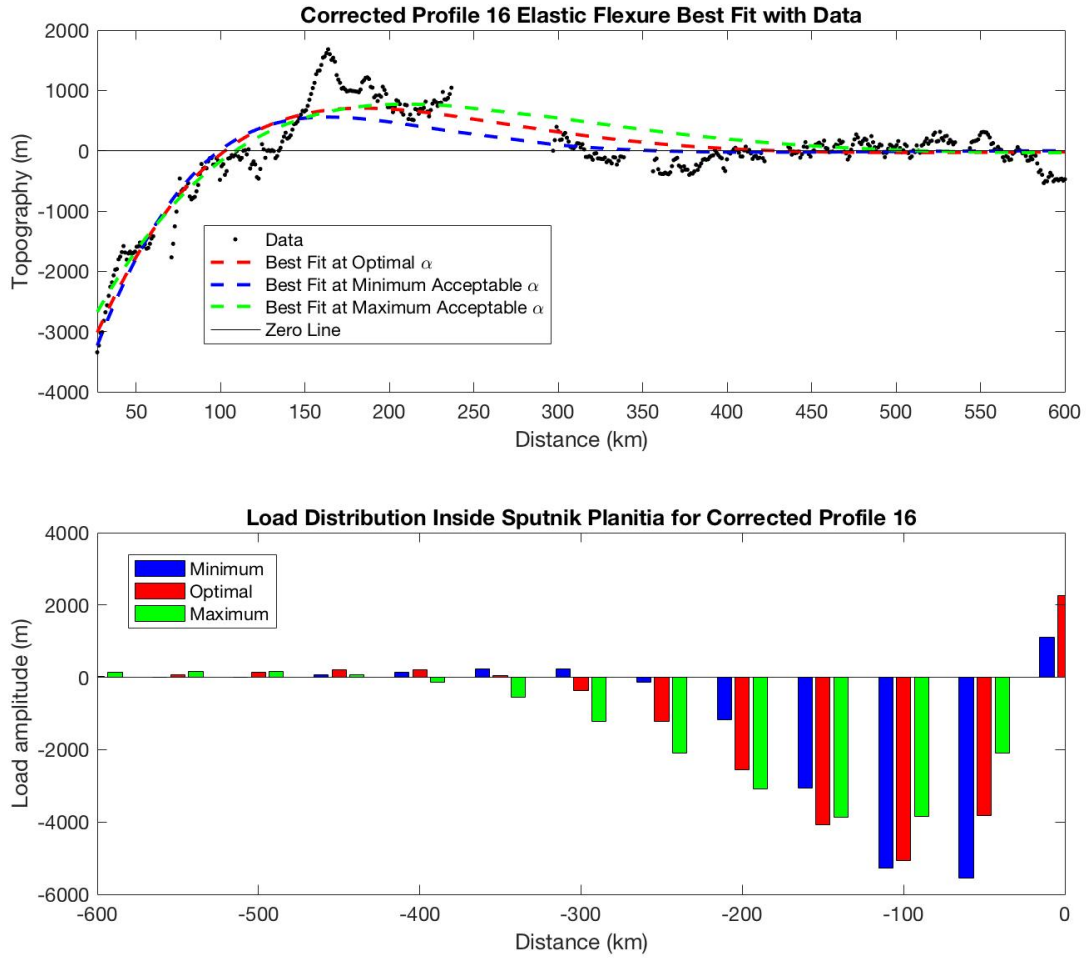


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

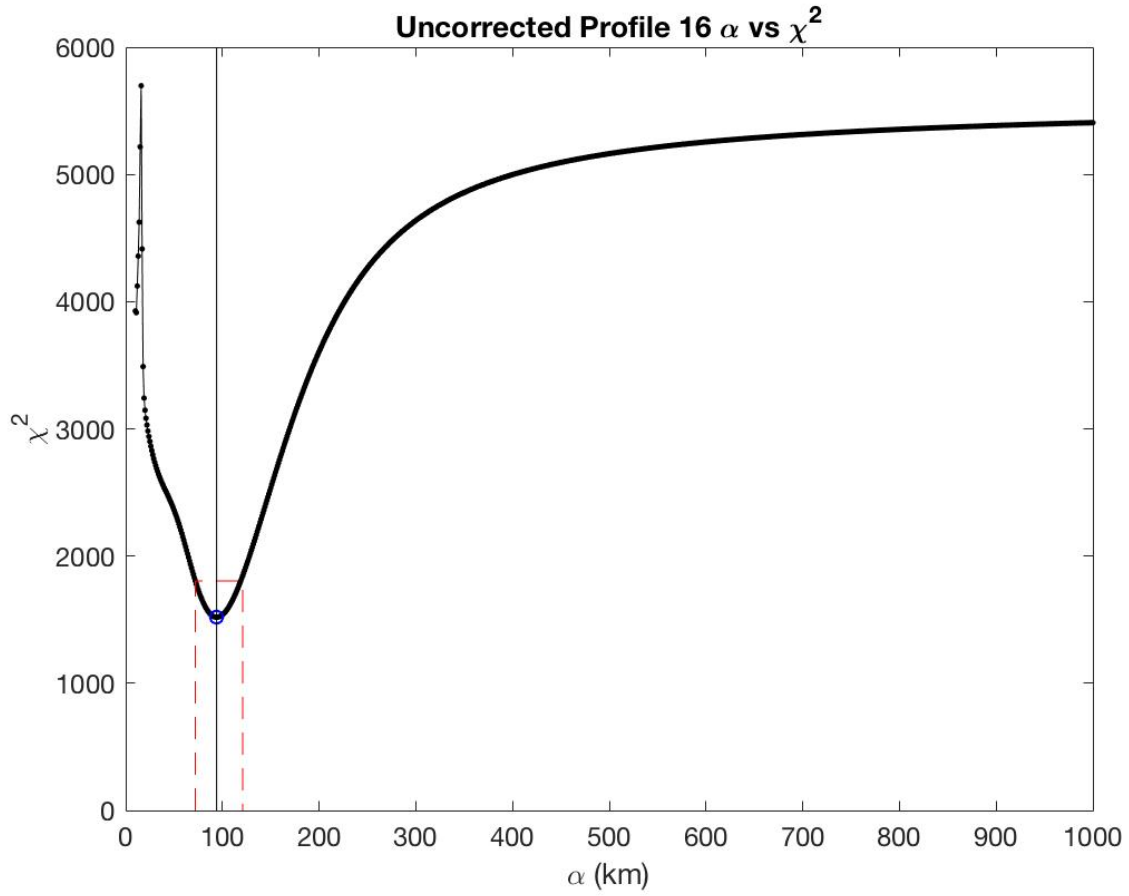
x. Profile 16's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



