Linking Volcano Morphology and Elastic Thickness on Mars

GEOL393

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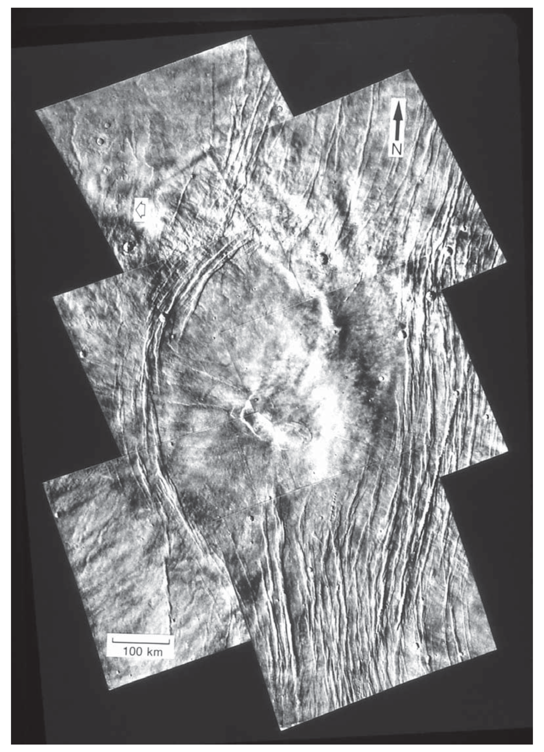
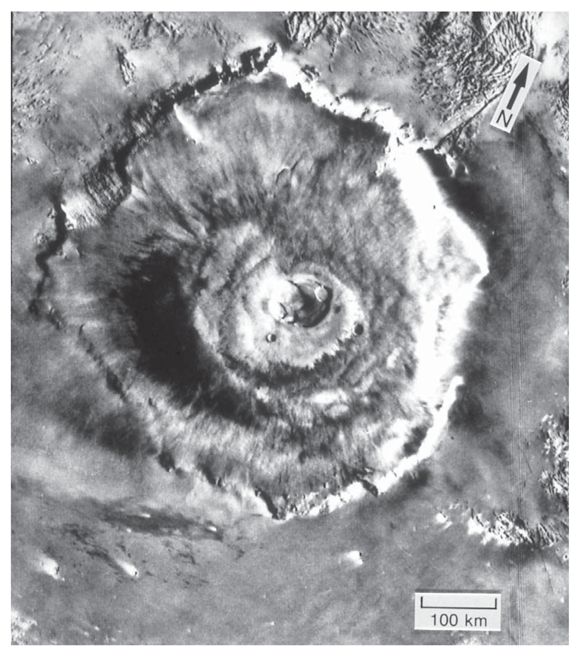
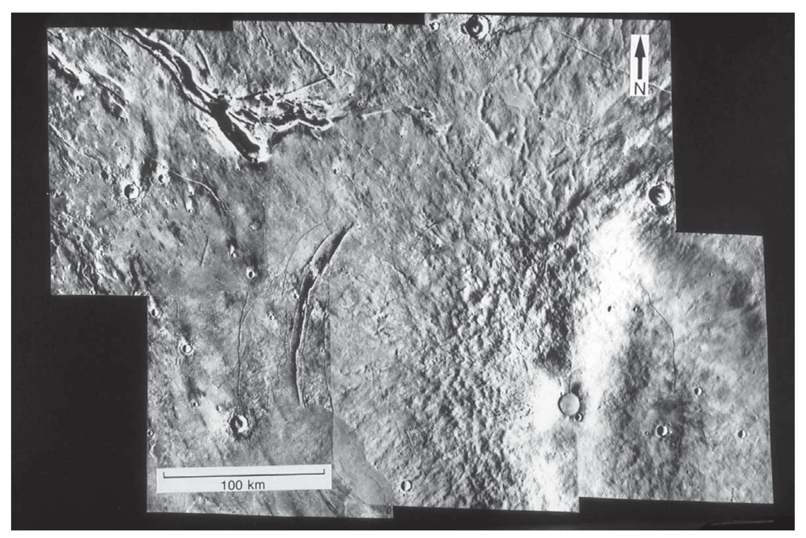
Volcano morphology



Alba Patera is an ancient (~3.7 Ma) Pyroclastic volcano, having a broad radial extent of ~1200 km, a topographic relief of only ~7 km, and a small slope of ~0.5°. This low topography and flat profile are characteristic of most patera, which can be classified as flat edifices.



