Thermal Evolution of an Early Magma Ocean

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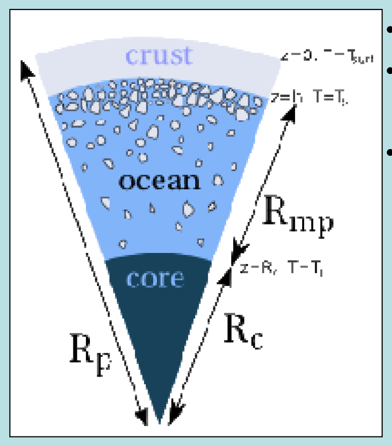
Goals and Objectives

Does a subsurface ocean exist within Triton's interior at present?

Hypothesis
- A subsurface ocean does exist within Triton.

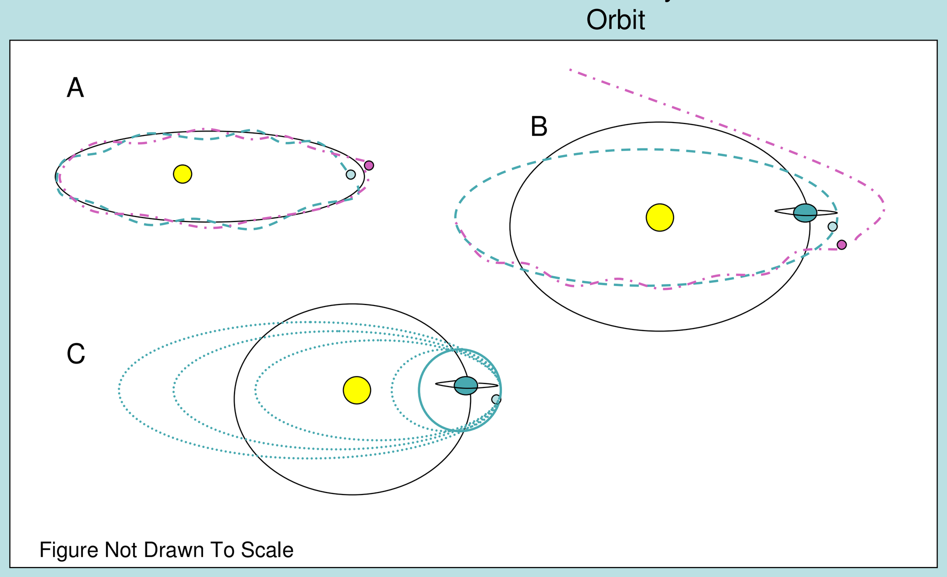
Immediate and Long-term Goals
- Discern the thermal structure of Triton's interior at present
- Determine the existence of a subsurface ocean within Triton's interior, its extent, and how it has been maintained
- Determine if internal heating is responsible for geologic activity on Triton's surface and if energy has been stored within Triton's interior?

Triton's Internal Structure and Orbital History



- Crust - Ice I shell
- Subsurface Cryomagma Ocean
  - Ammonia and Water Ice Mix
- Core - Silicate

A. Initially - Heliocentric Orbit
   Highly Eccentric
B. Capture by Neptune
   Binary Exchange
   Gas Drag
C. 'Evolutionary Event'
   Currently - Inclined and Circular Orbit