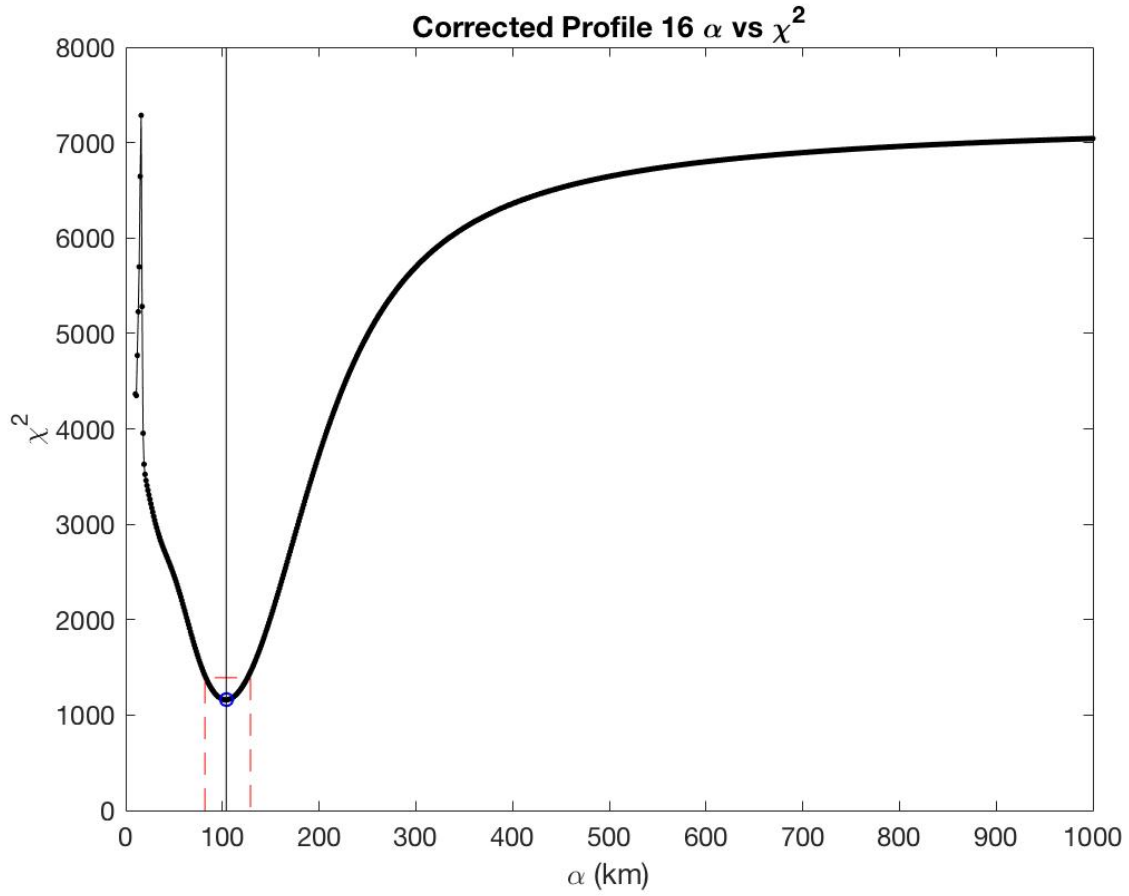
Track 16's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters present in the data. Lower panel: Profile 16's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .



Track 16's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data. Lower panel: Profile 16's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

