

Understanding the Material Properties of the Ice-Bed Interface of the
Greenland Ice Sheet: Subglacial Lakes or Pools of Sediment

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Abstract

For this project, the goal was to determine the material properties of the ice-bed interface of the Greenland ice sheet, specifically at location L2 in northwestern Greenland. There are 54 proposed subglacial lakes in Greenland, with two being active; meaning they fill in the winter and drain in the summer. L2 is one of the proposed active lakes. Data collected by Palmer et al. (2013) determined the existence of these 54 candidate lakes. Those data were collected using radio echo sounding. Radio waves from an airborne instrument were emitted into the ice sheet, and the reflections were returned and later processed, and the topography of the ice-bed interface was determined based on the reflectivity. It was also proposed that these candidate lakes were full of water, also based on the reflectivity.

In 2018, Dr. Schmerr and his team carried out active source experiments over top of L2. Hammers were used to strike a metal plate, and the seismic waves traveled down to the ice-bed interface and were reflected up to a series of geophones. These data were used to determine the nature of the interface below location L2, whether it be water or frozen sediment, or if the ice sheet is just frozen to the bedrock. The program Reflectivity (Fuchs & Muller, 1971) was used for this project. This program produced synthetic seismograms that were run through programs that has been written, called ComparisonCodeWater, ComparisonCodeSediment, ComparisonCodeBedrock, and TimePicking (in the appendix). The synthetic seismograms were then compared with the seismograms collected from the active source experiments by observing their misfit by using L2 normalization using said programs. This comparison was used to determine whether the data line up with the models for water, saturated dilatant sediment, or bedrock.

Synthetic seismograms have been produced for 66 water models, 275 sediment models, and 108 bedrock models, with the parameters used shown in tables 1 through 4. Using Reflectivity and ComparisonCode, it was determined that the material at the ice-bed interface is likely hypersaline water.

Plain Text Abstract

The goal of this project was to better understand the properties of the material at the bottom of the Greenland ice sheet in northwestern Greenland, and to determine what underlies this ice sheet at location L2.

Based on previous experiments that took place over the past decade, it has been proposed that there are 54 lakes underneath the Greenland ice sheet. Two of these proposed lakes have been determined to be filling up with water in the winter and draining in the summer. Those data were collected by Palmer et al. in 2013, and are radar data, meaning that an instrument was flown over Greenland in order to map the physical features of the layer directly beneath the ice sheet. The radar waves were bounced back to an instrument on the airplane that detected changes in the depth of the ice sheet, mapping out the layer discussed above. This is how the 54 proposed lakes were detected.

In 2018, experiments were carried out over top of location L2 by Dr. Nick Schmerr and Ross Maguire, as well as other members of their team. These experiments used human-induced

seismic waves by striking a plate on the surface of the ice. As a result, waves traveled down into the Earth to the bottom of the ice sheet and reflected up to the surface. These waves were detected and measured by instruments called geophones, which are devices that are stuck into the ice to gather wave data. Those data have been used to determine what underlies the ice sheet, whether it is water, wet rock or the ice sheet is directly frozen to the bedrock.

A program called Reflectivity (Fuchs & Muller, 1971) has been used to generate computer-generated waves. Through different programs that have been written called ComparisonCodeWater, ComparisonCodeSediment, ComparisonCodeBedrock, and TimePicking (in the appendix), these computer-generated waves have been examined against the collected data in order to see how closely they match up, which determined what lies underneath the Greenland ice sheet at location L2. The material was determined likely to be water with a high salt content.

Introduction and Background

In June 2018, active and passive source experiments were carried out in northwestern Greenland (location shown in figure 1) in order to better understand the subglacial properties of the Greenland ice sheet. Active and passive source experiments were used, which are methods used to obtain information about Earth's subsurface. Examples of active and passive seismic source experiments are using a hammer on the Earth's surface to create seismic waves in the interior and measuring their reflection, and measuring the waves produced by earthquakes, respectively (Maguire et al., 2021). Prior to these experiments, radio-echo sounding (RES) was used over the Greenland ice sheet with the intention of identifying subglacial lakes (Palmer et al., 2013). Radio-echo sounding is a technique that uses airborne radar devices that emit waves into the ice, and the reflection of these waves is measured to give insight into the properties of said ice.

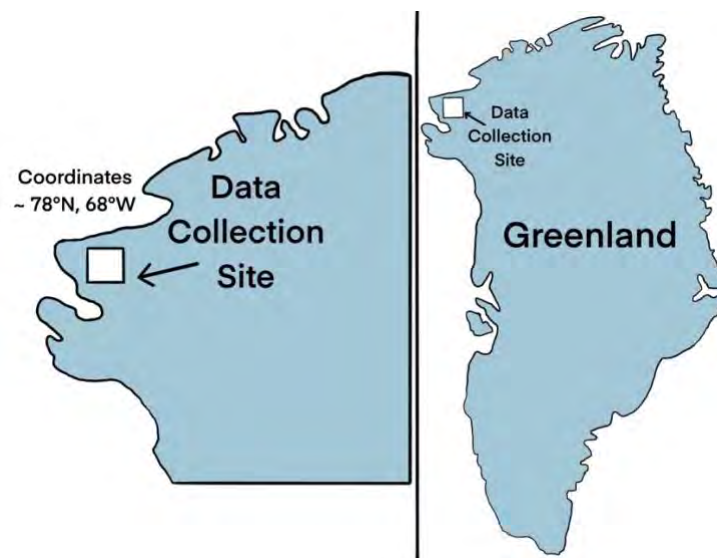


Figure 1.
Map of data collection site in northwestern Greenland.

Through the use of RES, 54 subglacial lakes were detected by Palmer et al. (2013). These lakes were found by analyzing the data from the RES and recognizing that there were “flat regions of anomalously high basal reflectivity below the northwestern Greenland ice sheet” (Maguire et al., 2021). This was determined by using the +10 to +20 dB amplitude anomalies of the ice-bed interface, which is the anomaly that subglacial lakes containing water are known to produce (Maguire et al., 2021). Frozen sediment produces a similar anomaly, with values ranging from -10 to -20 dB, which are the same absolute value, but with a different sign, so there could be a mistake in the initial identification of L2 as a lake full of water. This mistake could be due to overestimating the reflection coefficient due to uncertainties in the ice sheet, specifically the attenuation structure of the ice, which is the property of the wave that causes the amplitude to decrease as it passes through a medium.

A thin package of sediment underneath the lakes has been identified, as they present as a strong coda after the reflection of the lake bottom. These lakes were determined to lie under about 1-2 km of ice near the margins of the ice sheet. Two lakes were established to be active, meaning that they have seasonal draining and recharge cycles: they fill in the summer and drain in the winter (Bowling et al., 2019).

In comparison, in Antarctica, there have been over 400 subglacial lakes detected, and about 40% of them are considered to be active. The Antarctic subglacial lakes are larger than the Greenland lakes, and occur in different topographic settings, with a third of lakes in Greenland being found in small “bumps” of bedrock and a quarter surrounded by steep hills of bedrock. Alternatively, Antarctic lakes are found to lay in “tectonically controlled topographic depressions” (Bowling et al., 2019).

There is a possibility of fresh meltwater or hypersaline water, as well as frozen sediment underneath of L2. The possible hypersaline water under L2 may have been produced by the dissolution of a salt-bearing geological unit. This was observed in subglacial lakes under the Devon Glacier in Canada (Rutishauser et al., 2022). The Bay Fjord formation is the salt-bearing unit at the Devon Glacier that is predicted to outcrop at the ice-bed interface, providing the salt for the hypersaline water that makes up the subglacial lakes there. A similar situation may be what is happening under the Greenland ice sheet. It has been suggested that the lakes are products of storage of surface-derived meltwater (Palmer et al., 2015), which is water that is melting from the top of the ice sheet. Figure 2 shows an example of what a cross-section of the Greenland ice sheet might look like, showing how geothermal activity may be keeping new meltwater from freezing.

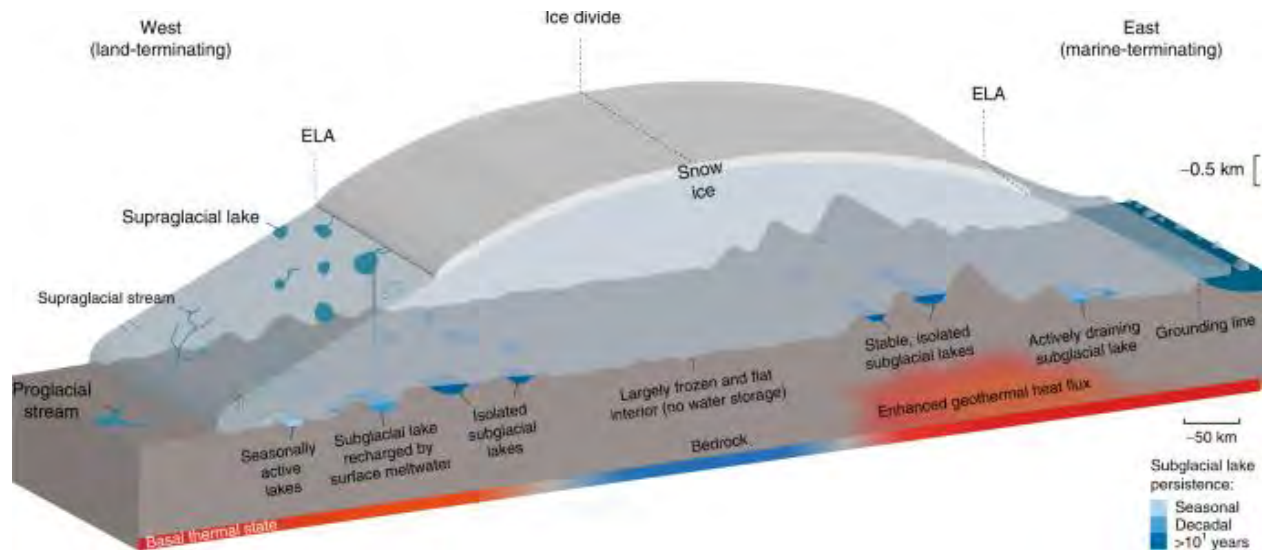


Figure 2.
Depiction of different types of lakes beneath the Greenland ice sheet, including active and isolated stable lakes (Figure from Bowling et al., 2019).

Underneath northwestern Greenland where L2 is located, the bedrock is the Precambrian shield, which was formed in the Paleoproterozoic Era. Knowing the time that the bedrock was formed can give us information on what type of rock it contains. The bedrock in this area is likely made up of granitoid and gneiss rocks (Dawes, 2009). The composition of the bedrock will have an impact on the velocity of P and S-waves that travel through it.

Objectives and Broader Impacts

For this thesis project, the null hypothesis is that there is no water or sediment at the ice-bed interface, and the ice sheet is frozen directly to the bedrock. The two alternative hypotheses are: the material at the ice-bed interface is water, and the material at the ice-bed interface is frozen sediment.

For the first hypothesis, there are different possibilities of the type of water that could be present. These types of water include hypersaline water, or warm meltwater. This water must be hypersaline or warmed by geothermal activity because the basal temperature is about -8°C (Palmer et al., 2013), and freshwater would freeze at that temperature, but hypersaline water remains liquid (Killingbeck et al., 2021).