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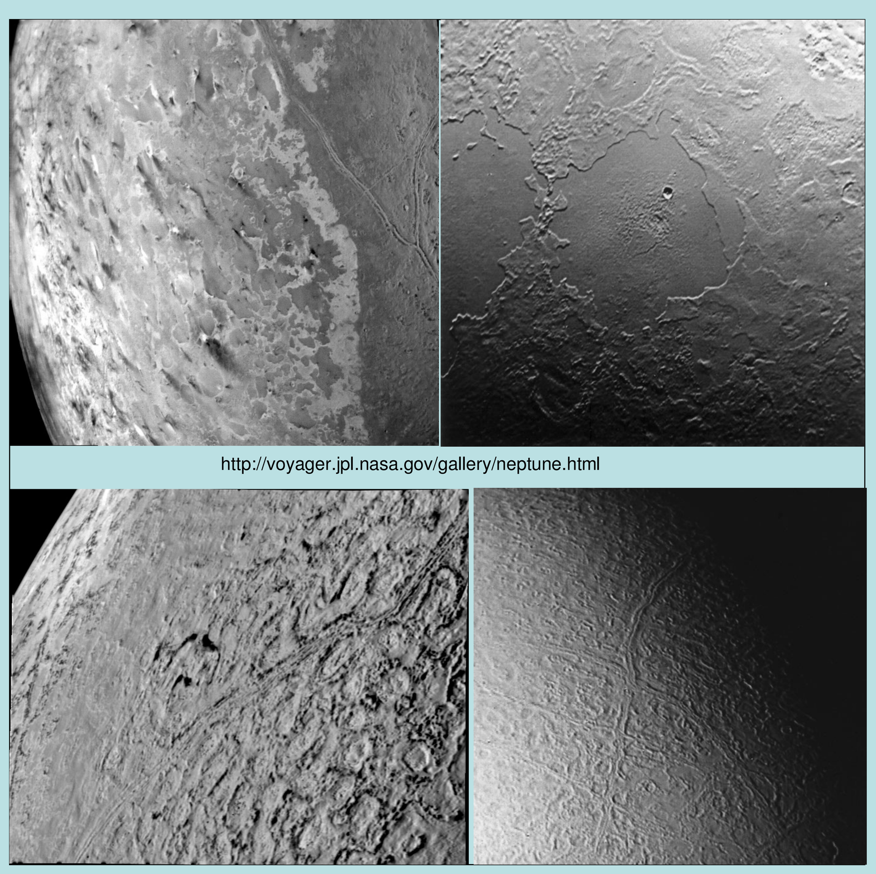
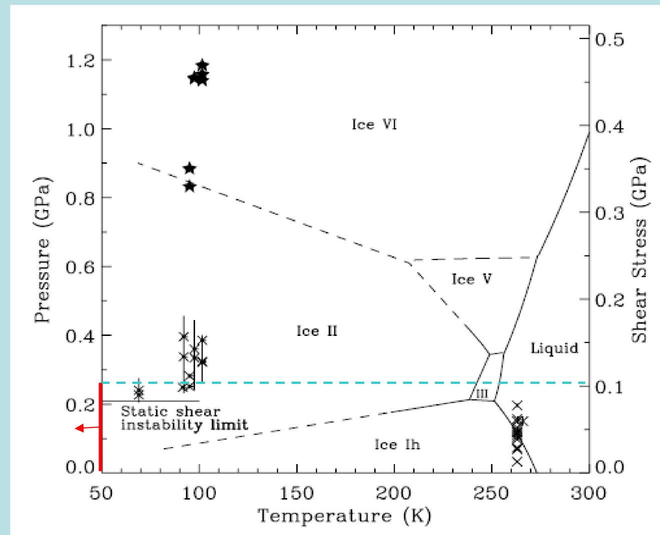
Geologic Activity and Surface Features on Triton


http://voyager.jpl.nasa.gov/gallery/neptune.html

Observation of Triton's surface have led researchers to believe that the satellite may still be geologically active.
- Surface Ridges
  - From Voyager 2 images of Triton's surface, researchers found ridges similar to those on Europa.
  - One hypothesis suggests these ridges were formed by tidal stresses from Triton's orbit (Prockter et al., 2005).
- Volcanic Plains
  - Observation also show volcanic plains overlying Triton's 'cantaloupe' terrain.
  - Researchers believe volcanic plains may be a result of thermal activity (Schenk and Zahnle, 2007).
  - Their presence suggest partial melting of the interior.
- Geysers
  - Eruptions have been observed on Triton's surface, releasing large amounts of N_2 into the atmosphere (Soderblom et al, 1990).
  - One explanation for these geysers is solar heating.
  - However, some researchers have begun modeling volatile transport on Triton's surface coupled with internal heat flow and have found internal heating to be a possible mechanism for these surface processes (Brown and Kirk, 1994).