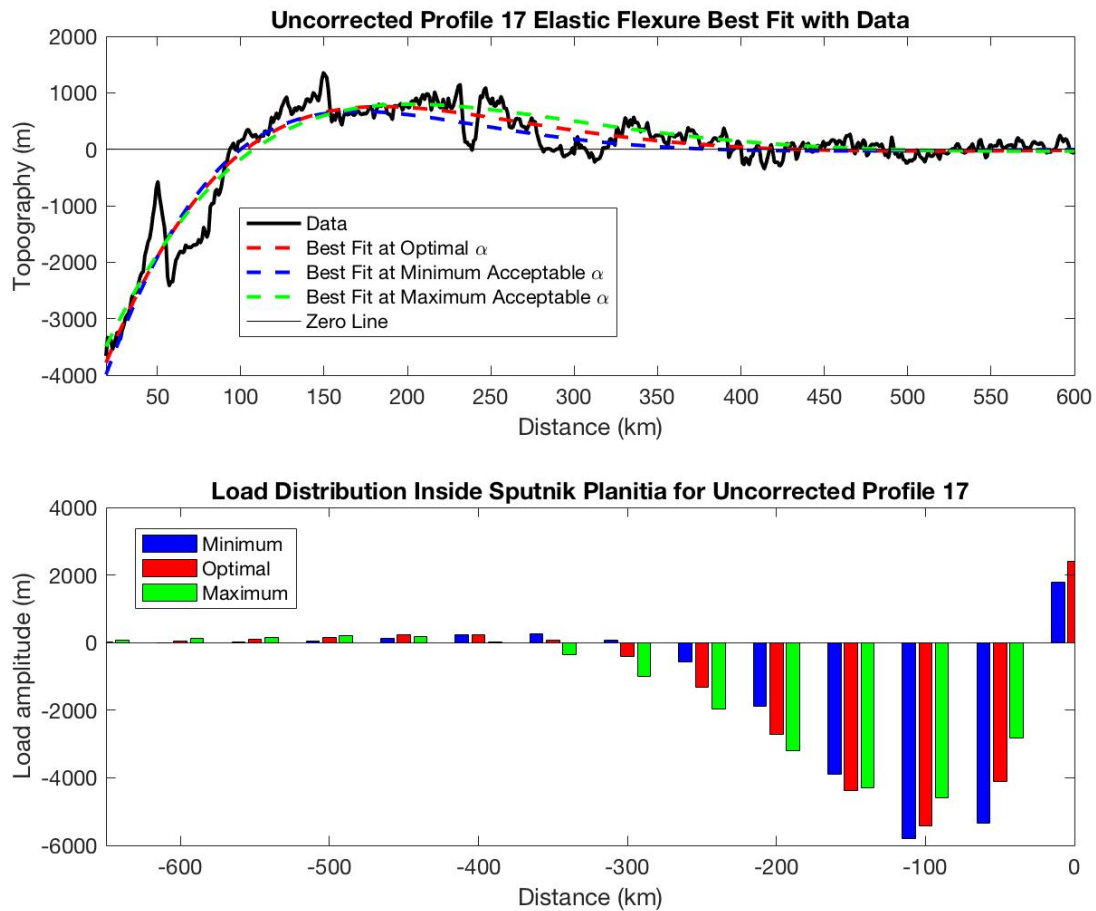


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

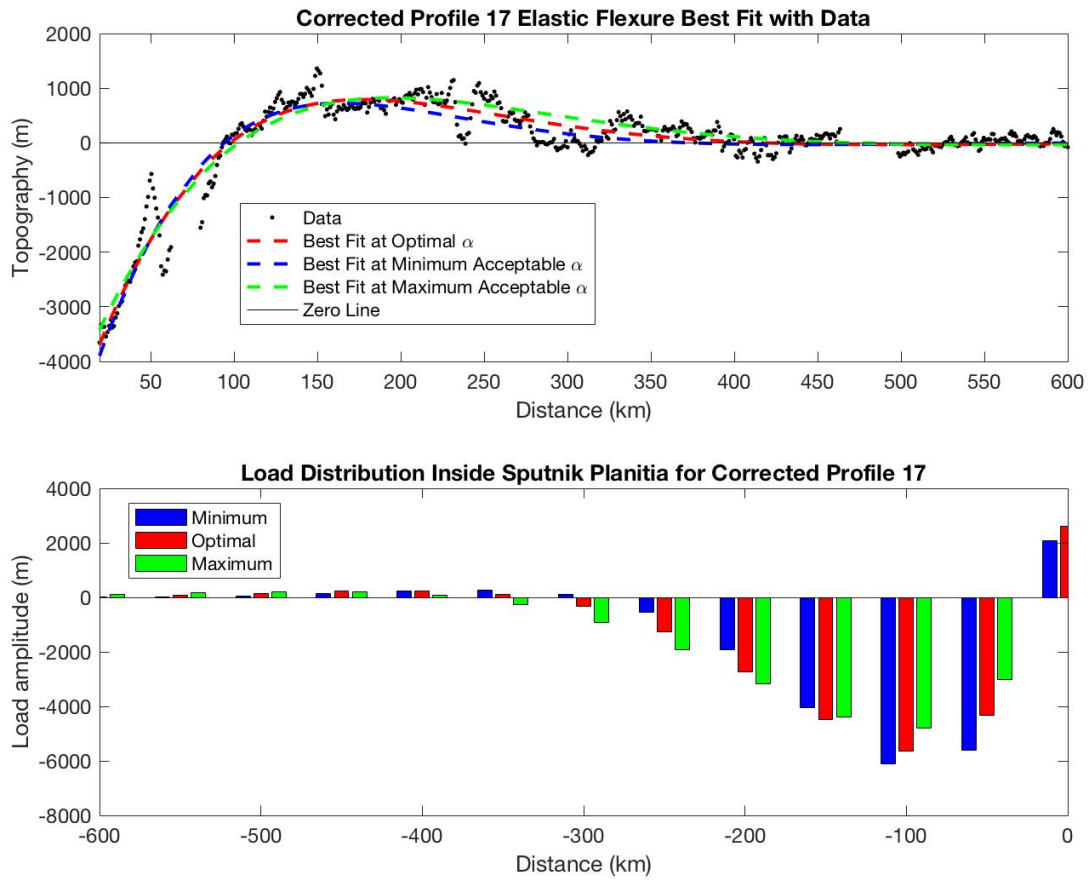


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

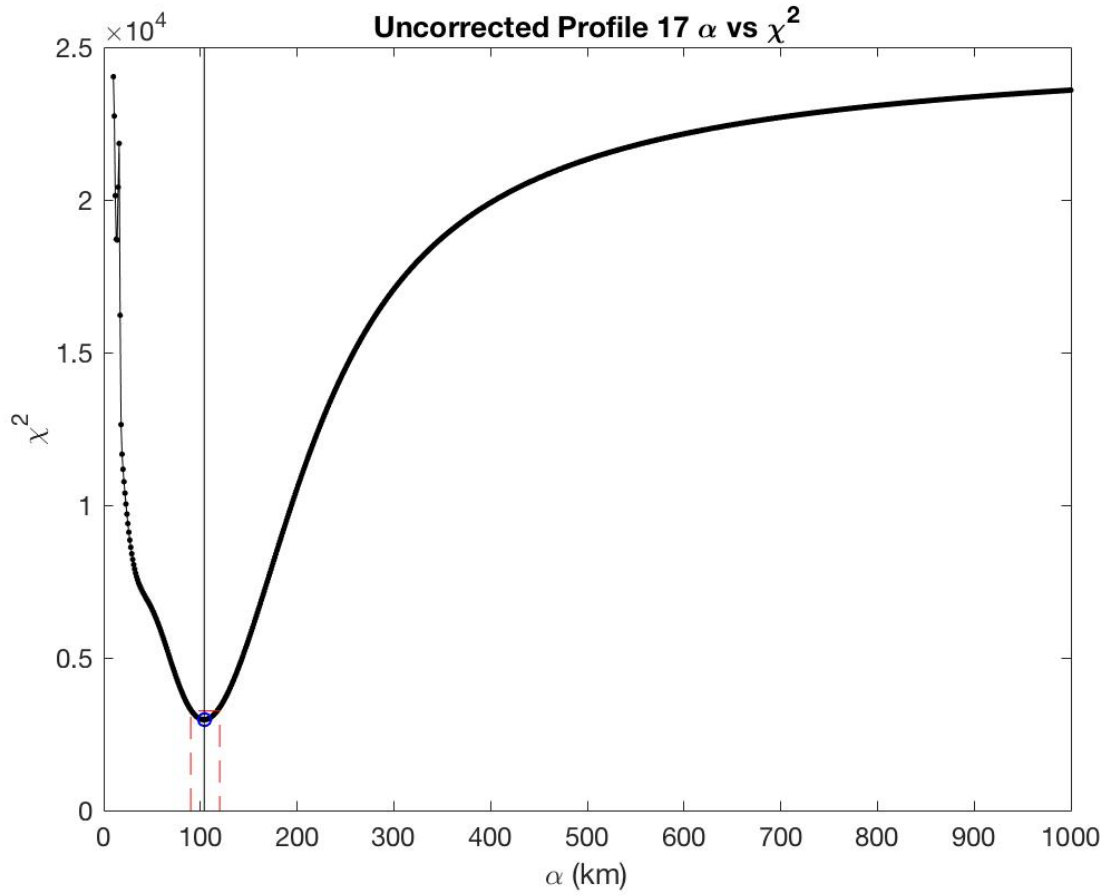