Subglacial lakes are newly detected features assumed to be associated with global climate change. As temperatures increase, ice melts, and this melt could fill in areas of depression beneath ice sheets. This new meltwater may be the cause of subglacial lakes. Ice sheets are not stationary; they move along the bedrock. Ice sheets are also split up into smaller pieces that move in different directions, and they meet in areas called ice divides. Water below the ice sheet causes the ice to move faster due to less friction (Willis et al., 2015). Given that different parts of the ice sheet move in different directions, some ice is headed towards the

ocean. If the amount of water under an ice sheet increases, the ice will move more quickly toward the ocean, and eventually, it will fall into the ocean, thus raising sea levels.

If the water beneath ice sheets is not caused by rising temperatures, then there is less cause for concern, as the amount of water should stay relatively constant and will not affect glacial flow. However, even though the situation may not seem as dire if the material is sediment and not water, the Greenland ice sheet is losing mass at an increasing rate, with about half of this loss due to surface meltwater runoff.

It is estimated that if the annual average temperature in Greenland increases by about 3°C , the average sea level could be raised 7 m over the course of 1,000 or more years (Gregory et al., 2004). If the temperature in Greenland increases by 8°C , the entire ice sheet could melt away over a 1,000 year or less time span.

Experiment Design

Data for this were collected by Dr. Schmerr and his team in 2018. These data were collected by the active source experiments briefly discussed above. For this experiment, twenty-four 40 Hz geophones were placed 5 m apart along a line. An 8 kg hammer was used to hit a 1.5 cm thick steel plate, in a process called a “shot”. There were 40 shot locations across a 2400 m traverse, with each location having been subjected to at least five shots, that were subsequently stacked into a single shot, a process that increases the signal-to-noise ratio. Because of the geophones being placed 5 m apart, there are reflection points at the ice bottom every 2.5 m across the traverse, which can be seen in figure 4. A geophone is pictured below in figure 5. The silver stake is placed in the ice, and the cable is used to transmit data. Figure 6 shows an image of an active source experiment.

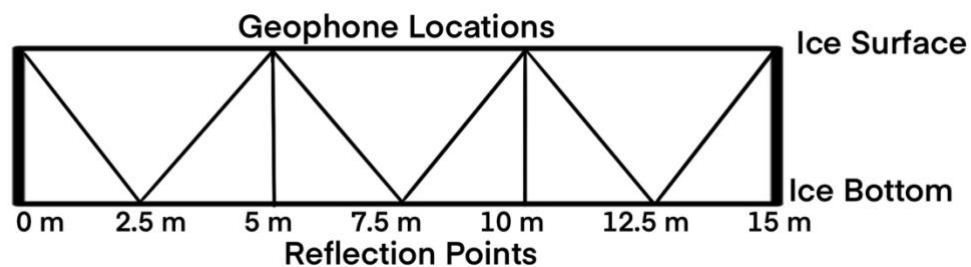


Figure 4.

An example of how reflection points are present every 2.5 m.



*Figure 5.
Example of a geophone used in the active source experiments.*



*Figure 6.
A member of Dr. Schmerr's team (Dr. Erin Pettit) conducting the hammer strike of the active source experiment.*

The active seismic experiment earlier discussed took place at a location above L2, one of the proposed active subglacial lakes. L2 is situated within a 980 km² drainage basin, is less than 10 km from the ice divide, and is within the accumulation area (the area in which snow and ice collect). Figure 7 illustrates a map of Greenland with the ground penetrating radar (GPR) profile shown in orange, and the first 10 shot locations for each geophone are plotted as green stars. Figure 8 shows the bed reflection for L2. The reflection starts at around 500 m on the y-axis which represents elevation, which is about 809 m below the surface of the ice. As the experiment moves northeast along the traverse, a change within the bed reflection can be seen; the distance below the ice surface decreases. Figure 9 shows an example of what the shape of the bed reflection could represent. Figure 10 depicts an example of the traverse location, as it moves from inside of L2 to outside. For figures 8, 9, and 10, the ice divide is located to the left, which is indicative of the west.

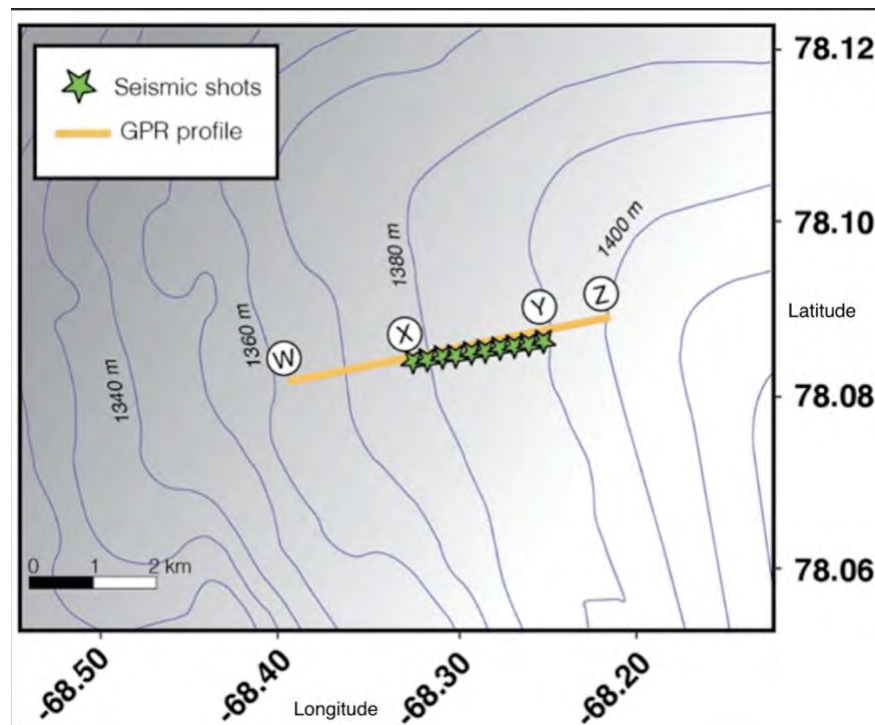


Figure 7.

Example of seismic shots along the Ground Penetrating Radar (GPR) survey in northwestern Greenland, with the orange line showing the survey, and the stars showing the seismic shots (Figure from Maguire et al., 2021). The points W, X, Y, and Z are shown as different locations along L2 in figure 7. W and Z are the ends of the GPR profile, and X and Y are the ends of the shot locations.

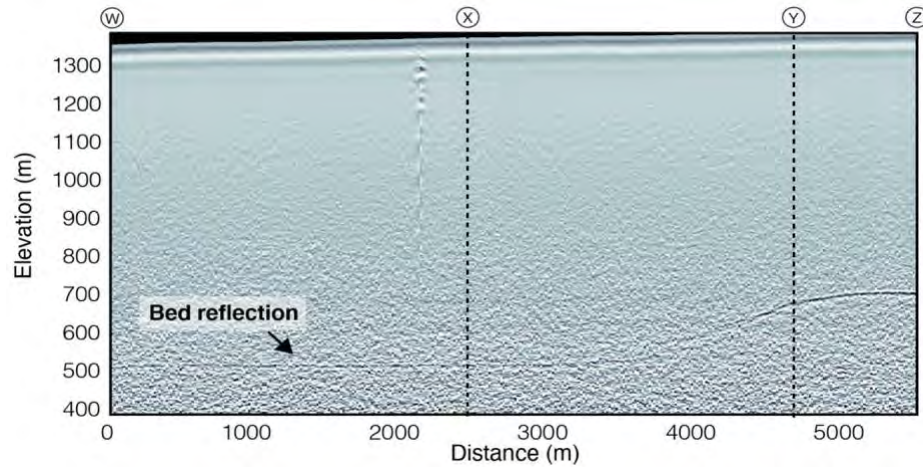


Figure 8.

GPR profile. Bed reflection is shown starting around 500 m (Figure from Maguire et al., 2021).

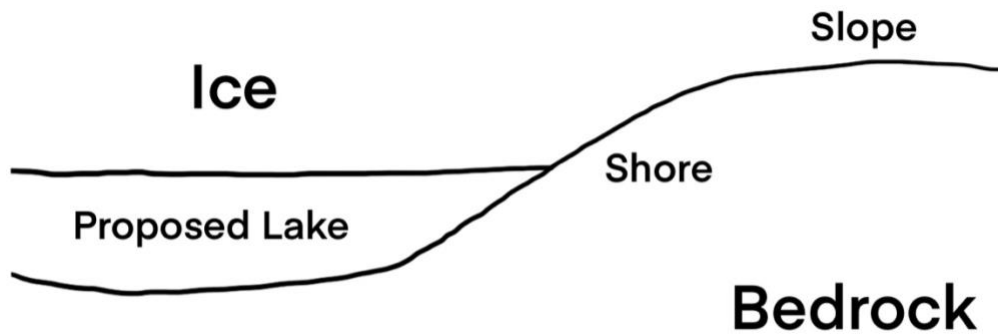


Figure 9.

Lake shape example.

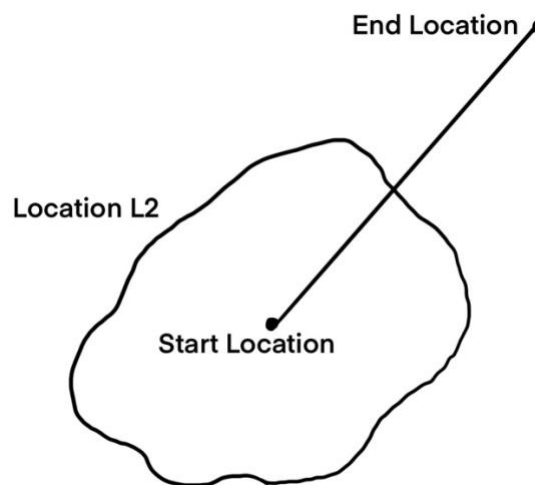


Figure 10.

Traverse example.

Due to the possible salinity of the proposed water underneath L2, a range of P-wave velocities were used to produce the water models created by Reflectivity. Thickness, P-wave velocity (vp), S-wave velocity (vs), and density were varied for sediment and bedrock models. These parameters can be seen in the tables below. There are two different types of bedrock used for the modeling, because there is a possibility for lithified sediment (sediment compacted due to pressure), or bedrock underneath L2. 66 models were run for the water parameters, 120 for the sediment parameters, and 108 for the lithified sediment and bedrock.

Water Input Parameters			
	min	max	Δ
vp (km/s)	1.500	1.800	0.030
vs (km/s)	0.000	0.000	0.000
thickness (km)	0.005	0.030	0.005
density (g/cm³)	1.000	1.000	0.000

Table 1.
Water input parameters.

Dilatant Sediment Input Parameters			
	min	max	Δ
vp (km/s)	1.600	1.800	0.050
vp/vs ratio (km/s)	1.750	2.050	0.100
thickness (km)	0.005	0.030	0.005
density (g/cm³)	1.700	1.700	0.000

Table 2.
Dilatant Sediment input parameters.

Bedrock Input Parameters (Lithified Sediment)			
	min	max	Δ
vp (km/s)	3.000	3.750	0.250
vp/vs ratio (km/s)	1.730	1.730	0.000
thickness (km)	1.000	1.000	0.000
density (g/cm³)	2.200	2.450	0.050

Table 3.
Lithified sediment input parameters.