Surface Age
- Triton's surface is estimated to be between 10 and 100 Myr old based on impact cratering density across the satellite's observed surface (Schenk and Zahnle, 2007).
- It is likely that the ridges found on Triton's surface are also young.

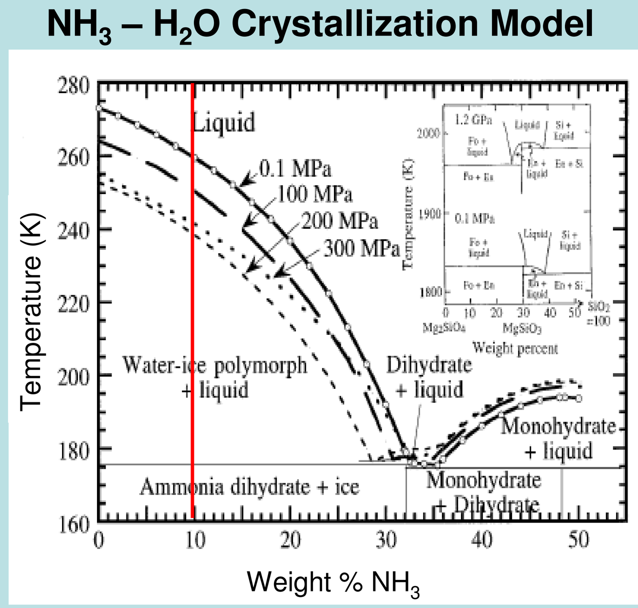
Crustal Composition



Based on Stewart and Ahrens, 2005

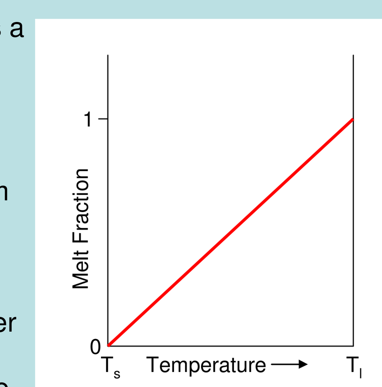
H_2O Ice I crust - low pressure form of ice
Assuming an entirely H_2O composition for shell and multiphase layer:
- Shell should thermally evolve according to the phase diagram
- Temperature should decrease, as pressure increases
- Temperature within the satellite would decrease below the surface temperature of the satellite as depth increases
- Incorporating a tidal heating of approximately 2 GW (Ross and Schubert, 1990), it would be difficult to maintain a pure H_2O ocean at present.
- Necessary to incorporate additional compositions