y. Profile 17's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



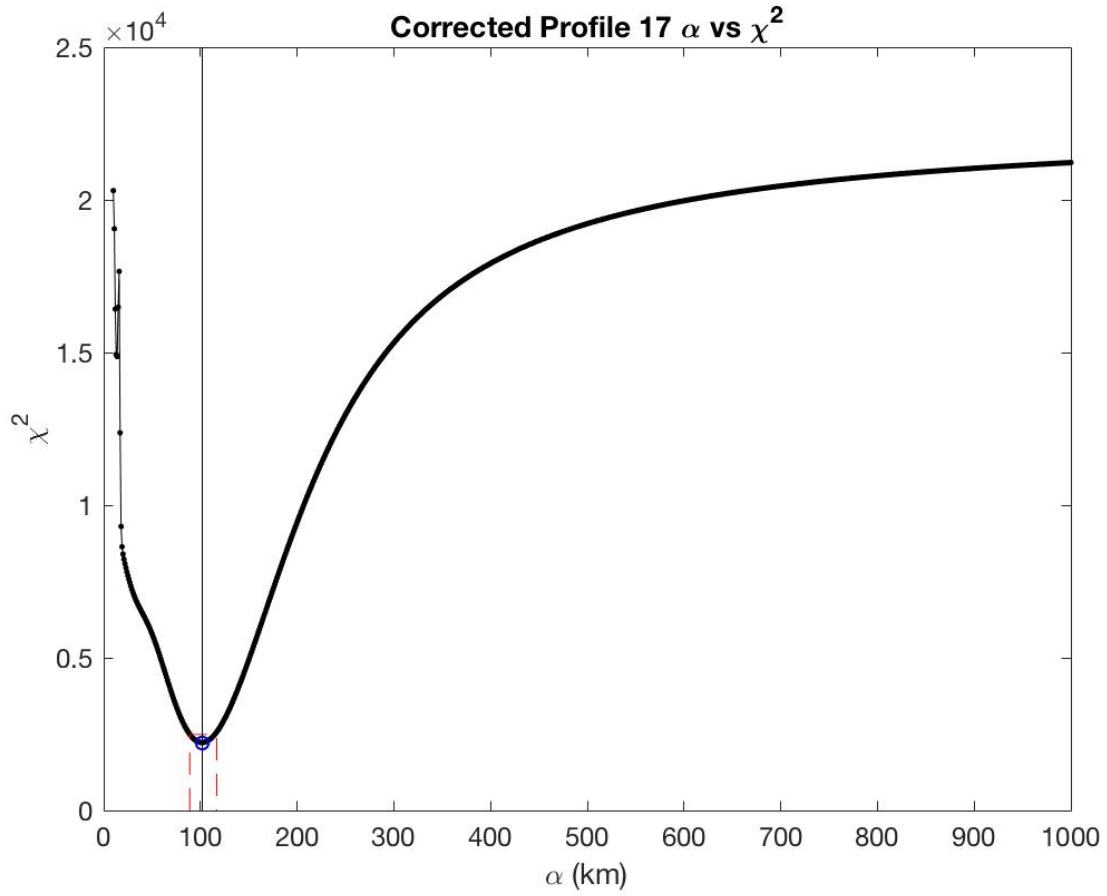
Track 17's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters present in the data. Lower panel: Profile 17's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .



