Thermal Evolution and Overpressurization of Europa's Subsurface Ocean

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Model Description

Thermal evolution model of the solid ice shell is governed by the heat conduction equation,

$$\frac{\partial T}{\partial t} = -\kappa \frac{\partial^2 T}{\partial z^2} + \kappa H_e$$

If heat is lost at the bottom of the shell, a layer of liquid water crystallizes so that the ice shell thickens according to

$$\frac{dz_m}{dt} = \frac{Q_m}{\rho L}\partial$$

T	Temperature
t	Time
z	Depth
Q_m	Temperature Gradient
T_s	Surface temperature
T_m	Melting temperature
κ	Thermal diffusivity
C_p	Heat Diffusivity
L	Latent heat of fusion
H_e	Heat Production

The boundary conditions are constant temperatures $T=T_s$ at the surface $z=0$ and $T=T_m$ at the base of the ice, $z=z_m$, which changes over time. The ice expands as water freezes at the bottom, increasing the excess pressure on the ocean at a rate of $\delta P_{ex}/\delta z$. P_{ex} is reduced on the ocean by radial displacement of the ice shell due to viscous deformation in the lower portion of the shell. The boundary between elastic deformation and viscous deformation is treated as the depth at which the ice temperature reaches 180K.

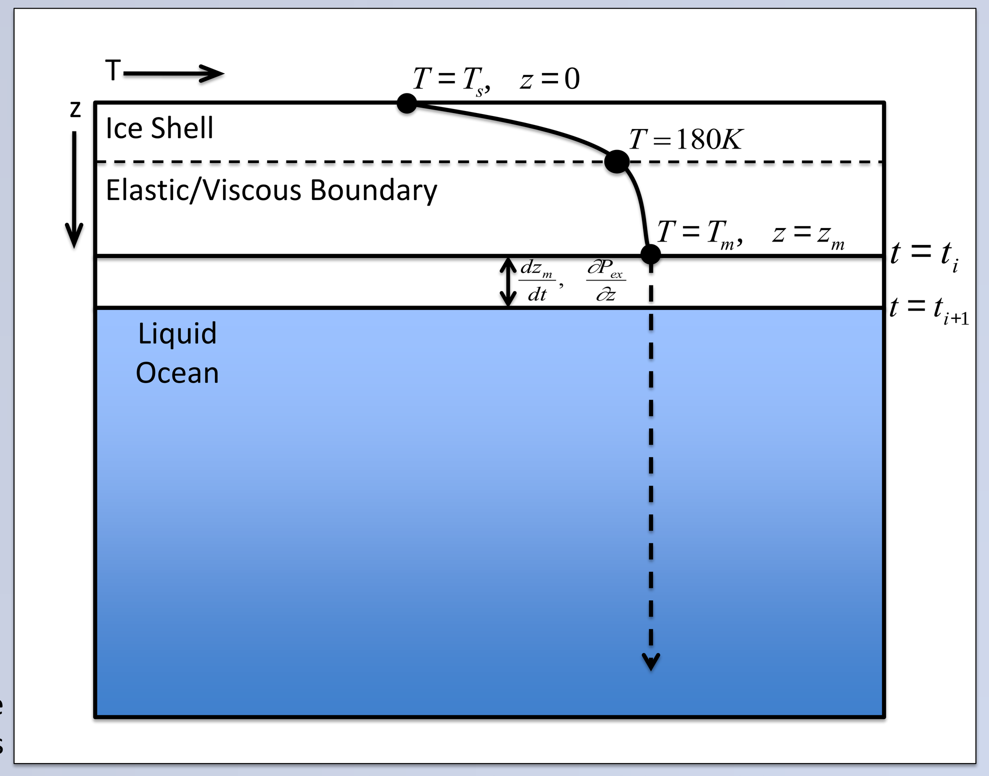

Figure 1: Schematic diagram of the model.

Tidal Heat Production

Tidal Heat is determined related to depth by expressing as a function of ice viscosity [4]:

$$H_e = \frac{2H_{Max}}{\left(\frac{\eta}{\eta_{Max}} + \frac{\eta_{Max}}{\eta}\right)}$$

with

$$H_{Max} = \frac{\mu\varepsilon^2}{\omega} \qquad \eta_{Max} = \frac{\mu}{\omega}$$

$$\eta = \eta_m \exp\left(-\gamma_t \frac{T-T_m}{\Delta T}\right)$$

η	Ice Viscosity at z
η_m	Viscosity at melting point
μ	Bulk shear modulus
ω	Orbital frequency
γ_t	Dimensionless material constant
ε	Average tidal strain rate

The Stefan Problem

If heat production is ignored, this model corresponds to a Stefan problem: the simultaneous cooling and solidification of a liquid.