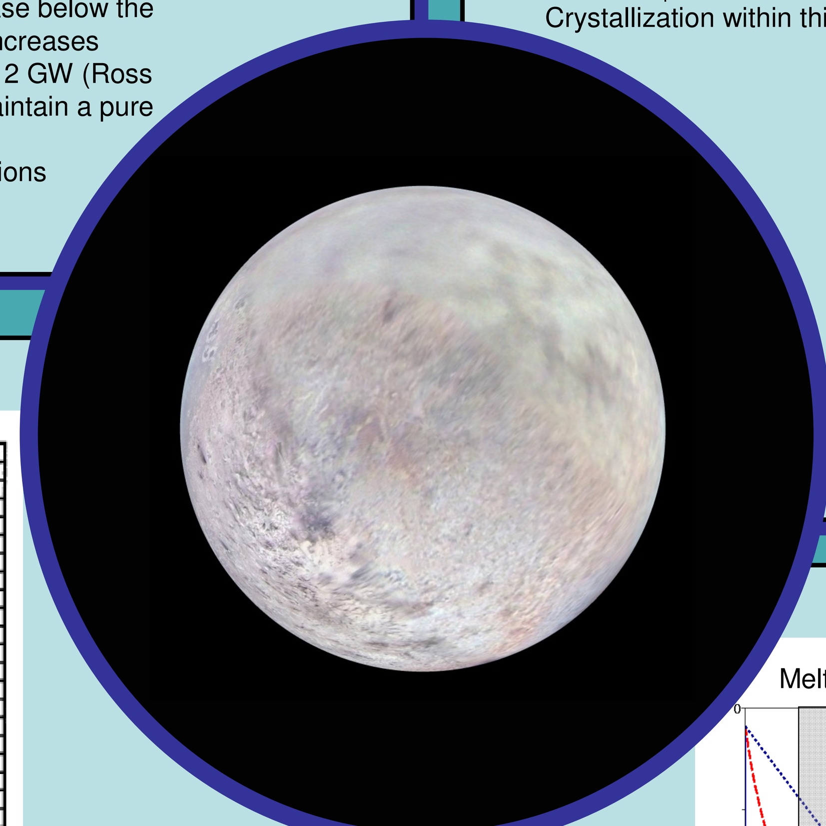
NH_3 – H_2O Crystallization Model



Hogenboom et al., 1997

Mixed Composition Model
- NH_3 wt. % estimates vary, but 10% NH_3 is a starting approximation
- Surface temperature of the crust (T_{surf}) ~38K (Brown and Kirk, 1994)
- Crustal thickness of 15 km (Ruiz, 2003)
- Crystallization according to phase diagram
- Solidus Phase – H_2O
- Liquidus Phase – NH_3 and H_2O
- $\rho_{solid} < \rho_{liquid}$: Crystals float
- Temperature at the crust - multiphase layer boundary is 176K, solidus temperature
- Temperature at the base of the multiphase layer is 240K, liquidus temperature
- Linear Crystallization Model





Governing Equations

Crust Boundary Conditions

At the surface:
z = 0
$T = T_{surf}$

At the base:
z=h
$T=T_s$

Temperature of the crust can be found as a solution to:

$$\rho c_p \frac{DT_c}{Dt} = k\nabla^2 T_c + \rho\Psi$$

where ρ is density, c_p is specific heat, k is thermal conductivity, and Ψ is tidal dissipation.

Assuming steady-state conditions:

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + v\frac{\partial T}{\partial z} \qquad \frac{DT}{Dt} = 0$$

The crust is not changing temporally or spatially.

$$k\frac{\partial^2 T_c}{\partial z^2} = -\rho\Psi$$

Multiphase Layer Boundary Conditions

At the interface:
z = h
$T = T_s$

At the base:
$z = R_c$
$T = T_l$

Temperature of the multiphase layer can be found as a solution to:

$$\frac{DT_m}{Dt} - \frac{T_m \Delta S}{c_p}\frac{DF}{Dt} = \frac{k}{\rho c_p}\nabla^2 T_m$$

where ΔS is entropy and F is melt fraction.

Crystallization within this layer is defined by

$$F = \left(\frac{T_m - T_s}{T_l - T_s}\right)^\beta$$

where β is equal to 1 for a linear model.
Steady-State Conditions:
System is not be varying temporally, only spatially as percolation occurs.

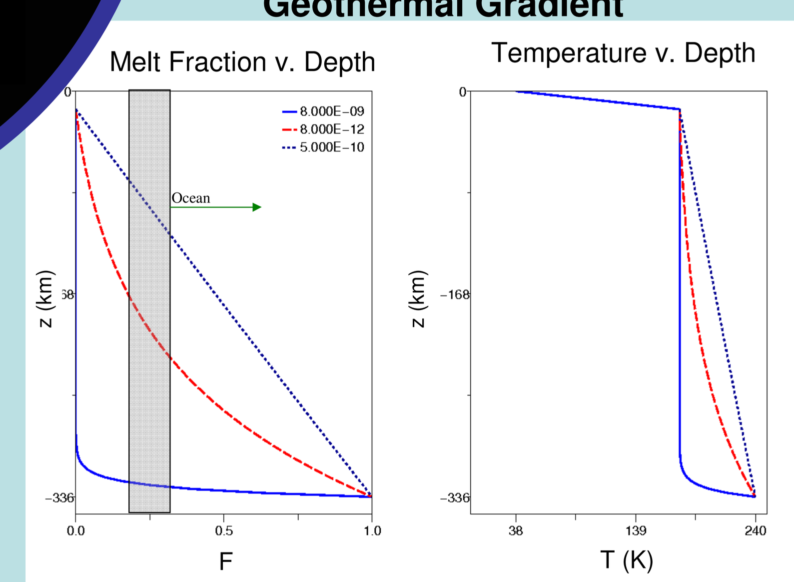
$$\frac{DT_m}{Dt} = v\frac{\partial T_m}{\partial z}$$