Track 17's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data. Lower panel: Profile 17's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

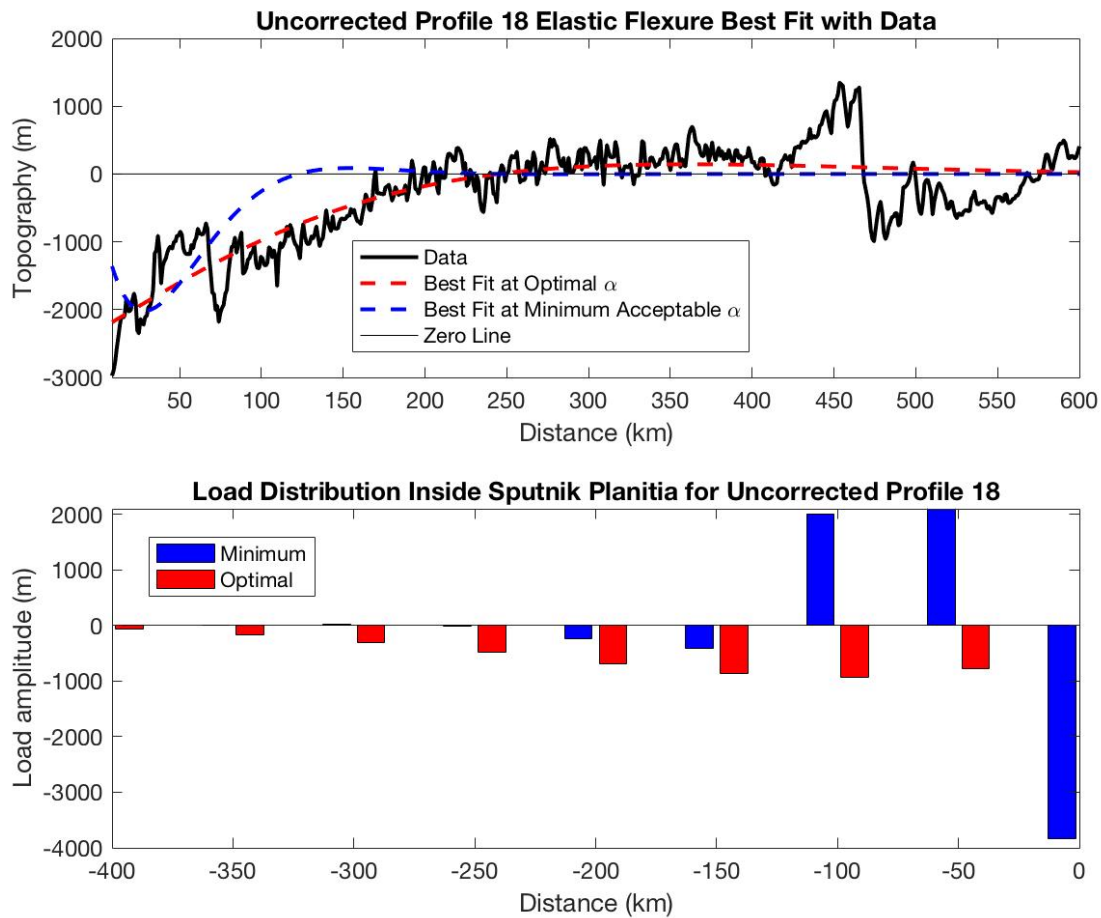


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

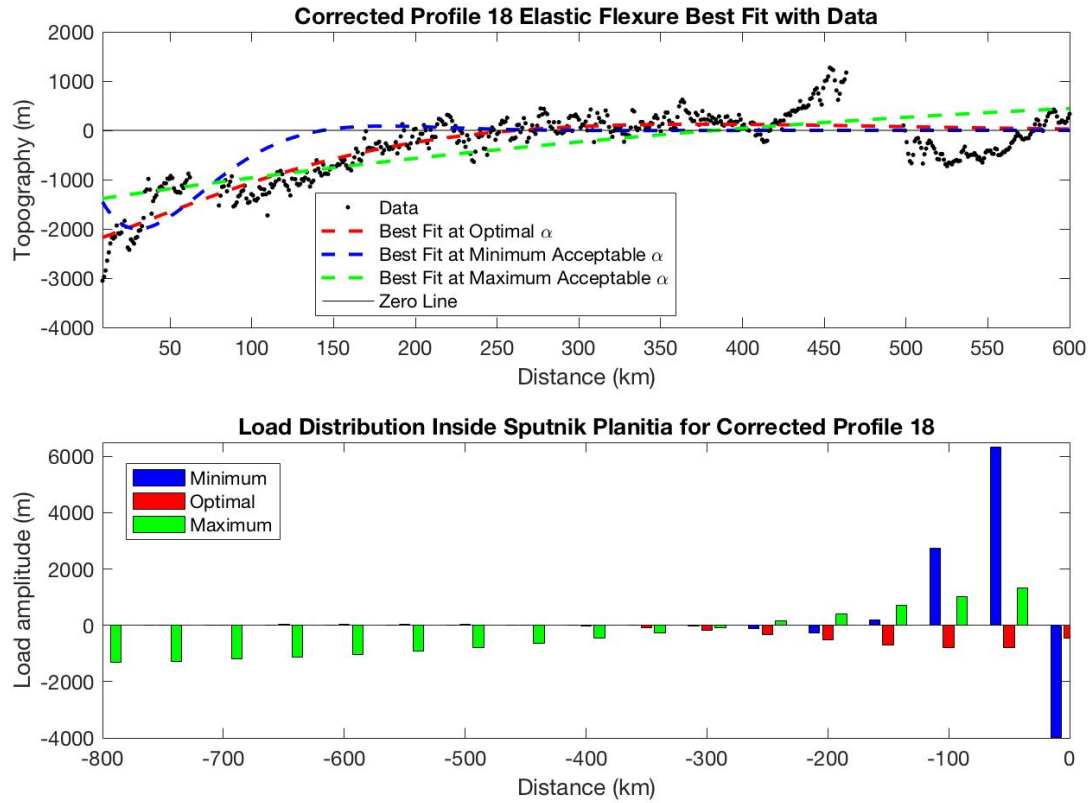


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

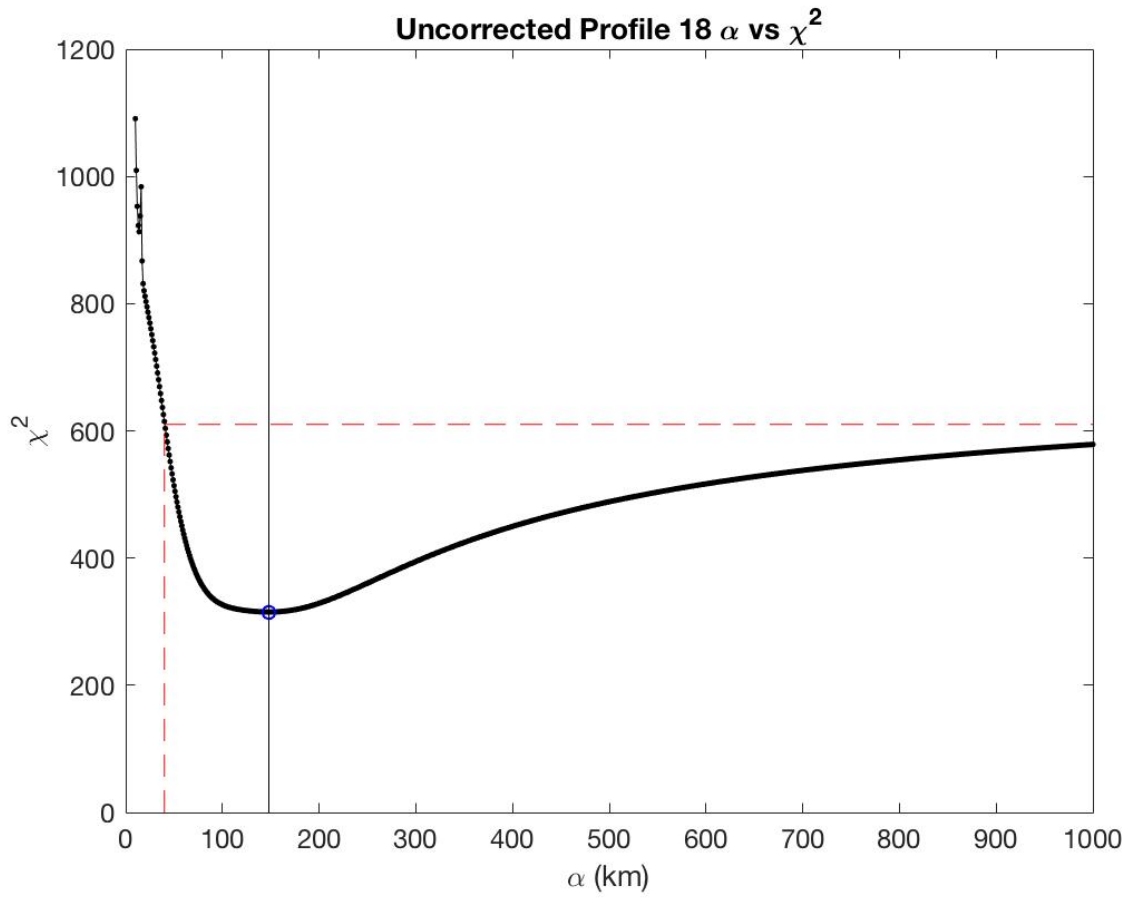