An analytical solution to this problem is well known. It is given by the following equation:

$$T = T_s + (T_m - T_s)\frac{\mathrm{erf}(\eta)}{\mathrm{erf}(\lambda)}$$

Where η and λ are determined by:

$$\eta = \frac{z}{\sqrt{\kappa t}}; \qquad \frac{L\sqrt{\pi}}{C_p(T_m - T_s)} = \frac{e^{-\lambda^2}}{\lambda\,\mathrm{erf}(\lambda)}$$

The analytical solution will serve to verify the accuracy of the numerical model that is needed to solve the thermal evolution of cooling and crystallizing ice shell in presence of heat dissipation.

Development of Overpressure

The increase in excess pressure P_{ex} on the ocean related to increasing ice shell thickness is determined by: [5]

$$\frac{\partial P_{ex}}{\partial z_m} = \frac{3(\rho_w - \rho_i)r_i^3}{\beta\rho_w(r_i^3 - r_c^3)}$$

However, the lower portion of this ice shell will deform viscously, causing an upward radial displacement u_r and reducing the overall excess pressure. The radius ξ at which the transition between elastic and viscous deformation occurs in the ice shell is taken as the depth the temperature in the ice shell reaches 180K. u_r is given by:

$$u_r = -\frac{\xi}{E}(\sigma_r - 2\upsilon\sigma_t)$$

Where σr and σt are given by:

$$\sigma_r = \frac{P_{ex}(z_m)}{\left(\frac{R}{\xi}\right)^3 - 1}\left[1 - \left(\frac{R}{\xi}\right)^3\right] \qquad \sigma_t = \frac{P_{ex}(z_m)}{\left(\frac{R}{\xi}\right)^3 - 1}\left[1 + \frac{1}{2}\left(\frac{R}{\xi}\right)^3\right]$$

This radial displacement and shell expansion will then decrease the overpressure by:

$$\partial P_{ex} = \frac{3u_r r_i^2}{\beta(r_i^3 - r_c^3)}$$

P_{ex} can then be solved for and used to determine the height H to which water will rise in the shell:

$$H = \left(\frac{\rho_w - \rho_i}{\rho_w}\right)(R - r_i) - \frac{P_{ex}}{\rho_w g}$$

R	Planetary Radius
r_i	Radius: base of shell
r_c	Radius: Ocean base
β	Compressibility
σ_r	Radial Stress
σ_t	Tangential stress
E	Young's Modulus
ν	Poisson Ratio
ρ_i	Density of ice
ρ_w	Density of water

Works Cited

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[4] Tobie, G., Choblet, G., and Sotin, C. 2003. Tidally heated convection: constraints on Europa's ice shell thickness. *Journal of Geophysical Research* ***108.***
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Overview

Several alternative hypotheses concerning the formation mechanism of ridges at the surface on Europa have been proposed. The best studied mechanism to date involves shear heating on a tidally-loaded crack [1]. However, cryovolcanic processes, especially the crystallization of a water intrusion inside the ice shell, may also form ridges [2, 3]. One issue with this model is that water intrusions are difficult to form due to the higher density of liquid water compared to ice. Intrusions may not form simply as the result of buoyancy-driven motion and would require the Europa's subsurface ocean to be pressurized and to force water partially into the ice shell.

The possibility of cracking the base of the ice shell is explored using a two-stage thermal/stress model constructed with MATLAB. The thermal evolution model considers an ice shell undergoing conductive cooling and heating by tidal flexure. If heat is lost at the base of the shell, the subsurface ocean crystallizes, thickening the ice layer. Estimates of mass transfer from the ocean to the ice shell is used in a stress evolution model to estimate the state of stress in the shell and evaluate the amount of vertical water transport in the shell.

Numerical Implementation

The heat equation is solved using Matlab's ODE solver ODE45. The spatial derivatives are computed by finite differences. However, the standard approach of using a time-independent grid for discretizing $T(z)$ lead to numerical issues as the position of the freezing front was often between grid points.