$$\frac{DF}{Dt} = v\frac{\partial F}{\partial z} = v\frac{dF}{dT}\frac{dT}{dz}$$

$$\left(1 - \frac{T_m \Delta S}{c_p \Delta T}\right)v\frac{dT_m}{dz} = \frac{k}{\rho c_p}\frac{d^2 T_m}{dz^2}$$

where z is depth.

Geothermal Gradient



Analytical Solutions

Temperature of the Crust:

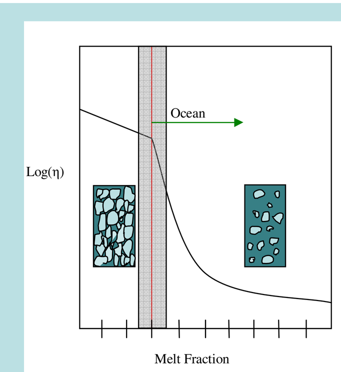
$$T_c = T_{surf}\left(1 - \frac{z}{h}\right) + T_s\frac{z}{h} + \rho\Psi z\frac{(h-z)}{2k}$$

Temperature of the Multiphase Layer:

$$T_m = \frac{\alpha(T_l - T_s)}{e^{\alpha(R-r_c)} - e^{\alpha h}}\frac{e^{\alpha z}}{\alpha} + \frac{1}{\alpha}\left(T_s\alpha - \frac{\alpha(T_l - T_s)}{e^{\alpha(R-r_c)} - e^{\alpha h}}e^{\alpha h}\right)$$

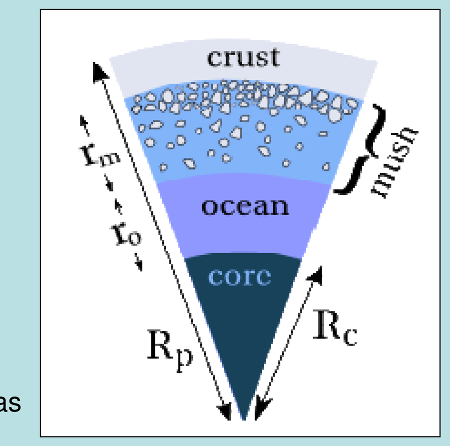
$$\alpha = \rho c_p \frac{v}{k}\left(1 - \frac{\Delta H}{\Delta T c_p}\right)$$

Segregation Velocity: $v = \frac{k}{\mu}\Delta\rho g$



Future Considerations

Coupled Thermal – Orbital Evolution:
- Triton will be subcategorized into four layers
  - Ice I crust
  - Mush layer
  - Cryomagma ocean
  - Silicate Core
- Model will numerically analyze for three temperatures
  - crust temperature, T_c
  - mush layer temperature, T_m
  - ocean temperature, T_o
- Model will become a moving boundary problem
- Tidal heating will be time dependent
- Melt fraction decreases in the mush layer as temperature decreases
- Radius of both the ocean, r_o, and much layer, r_m, will evolve