z. Profile 18's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



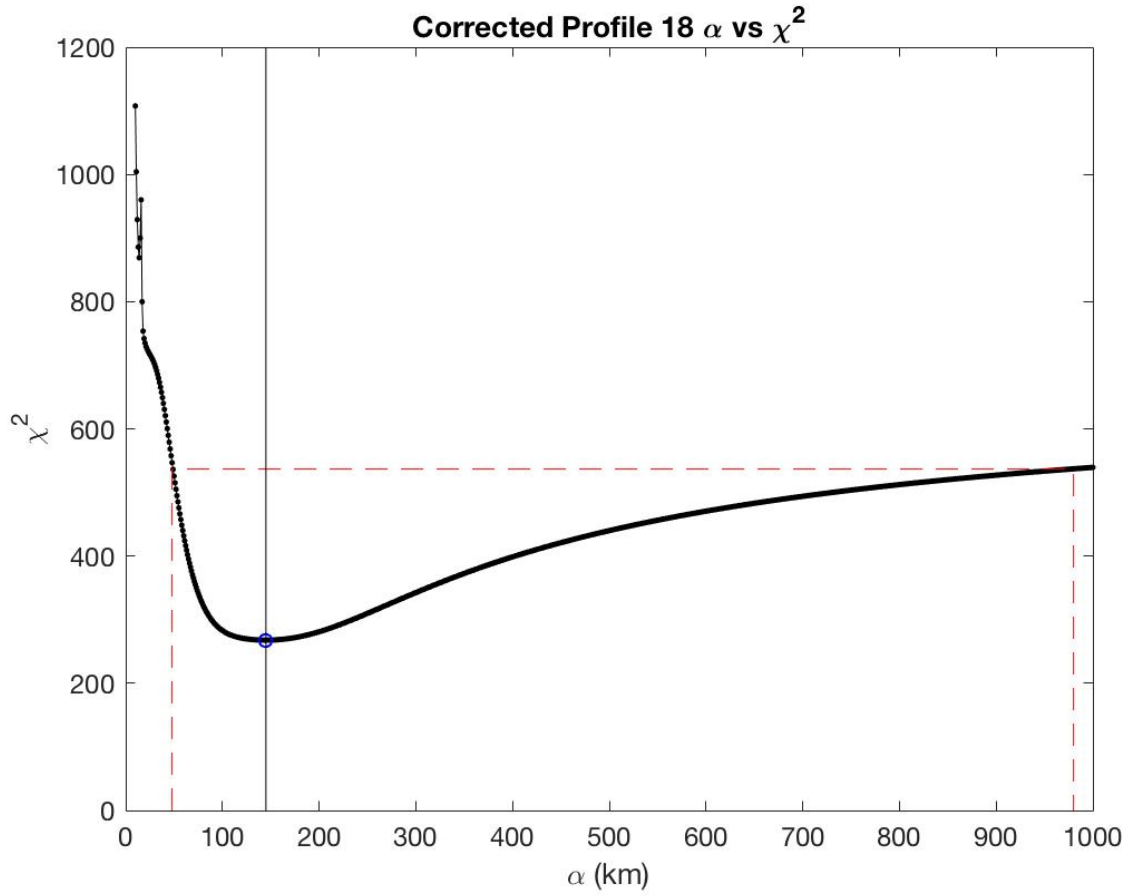
Track 18's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized, and the blue dashed line is the best fit with the minimum acceptable α . This plot has craters present in the data. Lower panel: Profile 5's corresponding load amplitudes of the two best fits that correspond to the acceptable range of α .