Bedrock Input Parameters			
	min	max	Δ
vp (km/s)	5.200	6.200	0.250
vp/vs ratio (km/s)	1.730	1.730	0.000
thickness (km)	1.000	1.000	0.000
density (g/cm ³)	2.700	2.800	0.020

Table 4.
Bedrock input parameters.

The program Reflectivity has been used to model synthetic seismograms for this project. The program calculates the reflection and transmission coefficients for the stack of layers from parameter input into the program. The program then returns a synthetic seismogram that predicts the reflection and gives the full waveform prediction including the amplitude and time of the wave (Fuchs & Muller, 1971).

Figure 11 shows a synthetic seismogram produced from Reflectivity. Given that this seismogram is assuming that the layer is water, the value inputted for the thickness is 0.015 km, the P-wave velocity is 1.500 km/s, and the S-wave velocity is 0.000 km/s because S-waves do not travel through water.

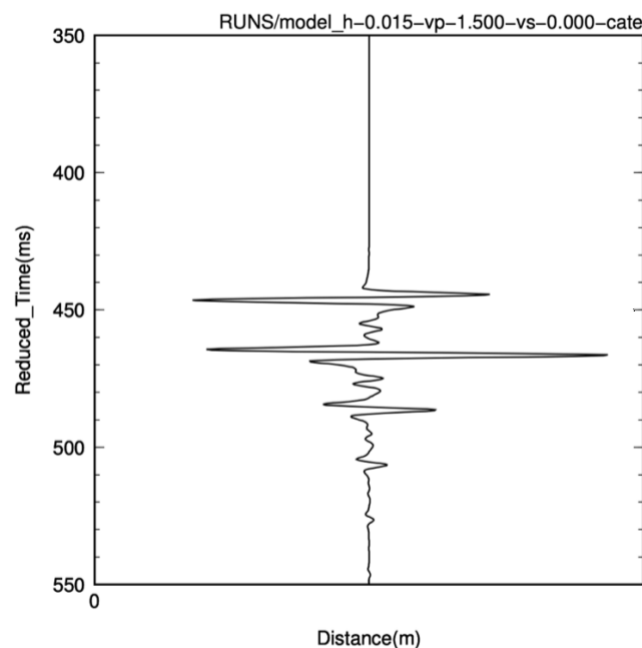


Figure 11.

Synthetic seismogram example from Reflectivity. The parameters for this model are 0.015 km for thickness, 1.500 km/s for vp, 0.000 km/s for vs, and 1.000 g/cm³ for density.

P-wave velocities will vary in the material that is assumed to be water in this model, depending on salinity. P-wave velocity will also vary in the bedrock depending on composition, which as stated before, it is known to be a mixture of gneiss and granitoid rock (Dawes, 2009). The value in this figure for water is representative of freshwater, and the bedrock velocity is a general estimation. Figure 12 depicts the model generated depth, velocity, and density profile of the parameters discussed above, the v_p is set to 1.500 km/s, the S-wave velocity is 0.000 km/s, and the thickness of the layer is 0.015 km.

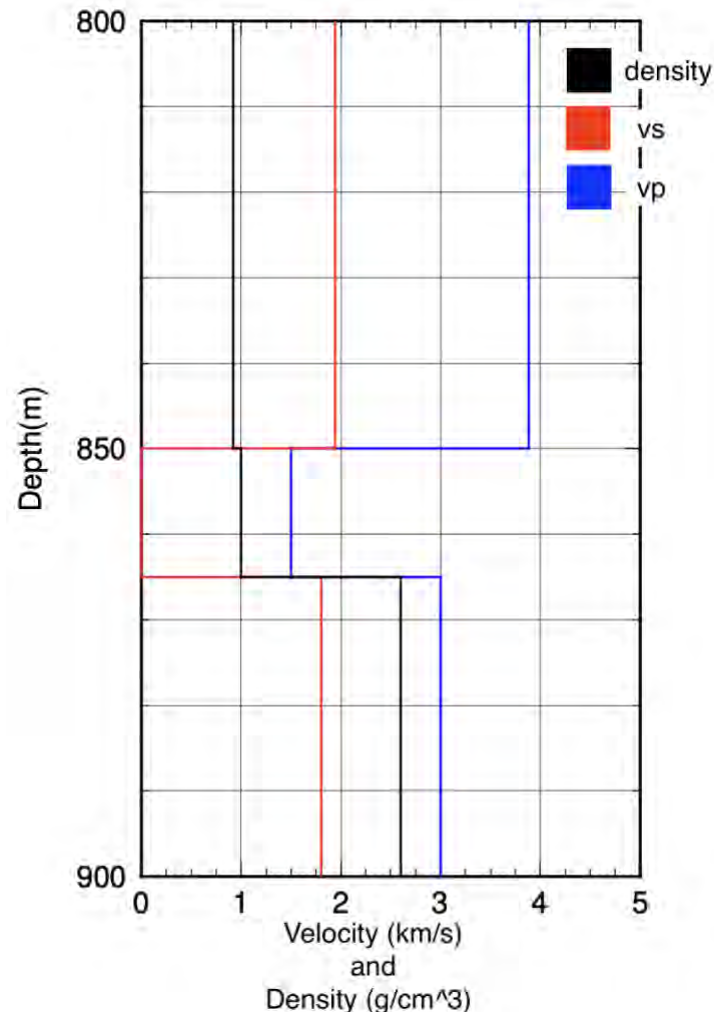


Figure 12.

Depth, velocity, and density profile of a water model with thickness of 0.015 km, v_p of 1.5 km/s, v_s of 0.000 km/s, and a density of 1.000 g/cm³. The red line represents S-wave velocity, the black represents density, and the blue represents P-wave velocity.

Results

In order to determine what the material is under location L2, the models must be compared to the collected data. The comparison has been carried out using a program that has been written called ComparisonCode that evaluates the misfit between data and synthetic seismograms using L2 normalization. There are three different iterations of ComparisonCode, one for each hypothesis. They are: ComparisonCodeWater, ComparisonCodeSediment, and ComparisonCodeBedrock.

The amplitudes of the different traces were compared with the seismograms at different times. The time was clipped out of the full seismograms by displaying the full seismograms using the program TimePicking that was written, and then using a tool in MATLAB called data tips to select the arrival of the P-wave. The seismograms were then clipped so the arrivals on the seismogram from the data and the synthetic seismogram match up.

There is some uncertainty in this because the time is picked by hand. The data tips tool gives a value only to the ten millionths place. The values were reported to the ten thousandths place due to time constraints.

To properly analyze the seismograms, they had to be scaled, because the amplitudes can vary greatly (as can be seen in figures 16 and 18). This was done by dividing the amplitude by the maximum amplitude of the seismogram. This is also known as normalizing. The raw seismogram from data and the seismogram from data that has been clipped and normalized can be seen in figures 13 and 14, respectively.

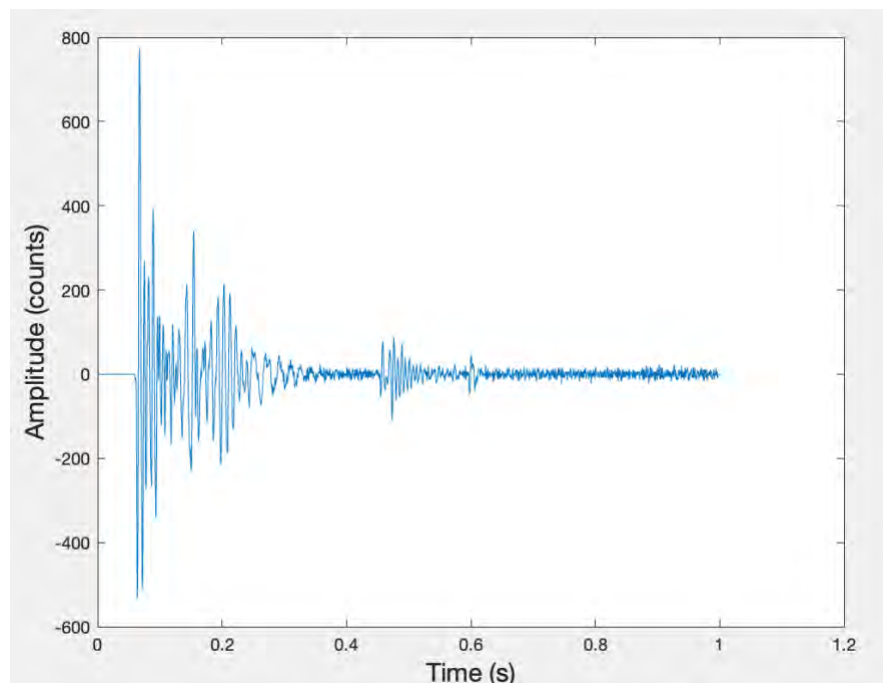


Figure 13.
Raw seismogram from data.

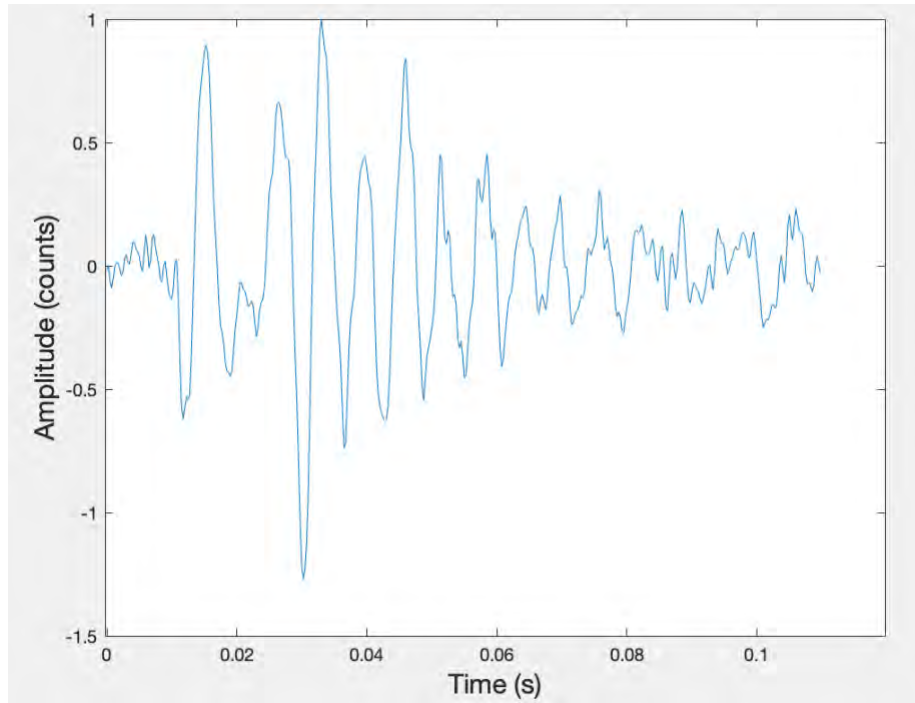


Figure 14.

Seismogram from data that has been clipped to the arrival of the P-wave and normalized to have 1 as the maximum.