Governing Equations

Equation for thermal evolution of the crust:

$$\rho c\frac{\partial T_c}{\partial t} = -\nabla\cdot q_c + \alpha_c\psi$$

Equation for thermal evolution of the mush layer:

$$\rho c\left(\frac{\partial T_m}{\partial t} + u_m\cdot\nabla T_m\right) = -\nabla\cdot q_m + \alpha_m\psi + L$$

$$L = \rho T_m \Delta s\left(\frac{\partial F}{\partial t} + u_m\cdot\nabla F\right)$$

Equation for thermal evolution of the cryomagma ocean:

$$\rho c\left(\frac{\partial T_o}{\partial t} + u_o\cdot\nabla T_o\right) = -\nabla\cdot q_o + \alpha_o\psi$$

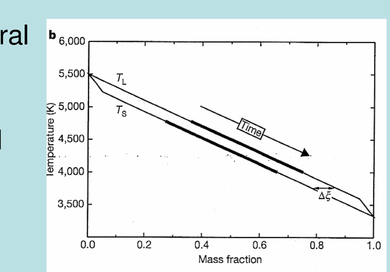
Ψ - tidal heating within each layer of the satellite
α – tidal heating coefficient introduced to control tidal heating distribution throughout satellite
u_m - represents the percolation or segregation velocity of the crystallizing mush layer, which serves as a means of heat transfer within that layer
u_o - represents the turbulent velocity within the cryomagma ocean, which affects heat transfer throughout the ocean

393H Research

Linear v. Nonlinear Crystallization In a Terrestrial Basal Magma Ocean

Can a linear crystallization model be used to approximate thermal evolution of a magma ocean?

Model of Coupled Thermal and Internal Structural Evolution
- Model incorporates evolving mantle temperature, magma ocean temperature, and magma ocean thickness
- Linear model for crystallization in a magma ocean



Linear Model

$$\frac{dT_L}{dt} = \frac{1}{M_m C_{pm} + M_c C_{pc}}\left[\frac{-4\pi a^2 k(T_L - T_M)}{\delta} + H(t) - 4\pi a^2\rho\Delta S T_L\frac{da}{dt}\right]$$

$$\frac{da}{dt} = \frac{a^3 - b^3}{3a^2\Delta\xi(T_A - T_B)}\frac{dT_L}{dt}$$

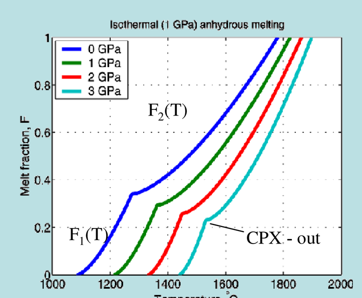
- Nonlinear model for crystallization in a magma ocean

$$F = \frac{a^3 - b^3}{a_0^3 - b^3} \qquad F = g_1(T_c - T_L)F_1(T_L) + g_2(T_L - T_c)F_2(T_L)$$

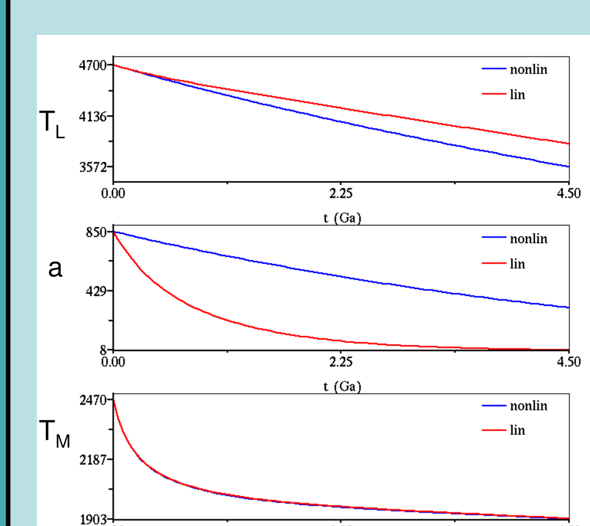
$$\frac{dF}{dt} = \frac{3a^2}{a_0^3 - b^3}\frac{da}{dt} \qquad \frac{da}{dt} = \frac{a_0^3 - b^3}{3a^2}\frac{dF}{dT_L}\frac{dT_L}{dt}$$