Track 18's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data. Lower panel: Profile 18's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .

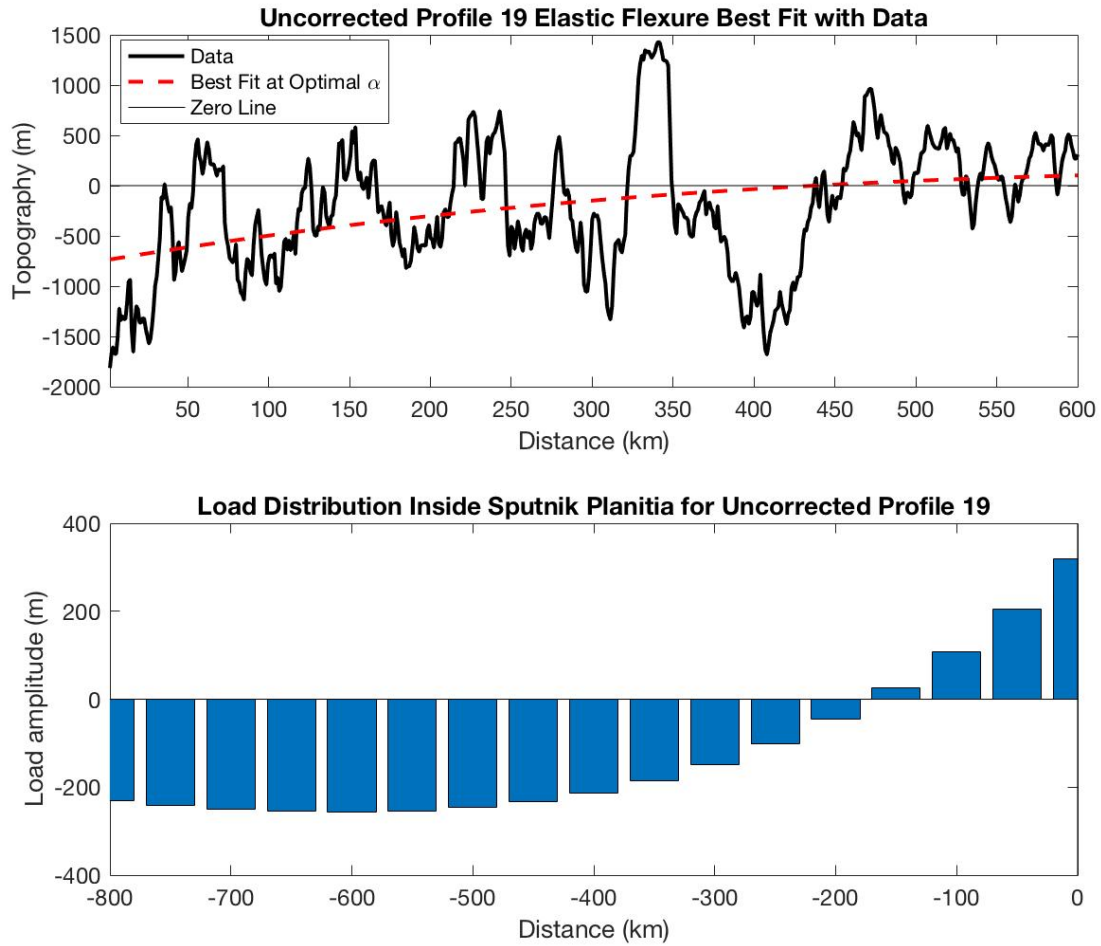


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

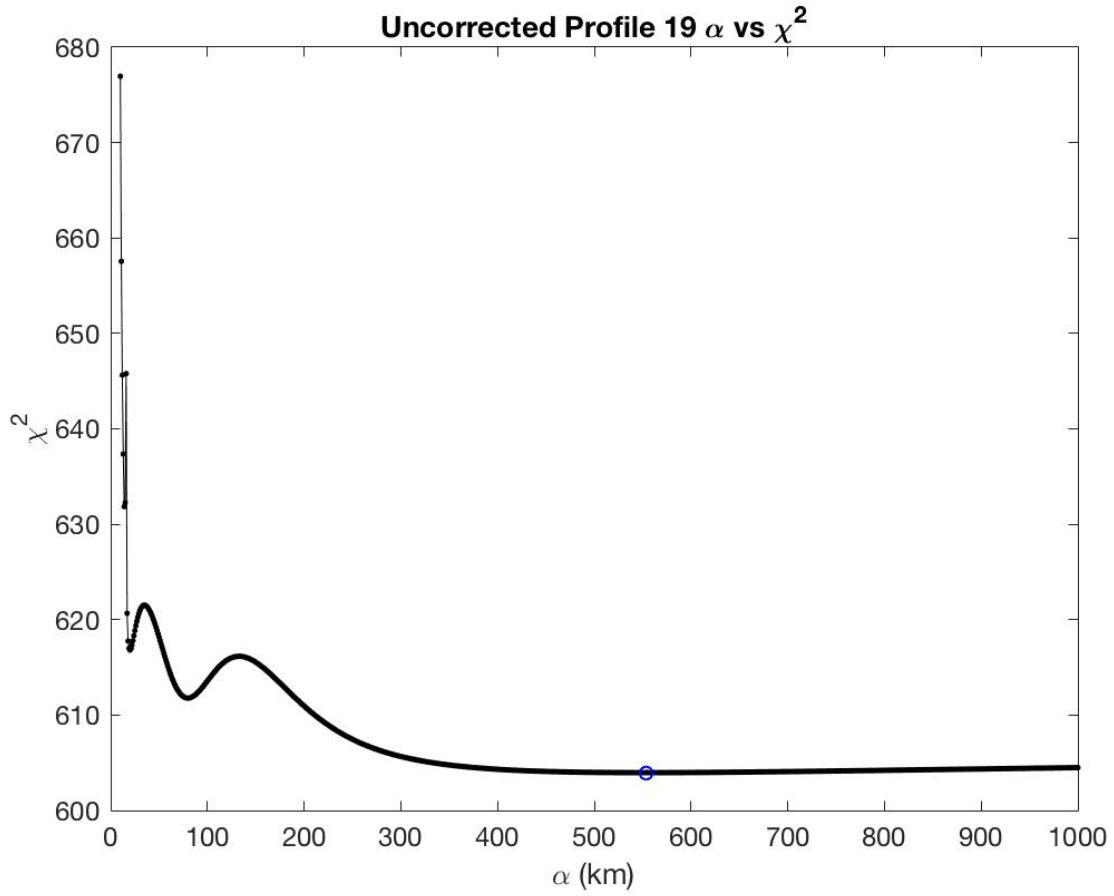


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

aa. Profile 19's Uncorrected, Load Amplitude Plot, and α Versus χ^2 Plot

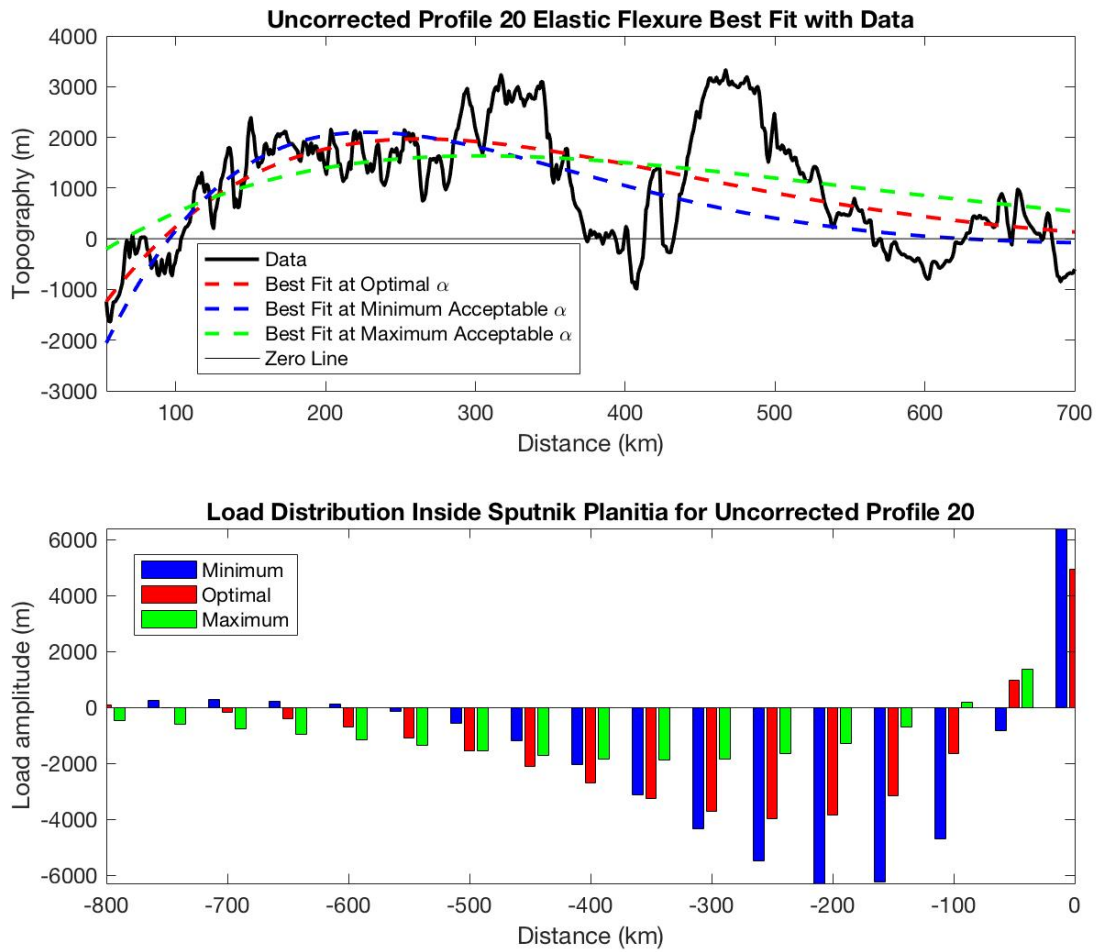


Track 19's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized. This plot has craters present in the data. Lower panel: Profile 19's corresponding load amplitudes of the best fit that corresponds to the acceptable range of α .