Olympus Mons is the largest volcano in the solar system. This shield volcano has a radial extent of ~600 km a high topographic relief of ~22 km, and a moderate slope of ~5°. This moderate slope and high elevation are characteristic of domical edifices.



Elysium Mons has a moderate radial extent of ~240 km, and a topographic relief of ~14 km. The edifice has a steep slope of~10°.

This steep slope and moderately high topographic relief are characteristic of conical edifices.

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Background

An overlying load (i.e. mountain, glacier, volcano) causes displacement on the underlying lithopshere, which must flex and deform to compensate for the mass of the load. A stress field is generated from the flexure, and on local scales, the lithosphere can be regarded as a thin elastic shell.

In this project, I will model the growth of a volcano by placing incremental loads atop plates of varying thicknesses. Flexure induced stress fields will be studied, magma ascent criteria will be applied, and resultant edifice morphology will be observed.

Hypothesis

After McGovern et al. (2013), I predict that conical volcanoes will be produced on thick elastic plates, flat volcanoes on thin elastic plates, and domical volcanoes on intermediate thicknesses. Terrestrial lithospheres are thought to consecutively cool and thicken over time. Implications for Martian thermal evolution can be drawn from the rate of cooling and the volcanic diversity across the planet.

Axisymetric solution for point load

w=w0 at x=0 and w=0 at x=±∞

$$w = w_k \mathrm{kei}\left(\frac{r}{\alpha}\right)$$

$$w_k = w_0/\mathrm{kei}(0)$$

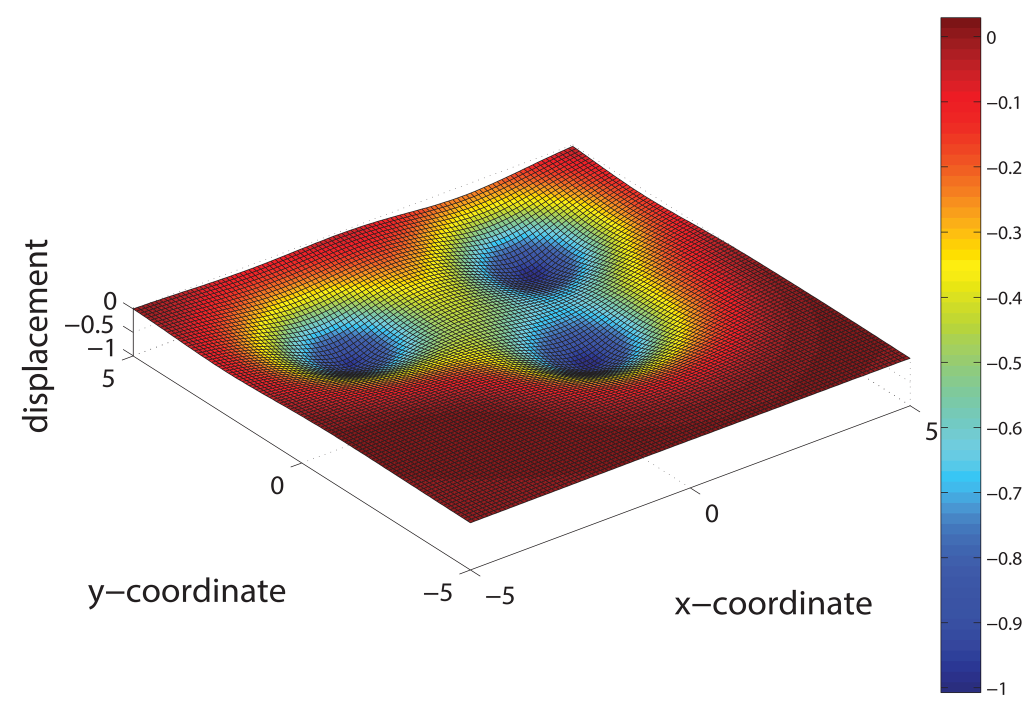
Kei: Imaginary part of modified Bessel function of the 2nd kind.