To better account for the movement of the freezing front, the grid where the temperature solution is defined is updated at each time step to conform to the change in ice thickness z_m (Fig. 2). The changes in the depth of each grid point generates a new term in the discretized heat equation, which becomes:

$$\frac{dT_i}{dt} = k\frac{Q_{i+1} - Q_{i-1}}{z_{i+1} - z_{i-1}} + \left(\frac{T_i - T_{i-1}}{z_i - z_{i-1}}\right)\frac{T_m - T_{m-1}}{(z_m - z_{m-1})\rho L} + kH_e$$

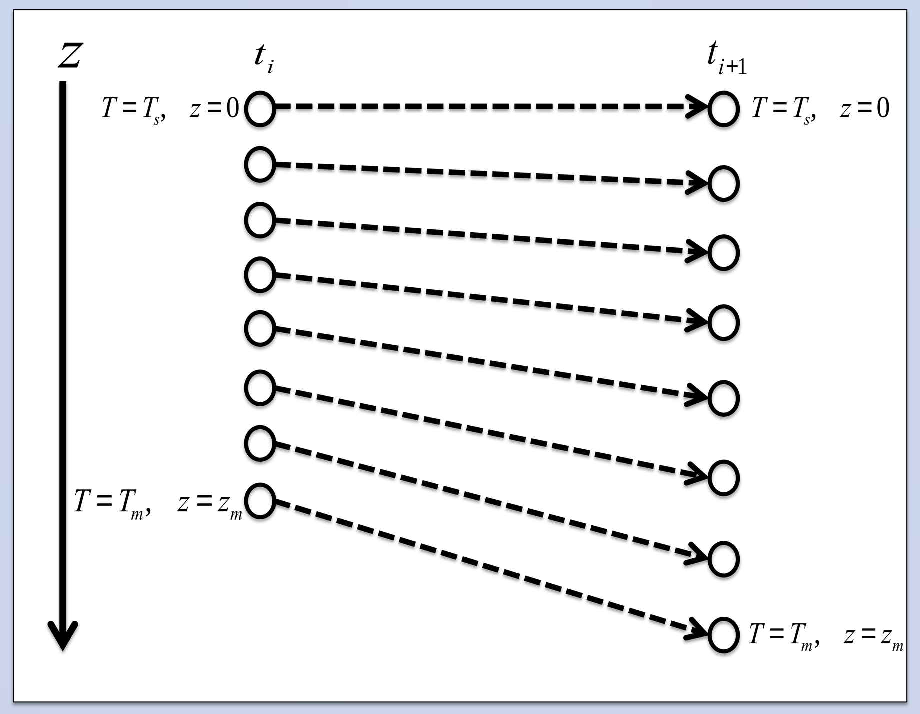

Figure 2: Moving coordinate procedure. At time step t_i, a depth profile with boundaries at $z=0$ and $z=z_m$ is defined. The temperature gradient is computed by finite difference and dz_m/dt is determined form Q_m at $z=z_m$. The depth of each node is updated at the next time step t_{i+1} to form a regular grid between the surface and the value of z_m.

Due to an amount of circularity inherent in the solving for P_{ex}, σ_r, and σ_t, the stress evolution equations must be combined into one numerical relation to accurately calculate overpressure. The combined equation then becomes:

$$P_{ex}\left\{\frac{\beta(r_i^3 - r_c^3)}{3r_i^2} - \frac{\xi}{E\left(\left(\frac{R}{\xi}\right)^3 - 1\right)}\left[1 - 2\upsilon - (1+\upsilon)\left(\frac{R}{\xi}\right)^3\right]\right\} = \frac{\rho_w - \rho_i}{\rho_w}\partial z_m$$

The stress evolution model is then coupled with the thermal evolution model and uses temperature results to determine viscoelastic deformation boundary depth ξ and z_m values returned by the model to determine r_i at every time step.

Thermal Evolution Model

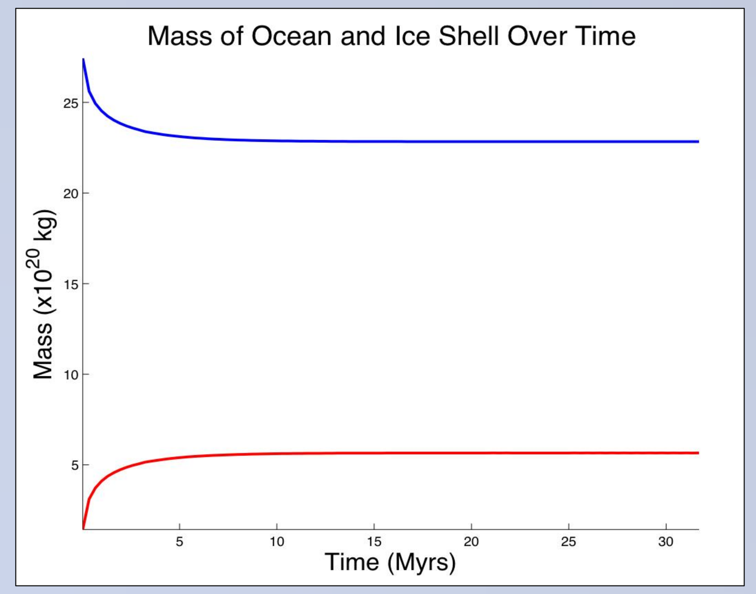

Figure 3: Time evolution of the mass estimate of the ocean (In blue) and the ice shell (In red). Ice thickness was converted into mass assuming that the ice shell corresponds to the upper layer of a 120-km thick water/ice spherical shell on a body of Europa's radius.