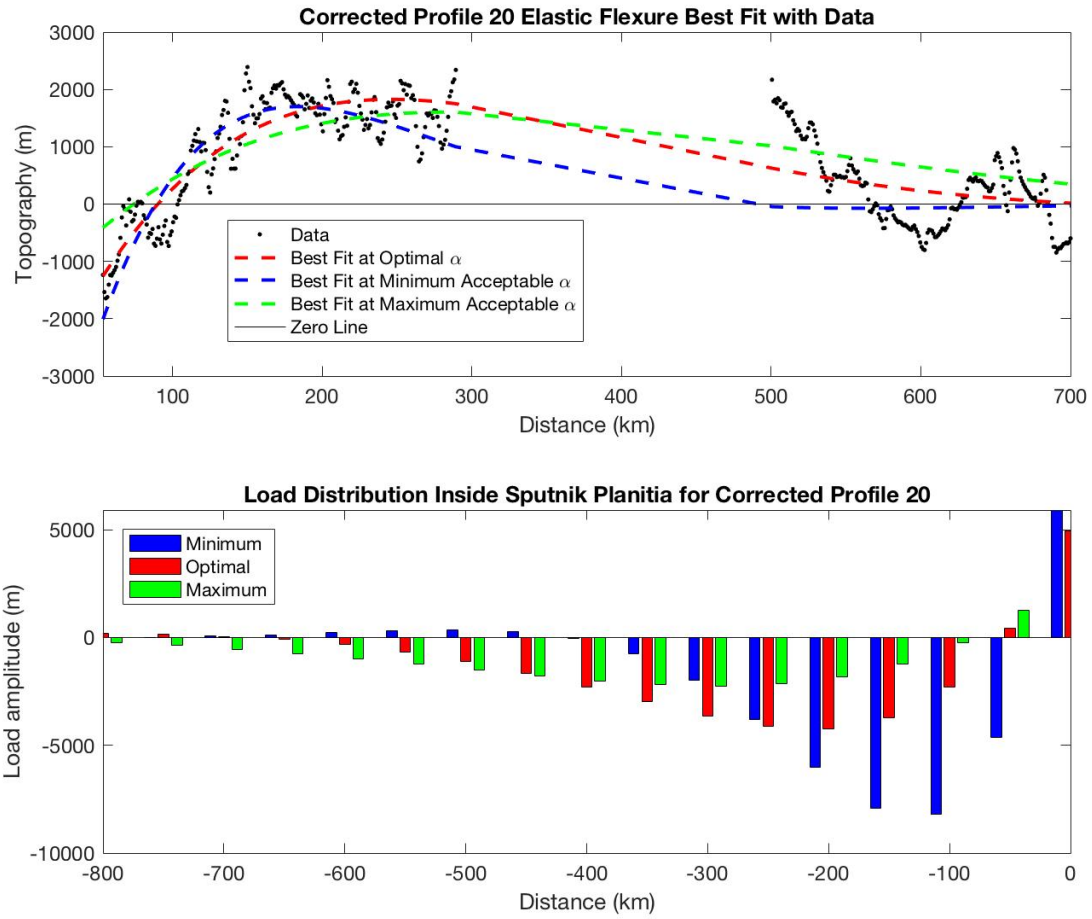


χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

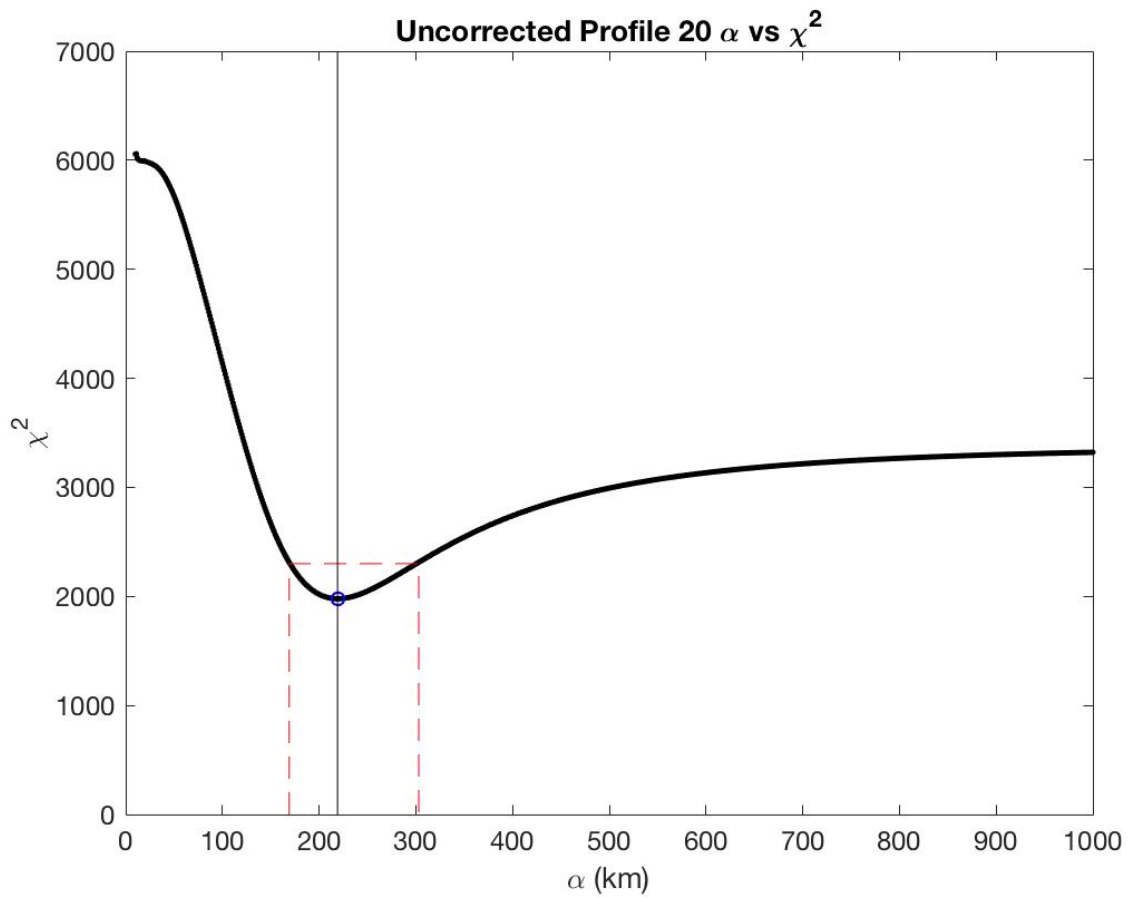
bb. Profile 20's Uncorrected and Corrected Profiles' Best Fit Plot, Load Amplitude Plot, and α Versus χ^2 Plot



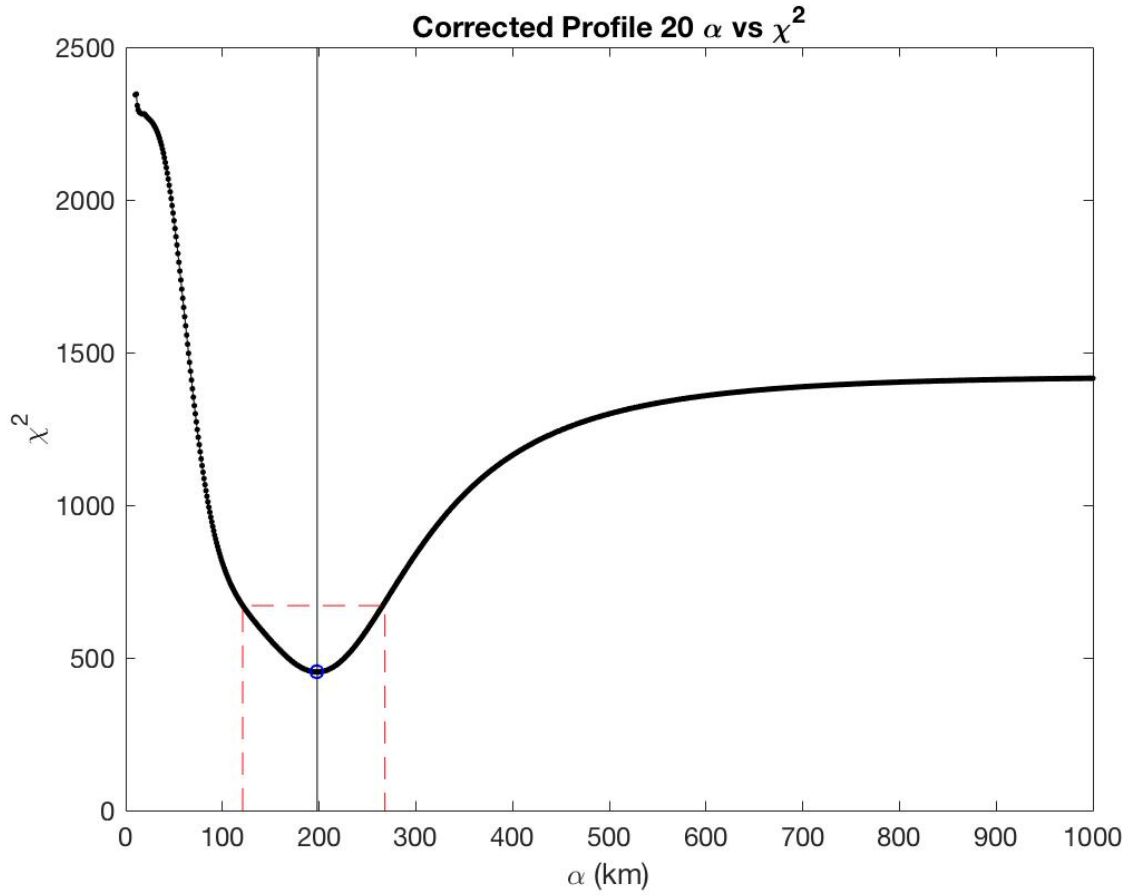
Track 20's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters present in the data. Lower panel: Profile 20's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .



Track 20's profile with the best fits from the acceptable range of α where the red dashed line is the best with the optimal α where χ^2 is minimized; the blue dashed line is the best fit with the minimum acceptable α ; and the green dashed line is the best fit with the maximum acceptable α . This plot has craters removed in the data. Lower panel: Profile 20's corresponding load amplitudes of the three best fits that correspond to the acceptable range of α .



χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.



χ^2 versus flexural parameter plot of track 9 where the χ^2 threshold is marked with a horizontal dashed line, and the acceptable flexural parameters within this threshold are within the dashed vertical lines. The black vertical line represents the best fit flexural parameter when χ^2 is minimized.