→ Uplift at x=± πα

→ Deflection negligible at|x|≥2πα

Flexure of a thin elastic shell

$$\nabla^4 w + \beta^{-4} w = q/D$$



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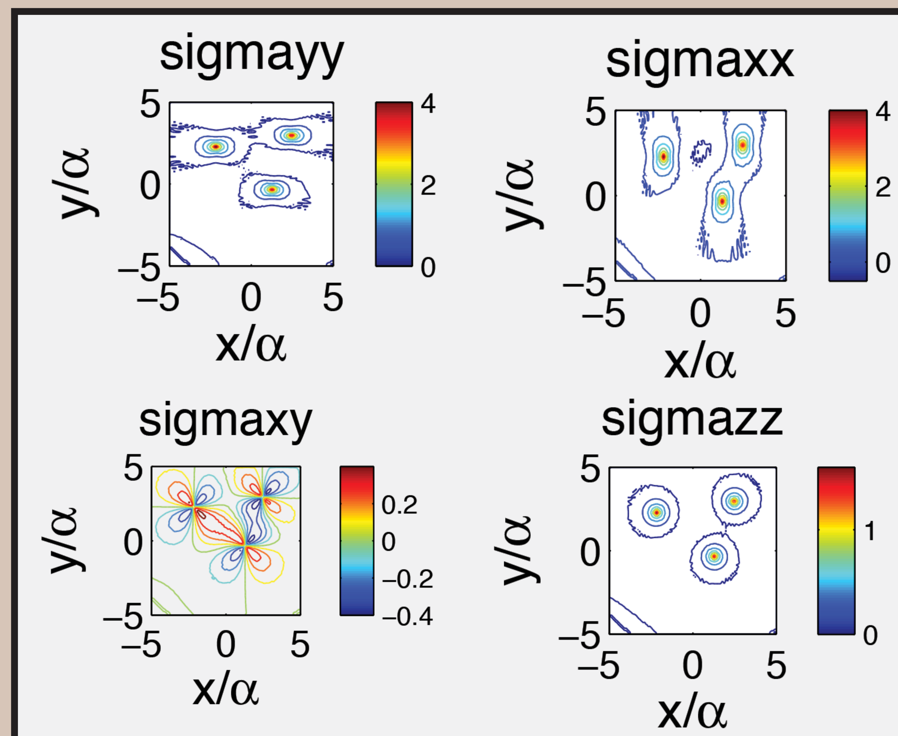
Flexure induced stress field

Symmetric nine-component stress tensor

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$



Governing stress equations

$$\sigma_{xx} = z_f \left[(\lambda + 2\mu)\frac{{}^2 w}{\partial x^2} + \lambda \frac{\partial^2 w}{\partial y^2}\right]$$

$$\sigma_{yy} = z_f \left[\lambda \frac{\partial^2 w}{\partial x^2} + (\lambda + 2\mu)\frac{\partial^2 w}{\partial y^2}\right]$$

$$\sigma_{xy} = \sigma_{yx} = 2\mu z_f \frac{\partial^2 w}{\partial x \partial y}$$

$$\sigma_{zz} = \rho_v g h_v + \rho_l g\left(\frac{H}{2} - z_f\right) + z_f \lambda \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$\mu = \frac{E}{2(1+\nu)} \qquad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

μ and λ are known as Lamé parameters that parameterize elastic modulii of homogenous isotropic solids

Numerical Implications

$$\left.\frac{\partial w}{\partial x}\right|_{i,j} = \frac{w_{i+1,j} - w_{i-1,j}}{2h}$$

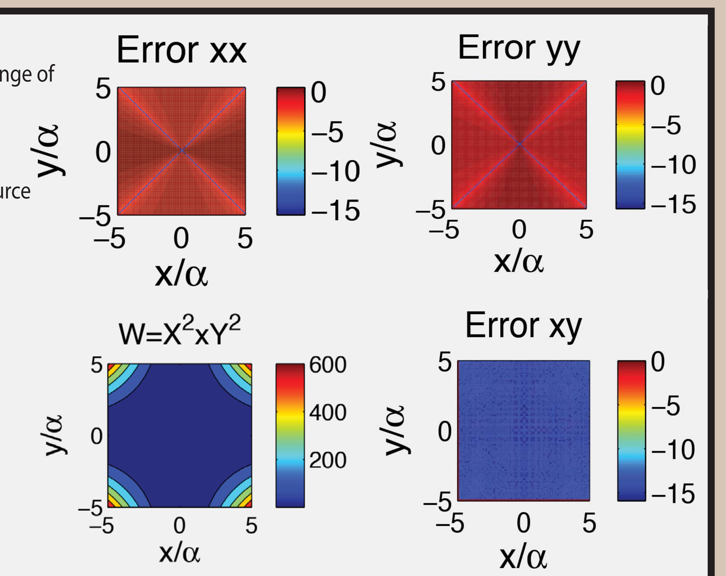
$$\left.\frac{\partial w}{\partial y}\right|_{i,j} = \frac{w_{i,j+1} - w_{i,j-1}}{2h}$$