Results

When applied to Europa, the thermal evolution model that the ice shell would reach a steady-state thickness of approximately 20 km in approximately 13 million years. If the initialization of crystallization is taken at the time of Europa's formation, 4.5 Ga, this result makes overpressure an unlikely cause of recently formed surface features on Europa, as viscous flow at the base of the ice shell would probably have dissipated the pressure generated early on in the satellite's history. However, Europa almost certainly has undergone orbital evolution throughout its history. As shown in Fig. 7, changes in tidal strain rate have a large effect on ice thickness and crystallization speed. A doubling in tidal strain, due for example to increased eccentricity, would cause the ice shell to thin by a factor of 4. The crystallization and pressurization process would restart when Europa adopted its current orbital parameters.

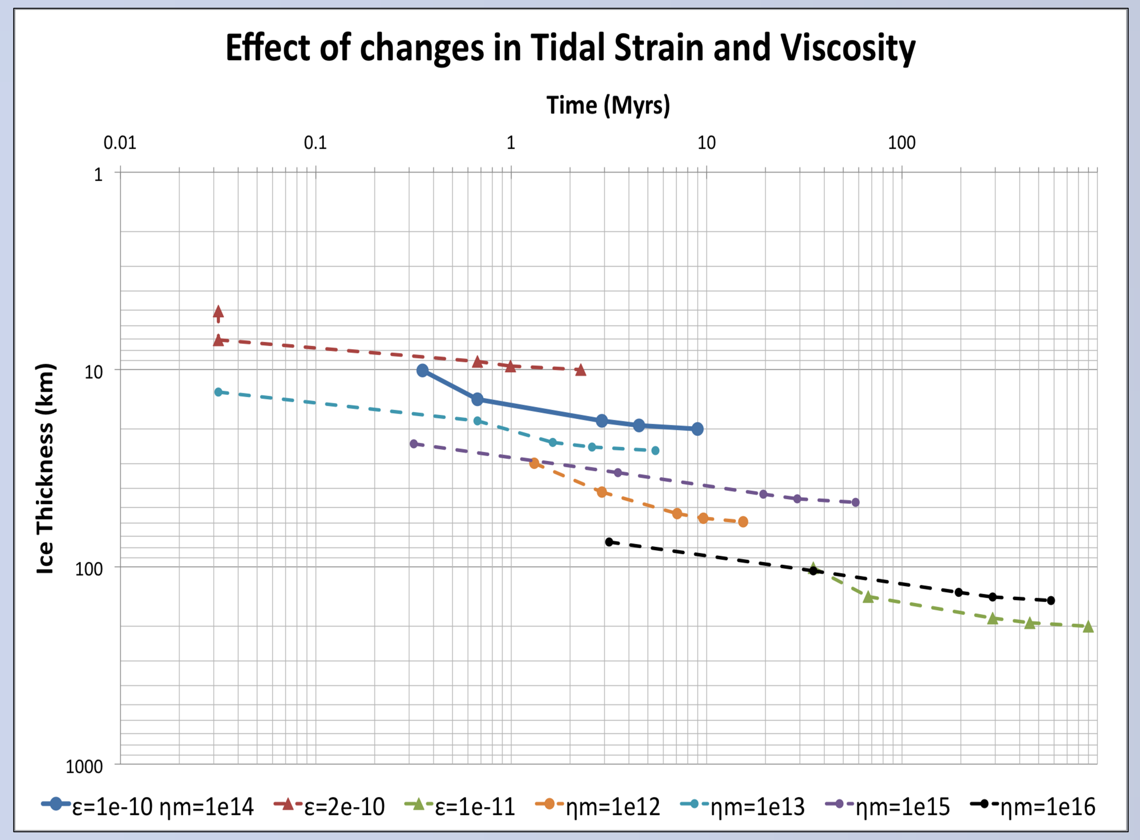

Figure 4: Time needed for the the ice to reach a certain thickness. Each marker represents ice at 50%, 70%, 90%, 95%, and 99% of the maximum thickness. The solid line represents the conditions found on Europa $\varepsilon=10^{-10}$ s^{-1}, $\eta_m = 10^{14}$ Pa s . The series marked with triangles assume different tidal strain, and the series marked with circles assume a different ice viscosity.