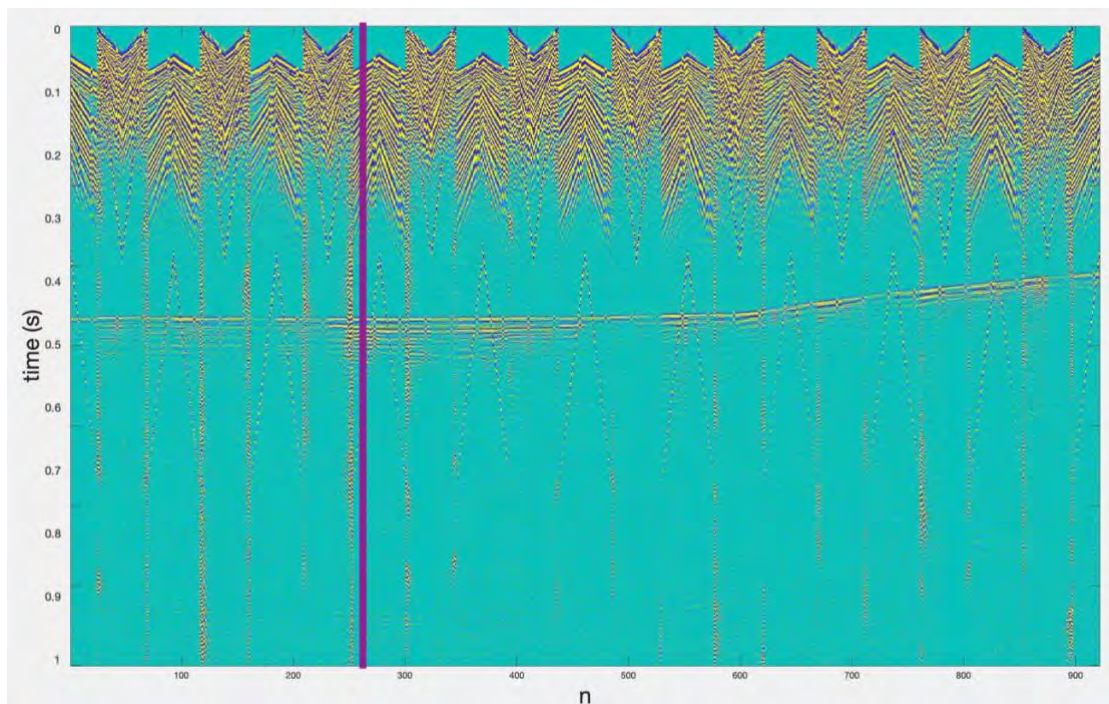


Figure 15.

Data from the active source experiment. N is the index of the seismograms. The bed reflection can be seen starting at around 0.45 seconds on the y-axis. The purple line at $n=261$ is the index that the data was taken from for this project.

In order to analyze the closeness of the two seismograms, a L2 normalization was used. This is called analyzing the misfit. When the misfit is zero, that means the two items being compared are equal, so the closer the misfit is to zero, the closer the items are to each other. This project will focus on the results from L2 normalization, which focuses on the difference in the synthetic to the data squared, so there is an over emphasis on the large differences.

For the water parameters, a best fitting model was determined. As can be seen in figure 16, there is a global minimum at model index 55. This means that the misfit is lowest at this point, indicating that this is the best-fitting model for water. The parameters input for this model are as follows; thickness of 0.025 km, v_p of 1.800 km/s, v_s of 0.000 km/s, and density of 1.000 gm/cm³. The L2 misfit value for this model was determined to be 41.17.

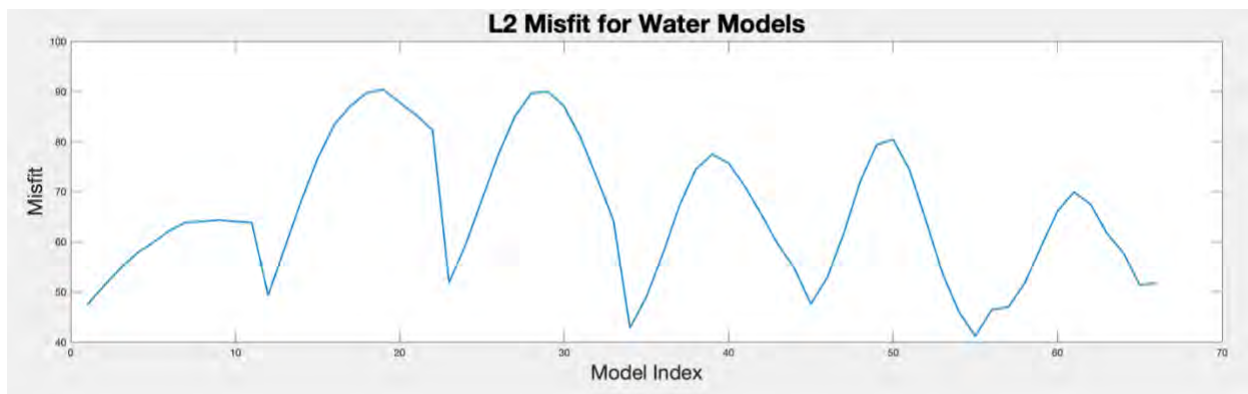


Figure 16.
Plot of L2 misfit values for index $n=261$ for the water models.

Another way to visualize the misfit is by looking at the normalized relative misfit. The misfit has been normalized, with the mean being 1. This can be seen in figure 17. The lighter the blue, the lower the relative misfit, so the lowest relative misfit can be seen as a thickness of 0.005 km and a v_p of 1.800 km. The lowest misfit is about 38% lower than the mean, and the highest misfit is about 37% higher than the mean. The highest and lowest misfits vary by about 75%.

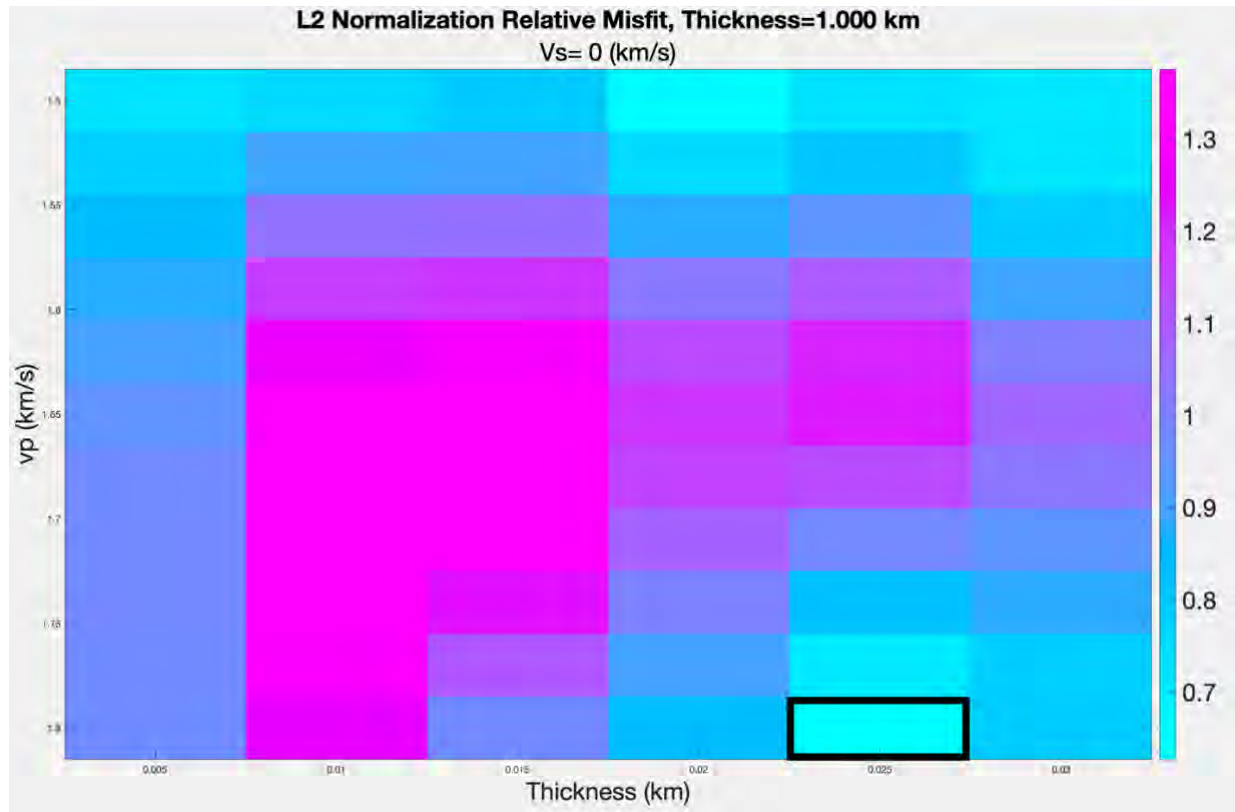


Figure 17.

Plot of relative L2 misfit values for index $n=261$ for water models. The lower the relative misfit, the lighter the blue, and the higher the relative misfit, the pinker it becomes. The best fitting model is outlined in black.

The synthetic seismogram for water model 55 and the seismogram from data can be seen plotted together in figure 18. The first arrival of the P-wave in the data is matched well with the model. This can be seen in figure 19; there is a low value of misfit at that point, before the time of 0.02 s. The misfit increases from that time until the second arrival, at around 0.04 s.

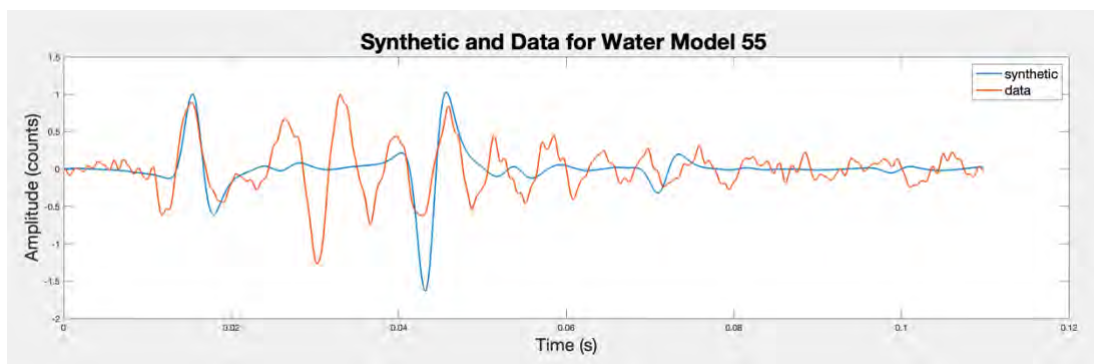


Figure 18.

Synthetic seismogram and seismogram from data plotted together at index $n=261$ for water model 55.

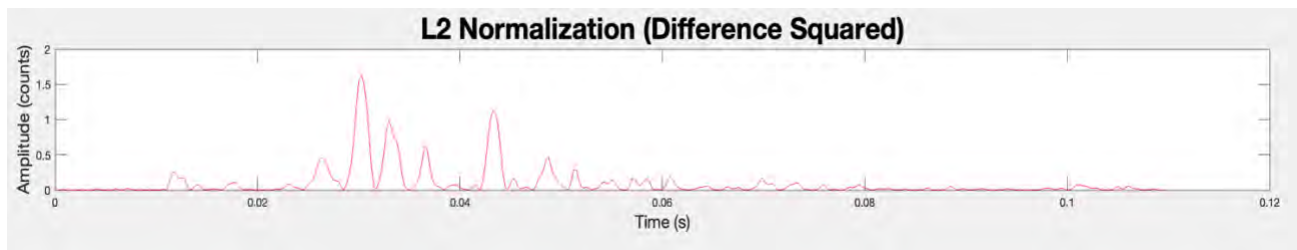


Figure 19.
L2 normalization for water model 55 at index $n=261$.