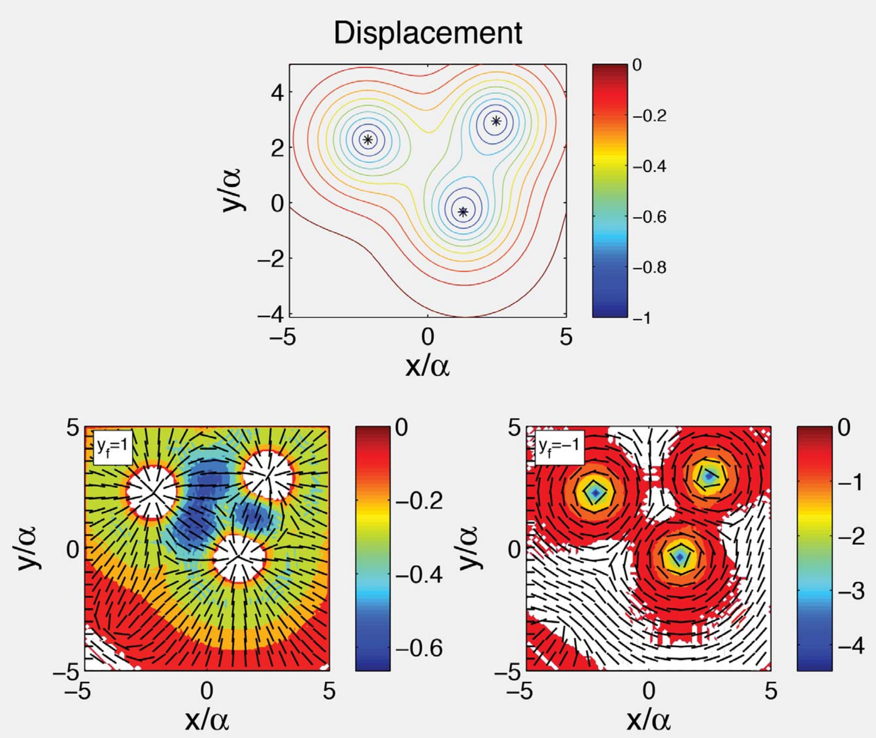
$$\left.\frac{\partial^2 w}{\partial x^2}\right|_{i,j} = \frac{\left.\frac{\partial w}{\partial x}\right|_{i+\frac{1}{2},j} - \left.\frac{\partial w}{\partial x}\right|_{i-\frac{1}{2},j}}{h} = \frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{h^2}$$

$$\left.\frac{\partial^2 w}{\partial y^2}\right|_{i,j} = \frac{\left.\frac{\partial w}{\partial y}\right|_{i,j+\frac{1}{2}} - \left.\frac{\partial w}{\partial y}\right|_{i,j-\frac{1}{2}}}{h} = \frac{w_{i,j+1} - 2w_{i,j} + w_{i,j-1}}{h^2}$$

$$\left.\frac{\partial^2 w}{\partial x \partial y}\right|_{i,j} = \frac{\left.\frac{\partial w}{\partial x}\right|_{i,j+1} - \left.\frac{\partial w}{\partial x}\right|_{i,j-1}}{2h} = \frac{w_{i+1,j+1} - w_{i-1,j-1} - w_{i+1,j-1} + w_{i-1,j-1}}{4h^2}$$

Relative error between numerical and analytical solutions is in the range of 10^{-4} (1/10000) for the sigma_xx and sigma_yy stress gradients. Sigma_xy is in the range of 10^{-12}. The bottom left plot shows the source function; $W=X^2*Y^2$ in the range of $-5<X<5$ and $-5<Y<5$.





Magma Propagation

Perpindicular to least principal stress

→ $\sigma_3 \perp \sigma_1$: Magma flows along σ_3

→ σ_3= Greatest principal stress

y_f=1: top of plate

→ Blue area experiences least amount of tension.

→ Predicts next magma emplacement

y_f=−1: bottom of plate

MATLAB 'eig' function

I would like to thank the UMD department of Geology for their comments, review, and time spent reading my work. I deeply appreciate the jokes that Dr. John Merck has told me. Dr. Doug Hamilton of the UMD Astronomy department has deeply provoked in me an interest in planetary sciences. Mark Larson has supported and criticized my work, expandig and furthering my knowledge and approach. Emily Holler provides a basis of support unlike any other. Danny Serven is a logician and engineer from anther world. And of course, Dr Laurent Montesi has shown constant support, in these endeavors, none of this work would have come to be without him.