$$\frac{dT_L}{dt} = \frac{1}{M_m C_{pm} + M_c C_{pc}}\left[\frac{-4\pi a^2 k(T_L - T_M)}{\delta} + H(t) - 4\pi a^2\rho\Delta S T_L\frac{da}{dt}\right]$$

$$\frac{dT_m}{dt} = \left(\frac{1}{M_m C_{pm}}\right)\left[He^{-\lambda t} - 4\pi R_m^2 k\frac{(T_m - T_s)^{\beta+1}}{R_m - a}\left(\frac{\alpha g}{\kappa\upsilon_0}\frac{(R_m - a)^3}{Ra_{cr}}\right)^\beta e^{\frac{-\beta A_0}{T_m}}\right]$$



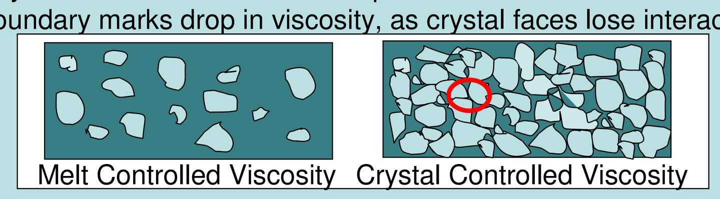
Nonlinear Model



- The results of this study showed that there is variation in trends produced from linear and nonlinear crystallization models.
- Results emphasize the necessity of using more realistic crystallization models.
- Ultimately, more realistic parameters will be used to constrain the Triton's ocean.
- The Triton models, like the basal magma ocean models, will eventually become time-dependent and will be solved numerically.

Results

Geothermal Gradient
- Illustrates the satellite's thermal structure for 3 different segregation velocities
  - Segregation velocity occurs as a result of density driven percolation of the heavy fluid through the crystal matrix.
  - Viscosity μ ~10^{-3} Pa s; Change in density is between 996 and 916 kg m^{-3}; g is approximately .78 m s^{-2}
  - 3 different thermal conductivities: 8 x 10^{-9}, 5 x 10^{-10}, 8 x 10^{-12}
- Melt Fractions greater than ~0.3 may constitute a magma ocean. This boundary has been marked within the plot.
  - Boundary marks drop in viscosity, as crystal faces lose interaction.



- The results from this plot demonstrate the influence of segregation velocity within the multiphase layer.
  - Faster velocity: Temperature and Melt Fraction undergo little change until the base of the layer. The sharp increase at the base suggests the existence of a smaller ocean with a higher melt fraction. Therefore, a faster velocity implies a thinner boundary layer.
  - Slower velocity: Temperature and Melt Fraction undergo gradual increases with depth. This encourages a significant internal ocean, with a larger crystal fraction.
  - In both cases, an internal ocean can exist, but its extent may vary.

- Presence of Ammonia
  - Ammonia reduces liquidus and solidus temperatures within the crust and multiphase layer.
  - Incorporating ammonia increases the likeliness of sustaining a cryomagma ocean at present.
  - Though ammonia constitutes a small portion of the ice composition, it should be included within the final model for the most accurate results.
- Boundary Condition Assumptions
  - NH_3 and H_2O model
    - The temperature at the base of the multiphase layer was prescribed at 240K, the liquidus temperature.
      - This forces the system to eventually obtain a melt fraction of 1 at the base of the multiphase layer.
      - This eliminates the option of obtaining a completely crystallized subsurface ocean.
    - Pre-determined crustal thickness at solidus temperature
      - Prevents crust from increasing (or decreasing) in thickness.
      - This can be corrected in a moving boundary problem.
- Tidal Heating Parameter
  - Within this model, tidal heating was prescribed a constant value.
  - A value for tidal heating derived from the satellite's orbital evolution would be more appropriate.
    - Incorporating orbital evolution will allow interior to respond more appropriately.

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