A best fitting model was also found for the sediment models. In figure 20, there is a global minimum at model index 100. This model's parameters are a thickness of 0.025 km, v_p of 1.800 km/s, v_s of 1.750 km/s, and a density of 1.700 g/cm³. The L2 misfit for this model is 43.82. The normalized relative misfit for sediment models is shown in figure 21. The values of misfit vary by about 65%.

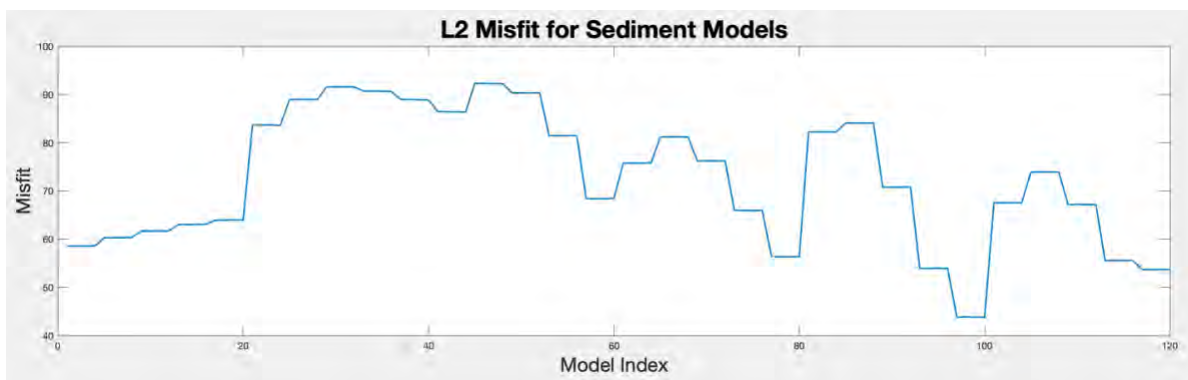


Figure 20.
Plot of L2 misfit values for index $n=261$ for the sediment models.

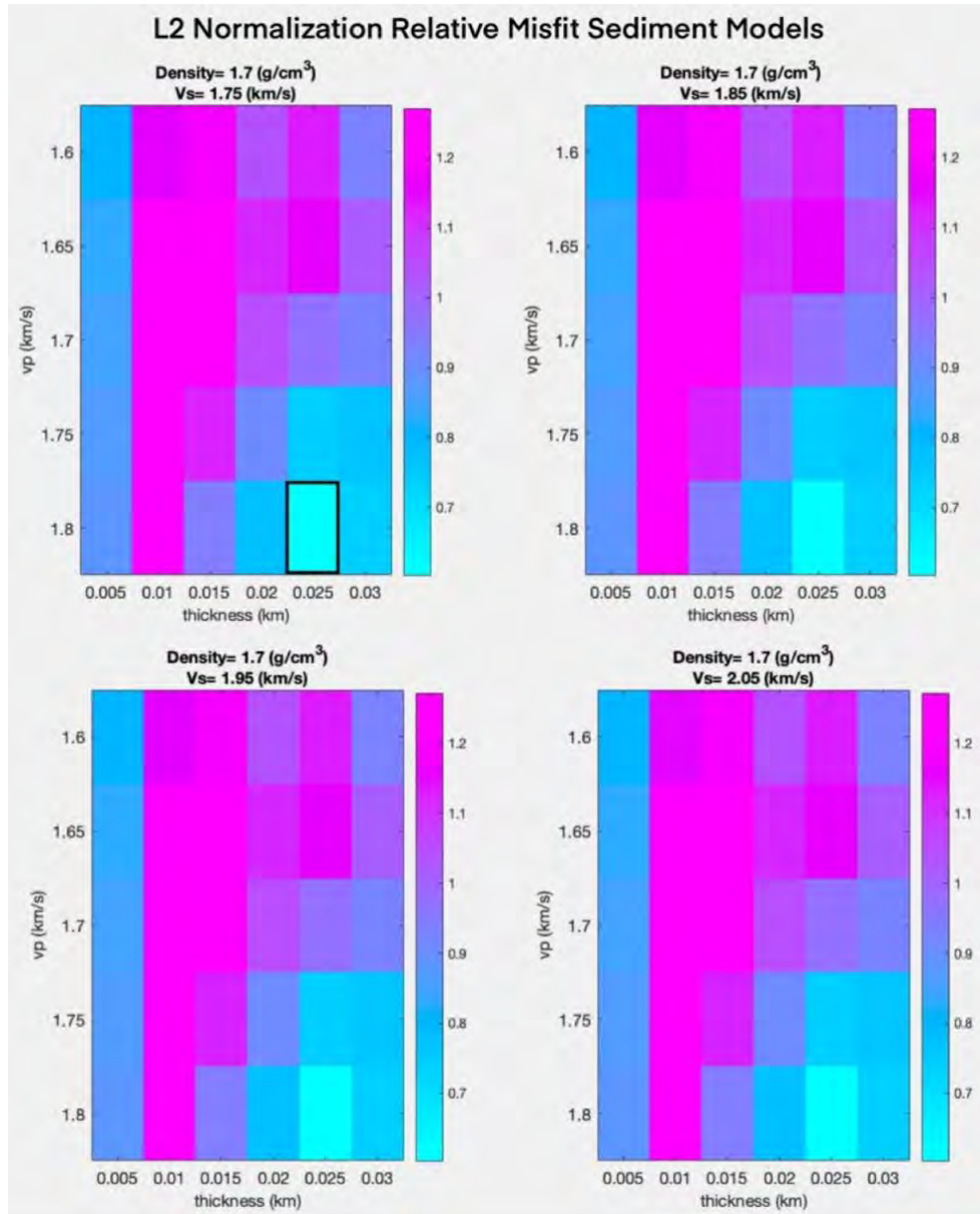


Figure 21.

Plot of relative L2 misfit values for index $n=261$ for sediment models. The lower the relative misfit, the lighter the blue, and the higher the relative misfit, the pinker it becomes. The best fitting model is outlined in black.

In figure 22, the synthetic seismogram for sediment model 100 and the seismogram from the data are plotted together. The L2 normalization plot follows a similar trend as the water model in figure 18. The misfit is higher at the first arrival because the model and data do not match up as well in comparison from figure 22 to figure 18; the amplitude in figure 22 is smaller.

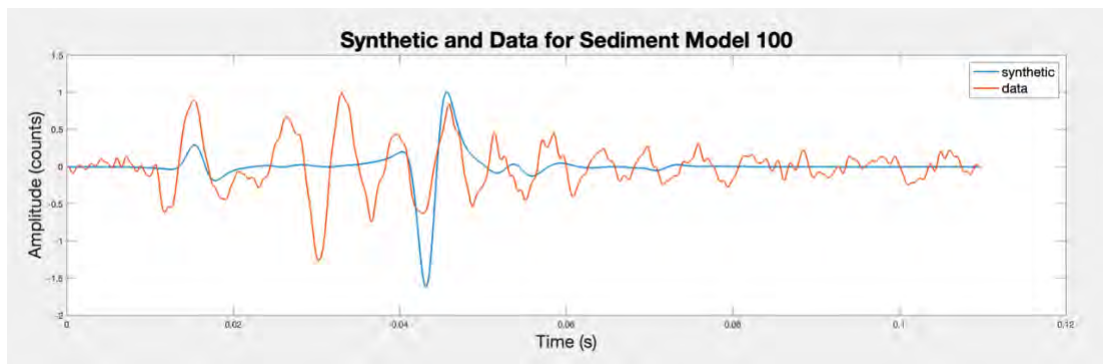


Figure 22.

Synthetic seismogram and seismogram from data plotted together at index $n=261$ for sediment model 100.

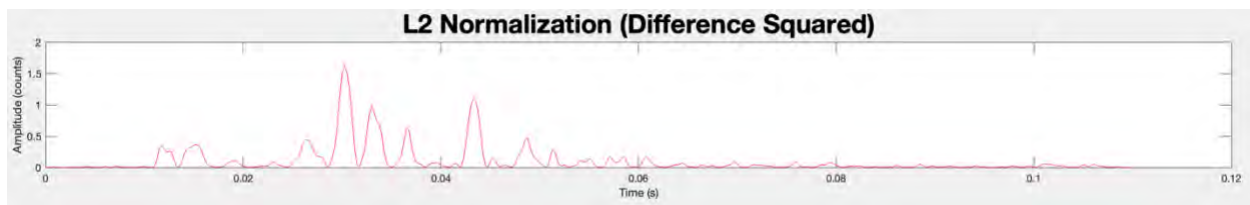


Figure 23.

L2 normalization for sediment model 100 at index $n=261$.

The best fitting model for the bedrock and lithified sediment models was determined to be model number 108, which can be seen as the global minimum in figure 24. Model 108 is a model for bedrock, not lithified sediment. The parameters for this model are a thickness of 1.000 km, v_p of 6.200 km/s, v_s of 1.7341 km/s, and density of 2.800 g/cm³. The L2 misfit for this model is 94.31. All of the bedrock models fall under a very similar value in misfit. The largest and smallest values vary by less than one hundred thousandth of a percent once normalized, which can be seen in figure 25.

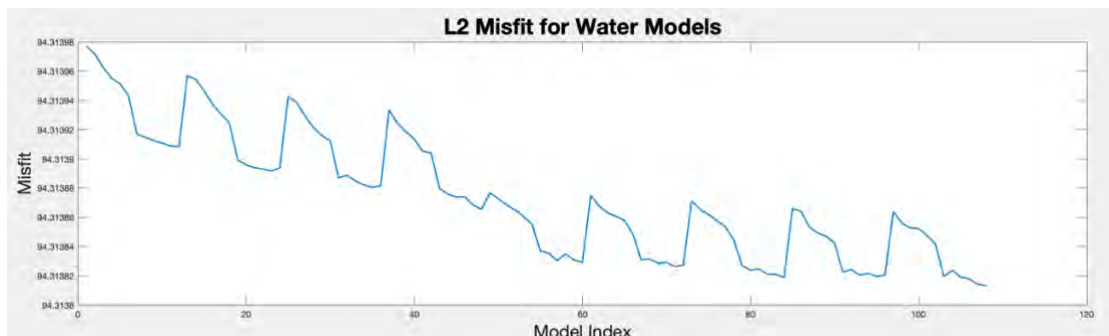


Figure 24.

Plot of L2 misfit values for index $n=261$ for the bedrock models.