Stress Evolution Model

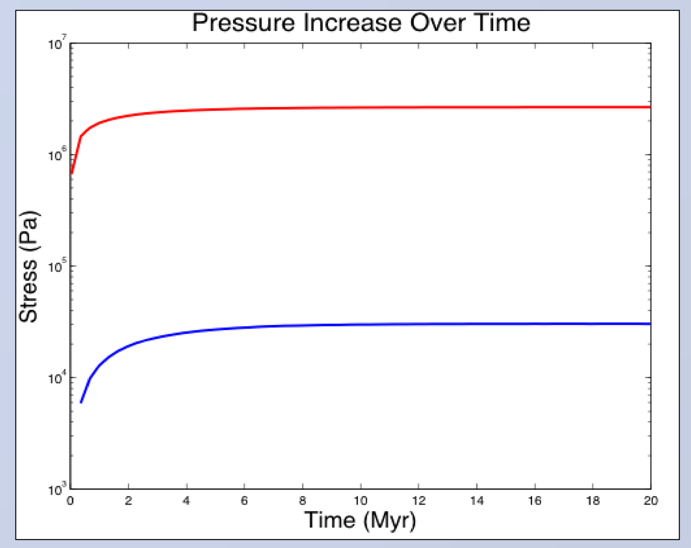

Figure 5: Comparison of results of stress evolution P_{ex} (In blue) with the amount of overpressure P_{crit} necessary for water extrusion (In red).

Results

These results indicate that the maximum excess pressure experienced by the ocean under the conditions of this model is approximately 30 kPa. Manga and Wang (2007) found that water was not able to erupt onto the surface of Europa by pressure alone, but the possibility may still exist that partial intrusion into the ice shell occurs. A comparison of the results of this model with a calculation of P_{crit}, the overpressure necessary for extrusive cryovolcainism, over time also shows that extrusion is impossible by overpressure alone. Water will still rise some distance through the ice shell, even though there is not enough pressure to extrude water on the surface. Over time, the depth to which water will rise increases from 91.0% of the ice shell's total thickness to 91.2% of the ice shell's total thickness over a period of 15 Ma. As the initial value of overpressure is treated as zero, it can be concluded that overpressure contributes to 0.2% of the relative rise through the ice shell, with the relative densities of water and ice accounting for the majority of penetration.

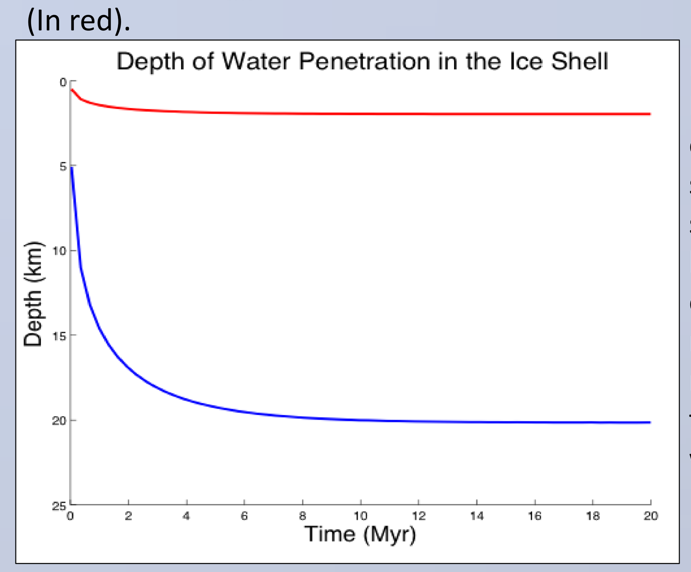

Figure 6: Final result: Depth profile of ice shell over time with the surface of Europa being set to 0 km. The blue line represents the base of the ice shell, and the red line represents the maximum depth above the base to which water will rise.

Conclusions

Overpressurization seems to have less of an impact on European cryovolcanism as was once thought. Other mechanisms of transporting water up the shell should be searched for. Although water does rise through a significant portion of the ice shell according to this model, that rise is chiefly accounted for by the relative densities of water and ice. There are several parameters considered in this model that are still uncertain on Europa, and these may impact overpressurization and water intrusion, but it seems that the effect would be either negligible or negative. Despite all this, these results ultimately indicate that cryovolcanic sills are still a possibility that merits future investigation.