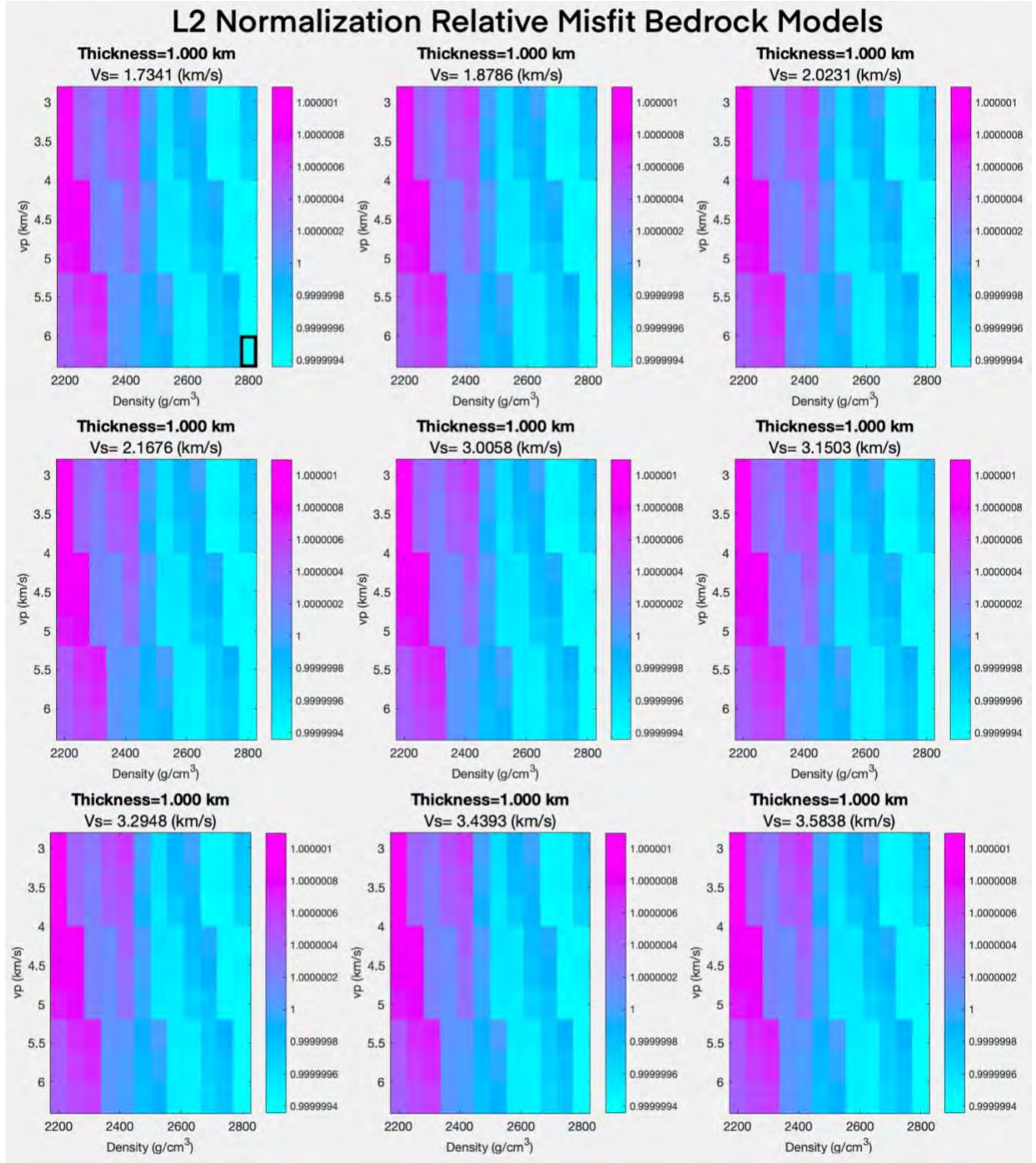


Figure 25.

Plot of relative L2 misfit values for index $n=261$ for bedrock models. The lower the relative misfit, the lighter the blue, and the higher the relative misfit, the pinker it becomes. The best fitting model is outlined in black.

Figure 26 shows the synthetic seismogram plotted with the seismogram from the data. The L2 normalization shown in figure 27 depicts a large amount of misfit at the time of the first arrival. This is because at the first arrival before the time of 0.02 s, the synthetic seismogram and the seismogram from the data are opposite each other.

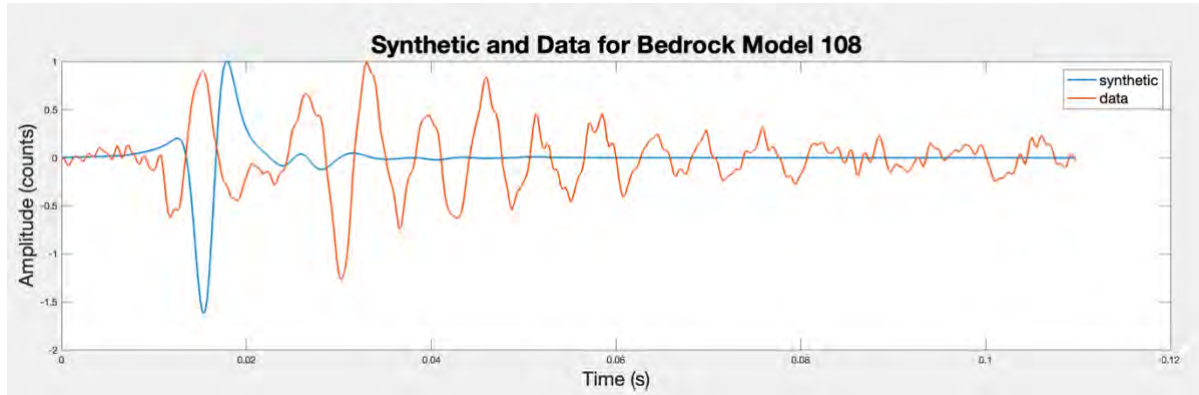


Figure 26.

Synthetic seismogram and seismogram from data plotted together at index $n=261$ for bedrock model 108.

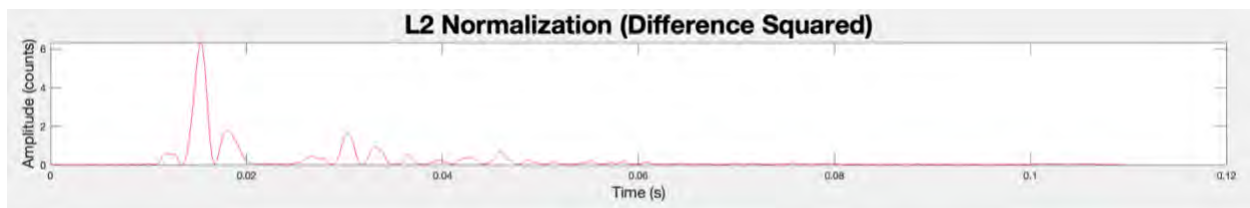


Figure 27.

L2 normalization for sediment model 108 at index $n=261$.

Based on the results from the water, sediment, and bedrock models, it can be concluded that the material at the ice-bed interface under location L2 is likely water. The closest matching water model had the lowest value for misfit. Because the v_p for the closest fitting water model is higher than 1.500, which is the v_p for freshwater, the water at the ice-bed interface is likely hypersaline. This is because the v_p for hypersaline water is higher than that of freshwater.

Suggestions for Future Work

Time constraint was the limiting factor in this project. Because of this time restraint, only 449 models were produced. The ideal number of models would be over 1000 to ensure that all of the parameters were tested.

An example of having to cut down on models and parameters being used is the density of the water. Fresh meltwater has a density of 1.000 g/cm^3 , but hypersaline water can have a density of over 1.010 g/cm^3 . Having a range of densities for water could lead to more accurate results. Also having a smaller step size in parameters would produce more models, because there would be more parameters in the same range. If there are more parameters, there is a higher accuracy in fitting the models.

Another change that may lead to more accurate results is changing the properties of the attenuation of the materials in the Reflectivity program. The attenuation of a material is a property of material that causes waves travelling through to dissipate as it passes through the material. This was not changed in the code when switching from water to sediment and bedrock, so all of the materials have the same attenuation as water.

Summary

The objective of this project is to determine what lies beneath the Greenland ice sheet, specifically what is below location L2. Data have already been collected, and models have been run in order to collect hundreds of synthetic seismograms with varying properties. These models have been compared against the previously collected data by using L2 normalization to determine which of the models closely match the data.

The null hypothesis that the ice is directly frozen to the bed has been refuted. The alternative hypothesis has been supported; the material at the ice-bed interface is likely water. The properties of the interface have been determined to likely be a thickness of 0.025 km , v_p of 1.800 km/s , vs of 0.000 km , and a density of 1.000 g/cm^3 .

The results of this project can help to expand the knowledge of the effects of climate change and even the knowledge of bodies in the solar system. If the material under the ice sheet is new meltwater, then that is likely due to a rise in global temperatures. As stated in an earlier section, when there is water beneath an ice sheet, basal friction decreases, and the ice can slide off into the ocean more readily, raising sea levels. If the material under the ice sheet is hypersaline water or water that is heated by geothermal activity, this can relate to icy bodies in our solar system such as Enceladus (Olsen et al., 2021), and can help further the knowledge about those bodies.

Acknowledgements

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Bibliography

- Bowling, J. S., Livingstone, S. J., Sole, A. J., & Chu, W. (2019). Distribution and Dynamics of Greenland Subglacial Lakes. *Nature Communications*, 10(2810).
<https://doi.org/10.1038/s41467-019-10821-w>
- Dawes, P. R. (2009). The Bedrock Geology Under the Inland Ice: the Next Major Challenge for Greenland Mapping. *Geological Survey of Denmark and Greenland Bulletin*, 17, 57–60.
<https://doi.org/10.34194/geusb.v17.5014>
- Fuchs, K., & Muller, G. (1971). Computation of Synthetic Seismograms with the Reflectivity Method and Comparison with Observations. *Geophysical Journal International*, 23(4), 417–433. <https://doi.org/10.1111/j.1365-246x.1971.tb01834.x>
- Gregory, J. M., Huybrechts, P., & Raper, S. C. (2004). Threatened Loss of the Greenland Ice-Sheet. *Nature*, 428(6983), 616–616. <https://doi.org/10.1038/428616a>
- Kapitsa, A. P., Ridley, J. K., de Q. Robin, G., Siegert, M. J., & Zotikov, I. A. (1996). A large deep freshwater lake beneath the ice of Central East Antarctica. *Nature*, 381(6584), 684–686.
<https://doi.org/10.1038/381684a0>
- Killingbeck, S. F., Dow, C. F., & Unsworth, M. J. (2021). A Quantitative Method for Deriving Salinity of Subglacial Water Using Ground-based Transient Electromagnetics. *Journal of Glaciology*, 68(268), 319–336. <https://doi.org/10.1017/jog.2021.94>
- Li, Y., & Hewett, B. (2016). Measurement of seawater average velocity using water bottom multiples from vertical seismic profile surveys. *Interpretation*, 4(4).
<https://doi.org/10.1190/int-2016-0022.1>
- Maguire, R., Schmerr, N., Pettit, E., Riverman, K., Gardner, C., DellaGiustina, D. N., Avenson, B., Wagner, N., Marusiak, A. G., Habib, N., Broadbeck, J. I., Bray, V. J., & Bailey, S. H. (2021). Geophysical Constraints on the Properties of a Subglacial Lake in Northwest Greenland. *The Cryosphere*, 15(7), 3279–3291. <https://doi.org/10.5194/tc-15-3279-2021>
- Olsen, K. G., Hurford, T. A., Schmerr, N. C., Huang, M., Brunt, K. M., Zipparo, S., Cole, H. M., & Aster, R. C. (2021). Projected Seismic Activity at the Tiger Stripe Fractures on Enceladus, Saturn, From an Analog Study of Tidally Modulated Icequakes within the Ross Ice Shelf, Antarctica. *Journal of Geophysical Research: Planets*, 126(6).
<https://doi.org/10.1029/2021je006862>
- Palmer, S. J., Dowdeswell, J. A., Christoffersen, P., Young, D. A., Blankenship, D. D., Greenbaum, J. S., Benham, T., Bamber, J., & Siegert, M. J. (2013). Greenland Subglacial Lakes Detected

by Radar. *Geophysical Research Letters*, 40(23), 6154–6159.
<https://doi.org/10.1002/2013gl058383>

Palmer, S., McMillan, M., & Morlighem, M. (2015). Subglacial Lake Drainage Detected Beneath the Greenland Ice Sheet. *Nature Communications*, 6(1).
<https://doi.org/10.1038/ncomms9408>

Rutishauser, A., Blankenship, D. D., Sharp, M., Skidmore, M. L., Greenbaum, J. S., Grima, C., Schroeder, D. M., Dowdeswell, J. A., & Young, D. A. (2018). Discovery of a hypersaline subglacial Lake Complex beneath devon ice cap, Canadian arctic. *Science Advances*, 4(4).
<https://doi.org/10.1126/sciadv.aar4353>

Rutishauser, A., Blankenship, D. D., Young, D. A., Wolfenbarger, N. S., Beem, L. H., Skidmore, M. L., Dubnick, A., & Criscitiello, A. S. (2022). Radar Sounding Survey over Devon Ice Cap Indicates the Potential for a Diverse Hypersaline Subglacial Hydrological Environment. *The Cryosphere*, 16(2), 379–395. <https://doi.org/10.5194/tc-16-379-2022>

Willis, M. J., Herried, B. G., Bevis, M. G., & Bell, R. E. (2015). Recharge of a subglacial lake by surface meltwater in Northeast Greenland. *Nature*, 518(7538), 223–227.
<https://doi.org/10.1038/nature14116>

Appendix

ComparisonCodeWater

```

clear;
clc;
close all;
addpath('/Users/cateondrusek/Desktop/geol 393/REFLECTIVITYwater/RUNS/')

load("reflectionresults");
%data

S=load("reflectionresults.mat","stack","T","midpts")
%data

%%

files=dir('REFLECTIVITYwater/RUNS/model_h*/ris_001.z');

A=[];
T=[];
H=[];

params = [];

for L = 1:length(files)

    file=[files(L).folder,'/',files(L).name];

    param = strsplit(files(L).folder,'-');
    params(L,1) = str2num(param{2}); % Thickness of model
    params(L,2) = str2num(param{4}); % Vp
    params(L,3) = str2num(param{6}); % Vs
    params(L,4) = 1.000;% Density

    D=rsac(file);
    %synthetic

    A(:,L)=D(:,2);
    T(:,L)=D(:,1);

```

```

H(:,L)=D(:,3);

end

%%

figure(1);
colormap gray;
maxA=max(max(A));
imagesc(1:1:size(files,1),T(:,1),A./maxA)
caxis([-1e-5,1e-5]); %amplitude scaling
colorbar;
ylim([0.3,0.6]);
xlabel("Model Index");
ylabel("Time (s)");
title("Model Space");

%%

T1=D(:,1);
vals = find(T1>=0.44-0.01 & T1<=0.44+0.1);
maxval1=max(D(vals,2));
A1=D(vals,2)/maxval1;
Q1=T1(vals)-min(T1(vals));

%%

n=261; %middle of lake
S.stack(:,n);
S.T(:);
S.midpts(n);

%%

Tpick=0.4531;
Twin=[Tpick-0.01,Tpick+0.1]; %time window to be clipped out for comparison

T2=S.T;

vals = find(T2>=Twin(1) & T2<=Twin(2)); %selecting out time window from Twin
maxval2=max(S.stack(vals,n)); %finding peak amp of data
A2=S.stack(vals,n)/maxval2; %clipped out time window for data and normalized amps

```

```

Q2=T2(vals)-min(T2(vals)); %new time vector

%%

L1=[];
L2=[];
Twin2=[0.44-0.01,0.44+0.1];
for i=1:length(files)

    T1=T(:,i);
    vals = find(T1>=Twin2(1) & T1<=Twin2(2));
    maxval1=max(A(vals,i));
    A1=A(vals,i)/maxval1;
    Q1=T1(vals)-min(T1(vals));

    A2q=interp1(Q2,A2,Q1); %interpolating data to have same sample rates

    L1(i)=sum(abs(A1-A2q))
    L2(i)=sum(abs(A1-A2q).^2)
end

%%

figure(2);
subplot(2,1,1);
plot(L2,'LineWidth',1.5);
title("L2 Misfit for Water Models",'FontSize',25);
xlabel("Model Index",'FontSize',20);
ylabel("Misfit",'FontSize',20);

subplot(2,1,2);
bestfit=(55);
T1=T(:,bestfit);
A1=A(vals,bestfit)/maxval1;
Q1=T1(vals)-min(T1(vals));
vals = find(T1>=Twin2(1) & T1<=Twin2(2));
plot(Q1,A1,'LineWidth',1.5);
hold on;
plot(Q2,A2,'LineWidth',1.5);
title("Synthetic and Data for Water Model 55",'FontSize',25);
xlabel("Time (s)",'FontSize',20);
ylabel("Amplitude (counts)",'FontSize',20);

```

```
legend('synthetic','data','FontSize',15);
```

```
%%
```

```
%water
```

```
nr=6;%thicknesses
```

```
nc=11;%vp
```

```
vpr=[1.500,1.800];
```

```
thr=[0.005,0.030];
```

```
rhos=[1.000];
```

```
vpvpsr=[1.000];
```

```
vp=unique(params(:,2));
```

```
rho=unique(params(:,4));
```

```
th=unique(params(:,1));
```

```
vs=unique(params(:,3));
```

```
X=th;
```

```
Y=vp;
```

```
Ylabel="vp (km/s)";
```

```
Xlabel="Thickness (km)";
```

```
nc=length(X);
```

```
nr=length(Y);
```

```
%%
```

```
% Figure out how many plots
```

```
k=length(rho);
```

```
l=length(vs);
```

```
figure('Position',[1 1 k*240,l*240]);
```

```
ct=1;
```

```
for i=1:k
```

```
    for j=1:l
```

```

rhoi = rho(i)
thi = th(i);
vsi = vs(j);

matches = find(params(:,4)== rhoi & params(:,3)== vsi);

L2mat= reshape(L2(matches),nr,nc);

subplot(k,l,ct);

colormap cool;
imagesc(X,Y,L2mat); hold on;
colorbar('FontSize',15);
title(['Thickness= ',num2str(thi),' (km) Vs= ',num2str(vsi),' (km/s)'],'FontSize',15);
xlabel(Xlabel,'FontSize',13);
ylabel(Ylabel,'FontSize',13);

ct=ct+1;
end
end
%%
L2n=L2mat./mean(L2)
figure;
colormap cool;
imagesc(X,Y,L2n); hold on;
colorbar('FontSize',25);
title("L2 Normalization Relative Misfit, Thickness=1.000 km",['Vs= ',num2str(vsi),' (km/s)'],'FontSize',25);
xlabel(Xlabel,'FontSize',25);
ylabel(Ylabel,'FontSize',25);

%%
clc;

figure;
subplot(4,1,3);
plot(Q1,A1,'-k','LineWidth',1);
title("Synthetic and Data",'FontSize',25);
xlabel("Time (s)");

```



```

ylabel("Amplitude (counts)");
hold on;
plot(Q2,A2,'-b','LineWidth',1);
xlabel("Time (s)","FontSize',15);
ylabel("Amplitude (counts)","FontSize',15);
legend('Synthetic','Data','FontSize',15);

subplot(4,1,4);
plot(Q2,abs((A1-A2).^2),'-r');
title("L2 Normalization (Difference Squared)","FontSize',25);
xlabel("Time (s)","FontSize',15);
ylabel("Amplitude (counts)","FontSize',15);

subplot(4,1,1);
plot(T1,D(:,2));
title("Synthetic","FontSize',25);
xlabel("Time (s)","FontSize',15);
ylabel("Amplitude (counts)","FontSize',15);
xlim([0.4,0.55]);

subplot(4,1,2);
plot(T2,S.stack(:,n));
title("Data","FontSize',25);
xlabel("Time (s)","FontSize',15);
ylabel("Amplitude (counts)","FontSize',15);
xlim([0.4,0.55]);

```

ComparisonCodeSediment

```

clear;
clc;
close all;
addpath('/Users/ateondrusek/Desktop/geol 393/REFLECTIVITYwater/RUNS/')

load("reflectionresults");
%data

S=load("reflectionresults.mat","stack","T","midpts")
%data

%%

files=dir("RUNSsediment/model_h*/ris_001.z");

A=[];
T=[];
H=[];

params = [];

for L = 1:length(files)

    file=[files(L).folder,'/',files(L).name];

    param = strsplit(files(L).folder,'-');
    params(L,1) = str2num(param{2}); % Thickness of model
    params(L,2) = str2num(param{4}); % Vp
    params(L,3) = str2num(param{6}); % Vs
    params(L,4) = 1.700;

    D=rsac(file);
    %synthetic

    A(:,L)=D(:,2);
    T(:,L)=D(:,1);
    H(:,L)=D(:,3);

end

```

```

%%
figure;
colormap gray;
maxA=max(max(A));
imagesc(1:1:size(files,1),T(:,1),A./maxA)
caxis([-1e-5,1e-5]); %amplitude scaling
colorbar;
ylim([0.3,0.6]);
xlabel("Model Index");
ylabel("Time (s)");
title("Model Space");

%%

T1=D(:,1);
vals = find(T1>=0.44-0.01 & T1<=0.44+0.1);
maxval1=max(D(vals,2));
A1=D(vals,2)/maxval1;
Q1=T1(vals)-min(T1(vals));

%%

n=261; %middle of lake
S.stack(:,n);
S.T(:);
S.midpts(n);

%%

Tpick=0.4531;
Twin=[Tpick-0.01,Tpick+0.1]; %time window to be clipped out for comparison

T2=S.T;

vals = find(T2>=Twin(1) & T2<=Twin(2)); %selecting out time window from Twin
maxval2=max(S.stack(vals,n)); %finding peak amp of data
A2=S.stack(vals,n)/maxval2; %clipped out time window for data and normalized amps
Q2=T2(vals)-min(T2(vals)); %new time vector

```

```

%%
L1=[];
L2=[];
Twin2=[0.44-0.01,0.44+0.1];
for i=1:length(files)

    T1=T(:,i);
    vals = find(T1>=Twin2(1) & T1<=Twin2(2));
    maxval1=max(A(vals,i));
    A1=A(vals,i)/maxval1;
    Q1=T1(vals)-min(T1(vals));

    A2q=interp1(Q2,A2,Q1); %%interpolating data to have same sample rates

    L1(i)=sum(abs(A1-A2q))
    L2(i)=sum(abs(A1-A2q).^2)
end
%%

figure(2);

subplot(2,1,1);
plot(L2,'LineWidth',1.5);
title("L2 Misfit for Sediment Models",'FontSize',25);
xlabel("Model Index",'FontSize',20);
ylabel("Misfit",'FontSize',20);

subplot(2,1,2);
bestfit=(100);
T1=T(:,bestfit);
A1=A(vals,bestfit)/maxval1;
Q1=T1(vals)-min(T1(vals));
vals = find(T1>=Twin2(1) & T1<=Twin2(2));
plot(Q1,A1,'LineWidth',1.5);
hold on;
plot(Q2,A2,'LineWidth',1.5);
title("Synthetic and Data for Sediment Model 100",'FontSize',25);
xlabel("Time (s)",'FontSize',20);
ylabel("Amplitude (counts)",'FontSize',20);
legend('synthetic','data','FontSize',15);

```

```

%%

%sediment
nr=[5];%vp
nc=[6];%thickness

params(:,5)=(round((params(:,2)./params(:,3))*100))/100;

vp=unique(params(:,2));
rho=unique(params(:,4));
th=unique(params(:,1));
vs=unique(params(:,5));

X=th;
Y=vp;

Ylabel="vp (km/s)";
Xlabel="thickness (km)";

nc=length(X);
nr=length(Y);
%%

% Figure out how many plots
k=length(rho);
l=length(vs);

figure('Position',[1 1 k*240,l*240]);

ct=1;
for i=1:k
    for j=1:l

        rhoi =rho(i);
        vsi  = vs(j);

        matches = find(params(:,4)== rhoi & params(:,5)== vsi);

```

```

L2mat= reshape(L2(matches),nr,nc);

subplot(k,l,ct);

colormap cool;
imagesc(X,Y,L2mat); hold on;
colorbar('FontSize',10);
title(['Density= ',num2str(rhoi),' (gm/cm^3) Vs= ',num2str(vsi),' (km/s)'],'FontSize',15);
xlabel(Xlabel,'FontSize',13);
ylabel(Ylabel,'FontSize',13);

ct=ct+1;
end
end
%%

figure('Position',[1 1 k*240,l*240]);

ct=1;
for p=1:k
    for o=1:l

        rhoi =rho(p);
        thi = th(p);
        vsi = vs(o);

        matches = find(params(:,4)== rhoi & params(:,5)== vsi);

        L2mat= reshape(L2(matches)./mean(L2),nr,nc);

        subplot(k,l,ct);

        colormap cool;
        imagesc(X,Y,L2mat); hold on;
        colorbar('FontSize',8);
        title(['Density= ',num2str(rhoi),' (g/cm^3)']
            ['Vs= ',num2str(vsi),' (km/s)'],'FontSize',10);
        xlabel(Xlabel,'FontSize',10);

```

```

ylabel(Ylabel,'FontSize',10);

ct=ct+1;

end
end

%%

clc;

figure;
subplot(4,1,3);
plot(Q1,A1,'-k');
title("Synthetic and Data",'FontSize',25);
xlabel("Time (s)");
ylabel("Amplitude (counts)");
hold on;
plot(Q2,A2,'-b');
xlabel("Time (s)");
ylabel("Amplitude (counts)");
legend('Synthetic','Data','FontSize',15);

subplot(4,1,4);
plot(Q2,abs((A1-A2).^2),'-r');
title("L2 Normalization (Difference Squared)",'FontSize',25);
xlabel("Time (s)");
ylabel("Amplitude (counts)");

subplot(4,1,1);
plot(T1,D(:,2));
title("Synthetic",'FontSize',25);
xlabel("Time (s)");
ylabel("Amplitude (counts)");
xlim([0.4,0.55]);

subplot(4,1,2);
plot(T2,S.stack(:,n));
title("Data",'FontSize',25);
xlabel("Time (s)");
ylabel("Amplitude (counts)");
xlim([0.4,0.55]);

```

ComparisonCodeBedrock

```

clear;
clc;
close all;
addpath('/Users/ateondrusek/Desktop/geol 393/REFLECTIVITYwater/RUNS/')

load("reflectionresults");
%data

S=load("reflectionresults.mat","stack","T","midpts")
%data

%%

files=dir("RUNSbedrock/model_h*/ris_001.z");

A=[];
T=[];
H=[];

params = [];

for L = 1:length(files)

    file=[files(L).folder,'/',files(L).name];

    param = strsplit(files(L).folder,'-');
    params(L,1) = 1.000; % Thickness of model
    params(L,2) = str2num(param{4}); % Vp
    params(L,3) = str2num(param{6}); % Vs
    params(L,4) = str2num(param{8}); % Density

    D=rsac(file);
    %synthetic

    A(:,L)=D(:,2);
    T(:,L)=D(:,1);
    H(:,L)=D(:,3);

```



```

end
%%
figure;
colormap gray;
maxA=max(max(A));
imagesc(1:1:size(files,1),T(:,1),A./maxA)
caxis([-1e-5,1e-5]); %amplitude scaling
colorbar;
ylim([0.3,0.6]);
xlabel("Model Index");
ylabel("Time (s)");
title("Model Space");

%%

T1=D(:,1);
vals = find(T1>=0.44-0.01 & T1<=0.44+0.1);
maxval1=max(D(vals,2));
A1=D(vals,2)/maxval1;
Q1=T1(vals)-min(T1(vals));

%%

n=261; %middle of lake
S.stack(:,n);
S.T(:);
S.midpts(n);

%%

Tpick=0.4531;
Twin=[Tpick-0.01,Tpick+0.1];%time window to be clipped out for comparison

T2=S.T;

vals = find(T2>=Twin(1) & T2<=Twin(2)); %selecting out time window from Twin
maxval2=max(S.stack(vals,n)); %finding peak amp of data
A2=S.stack(vals,n)/maxval2; %clipped out time window for data and normalized amps
Q2=T2(vals)-min(T2(vals)); %new time vector

```

```

%%
L1=[];
L2=[];
Twin2=[0.44-0.01,0.44+0.1];
for i=1:length(files)

    T1=T(:,i);
    vals = find(T1>=Twin2(1) & T1<=Twin2(2));
    maxval1=max(A(vals,i));
    A1=A(vals,i)/maxval1;
    Q1=T1(vals)-min(T1(vals));

    A2q=interp1(Q2,A2,Q1); %%interpolating data to have same sample rates

    L1(i)=sum(abs(A1-A2q))
    L2(i)=sum(abs(A1-A2q).^2)
end
%%

figure;
subplot(2,1,1);
plot(L2,'LineWidth',1.5);
title("L2 Misfit for Water Models",'FontSize',25);
xlabel("Model Index",'FontSize',20);
ylabel("Misfit",'FontSize',20);

subplot(2,1,2);
bestfit=(108);
T1=T(:,bestfit);
A1=A(vals,bestfit)/maxval1;
Q1=T1(vals)-min(T1(vals));
vals = find(T1>=Twin2(1) & T1<=Twin2(2));
plot(Q1,A1,'LineWidth',1.5);
hold on;
plot(Q2,A2,'LineWidth',1.5);
title("Synthetic and Data for Water Model 108",'FontSize',25);
xlabel("Time (s)",'FontSize',20);
ylabel("Amplitude (counts)",'FontSize',20);
legend('synthetic','data','FontSize',15);

```

```

%%

% %bedrock
nr=[9];%vp
nc=[12];%densities
%
vpr=[3.000,3.750,5.200,6.200];
thr=[1.000];
rhor=[2.200,2.450,2.700,2.800];
vpvpsr=[1.730];

vp=unique(params(:,2));
rho=unique(params(:,4));
th=unique(params(:,1));
vs=unique(params(:,3));

X=rho;
Y=vp;

Ylabel="vp (km/s)";
Xlabel="Density (g/cm^3)";

nc=length(X);
nr=length(Y);
%%

% Figure out how many plots

k=length(th);
l=length(vs);

figure('Position',[1 1 k*240,l*240]);

ct=1;

```

```

for i=1:k
    for j=1:l

        rhoi =rho(i);
        thi = th(i);
        vsi = vs(j);

        matches = find(params(:,1)== thi);

        L2mat= reshape(L2(matches),nr,nc);

        subplot(k,l,ct);

        colormap cool;
        imagesc(X,Y,L2mat); hold on;
        colorbar('FontSize',8);
        title("Thickness=1.000 km",["Vs= ",num2str(vsi),' (m/s)'],'FontSize',13);
        xlabel(Xlabel,'FontSize',10);
        ylabel(Ylabel,'FontSize',10);

        ct=ct+1;
    end
end
%%

figure('Position',[1 1 k*240,l*240]);

ct=1;
for p=1:k
    for o=1:l

        rhoi =rho(p);
        thi = th(p);
        vsi = vs(o);

        matches = find(params(:,1)== thi);

        L2mat= reshape(L2(matches)./mean(L2),nr,nc);

```

```

subplot(k,l,ct);

colormap cool;
title("L2 Normalization Relative Misfit, Thickness=1.000 km")
imagesc(X,Y,L2mat); hold on;
colorbar('FontSize',8);
title("Thickness=1.000 km",['Vs= ',num2str(vsi), ' (km/s)'],'FontSize',13);
xlabel(Xlabel,'FontSize',10);
ylabel(Ylabel,'FontSize',10);

ct=ct+1;
end
end

%%
clc;

figure;
subplot(4,1,3);
plot(Q1,A1,'-k','LineWidth',1);
title("Synthetic and Data",'FontSize',25);
xlabel("Time (s)");
ylabel("Amplitude (counts)");
hold on;
plot(Q2,A2,'-b','LineWidth',1);
xlabel("Time (s)",'FontSize',15);
ylabel("Amplitude (counts)",'FontSize',15);
legend('Synthetic','Data','FontSize',15);

subplot(4,1,4);
plot(Q2,abs((A1-A2).^2),'-r');
title("L2 Normalization (Difference Squared)",'FontSize',25);
xlabel("Time (s)",'FontSize',15);
ylabel("Amplitude (counts)",'FontSize',15);

subplot(4,1,1);
plot(T1,D(:,2));
title("Synthetic",'FontSize',25);
xlabel("Time (s)",'FontSize',15);
ylabel("Amplitude (counts)",'FontSize',15);
xlim([0.4,0.55]);

```

```
subplot(4,1,2);  
plot(T2,S.stack(:,n));  
title("Data",'FontSize',25);  
xlabel("Time (s)",'FontSize',15);  
ylabel("Amplitude (counts)",'FontSize',15);  
xlim([0.4,0.55]);
```

TimePicking

```

clear;
clc;

addpath('/Users/ateondrusek/Desktop/geol 393/REFLECTIVITYwater/RUNS/');

D=rsac('ris_001.z');

load("reflectionresults");

S=load("reflectionresults.mat","stack","T","midpts")

figure (1);
plot(D(:,1),D(:,2))

T1=D(:,1);
vals = find(T1>=0.44-0.01 & T1<=0.44+0.1);
maxval1=max(D(vals,2));
A1=D(vals,2)/maxval1;
Q1=T1(vals)-min(T1(vals));
xlabel("time (s)");
ylabel("amplitude (counts)");

figure(2);
plot(Q1,A1);
xlabel("time (s)");
ylabel("amplitude (counts)");

n=261;
S.stack(:,n);
S.T(:);
S.midpts(n);
T2=S.T;

figure(3);
plot(T2,S.stack(:,n));
xlabel("time (s)");
ylabel("amplitude (counts)");

```

```
figure(4);
imagesc(S.stack);
caxis([-50,50]);
xlabel("n",'FontSize',25);
ylabel("time (s)", 'FontSize',25);

vals = find(T2>=0.4531-0.01 & T2<=0.4531+0.1);
maxval2=max(S.stack(vals,n));
A2=S.stack(vals,n)/maxval2;
Q2=T2(vals)-min(T2(vals));

figure(5);
plot(Q2,A2);
xlabel("time (s)");
ylabel("amplitude (counts)");
```


Honor Pledge

I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination.