Elastic Flexure Model for Sputnik Planitia on Pluto

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Motivation

Sputnik Planitia

- Large teardrop-shaped basin located on Pluto at 20°N 180°E
- Size: 1300 km by 900 km
- Depth: 3-4 km (basin)
- Deposit of nitrogen ice
- Water ice basement

Formation Hypotheses

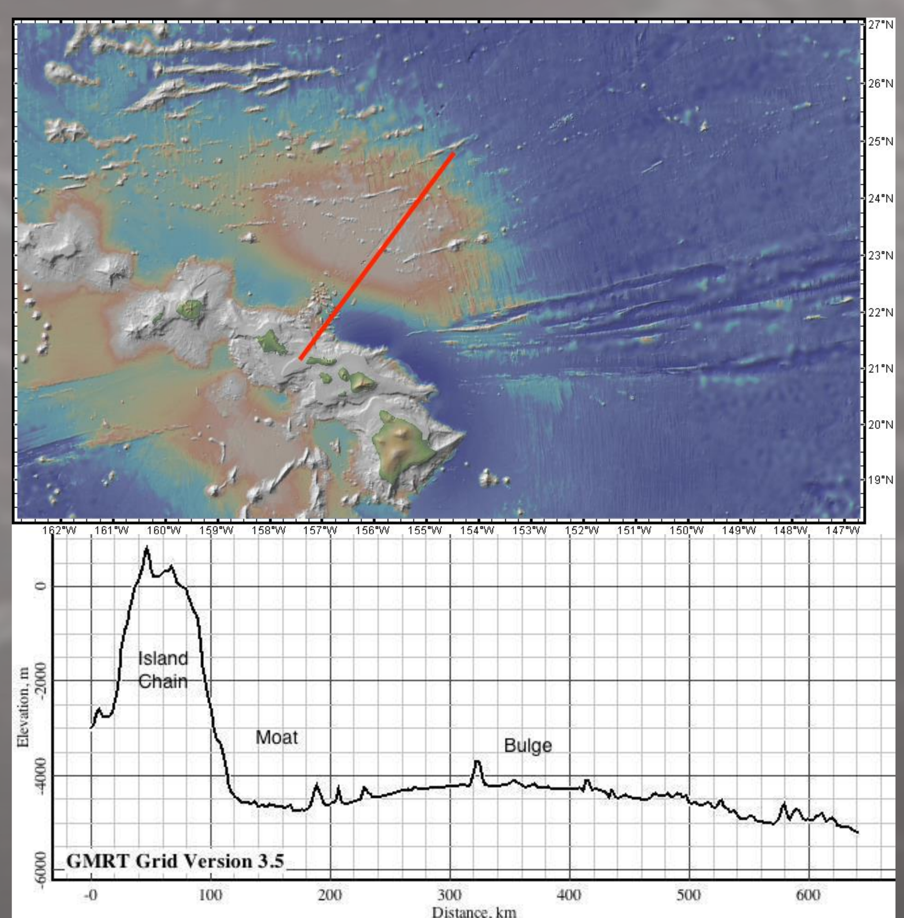
- An ancient impact basin created by an impactor later filled with N_2 ice. The feature would have moved to the current location through polar wander.
- Runaway deposition of N_2 ice due to albedo feedback at the ±30°. The depression is due to elastic flexure under the load of a thick N_2 ice cap. Most of the cap has since sublimated away.

Hypothesis

- **The topography of Pluto around Sputnik Planitia matches the prediction of an elastic plate flexed by a large load.**

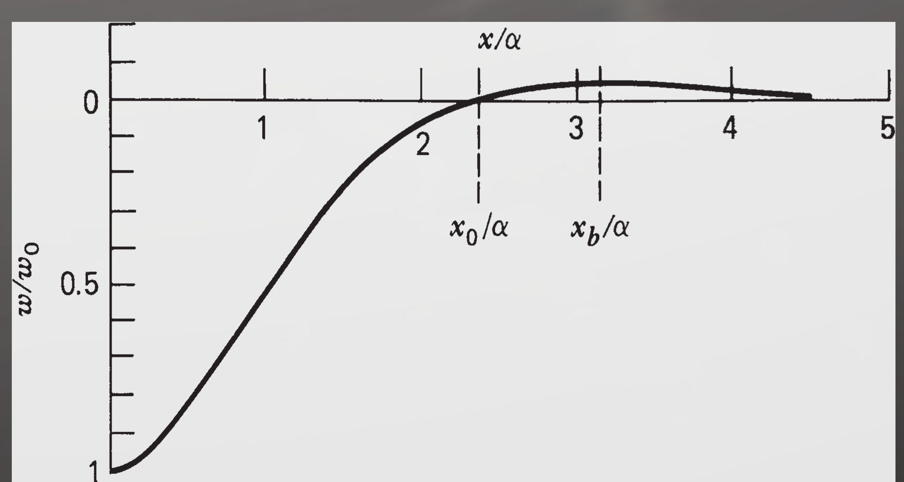
Expected results and implications

- What elastic thickness would explain the observe topography around Sputnik Planitia?
  - Was the interior relatively warm or cold? Did the deformation happen early or late in the history of Pluto?
  - Was the ice as flexible as expected if there is a water ocean?
- What load distribution and amplitude can explain the observed topography?
  - Is there evidence of a much larger N_2 ice layer as inferred today?



Example of plate flexure on Earth

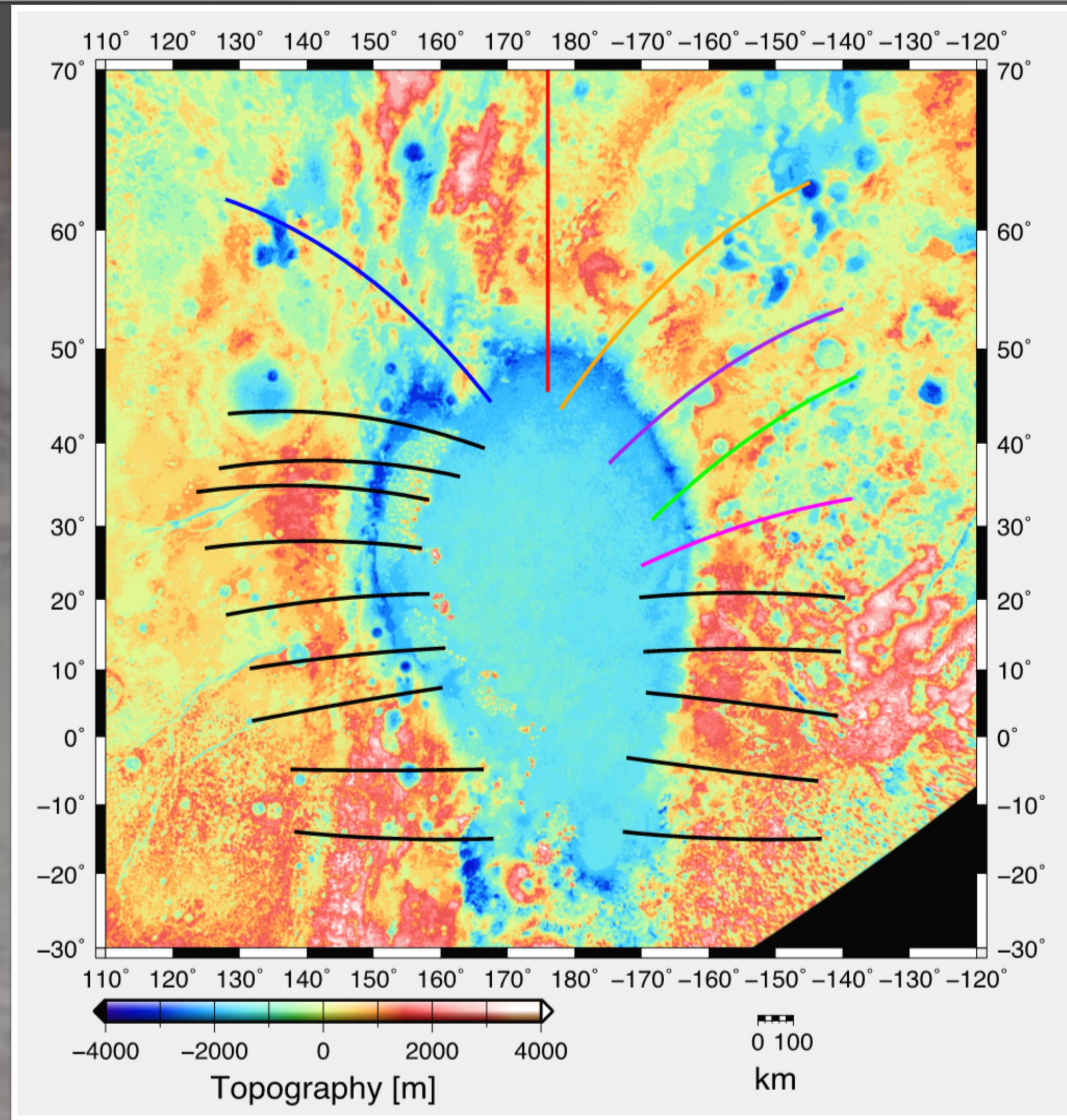
- The Hawaiian islands act as line load on the Pacific Plate.
- Flexure is expressed as a moat around the islands surrounded by a bulge.
- A similar signal may be present around Sputnik Planitia.
- Topographic map and profile taken from GeoMapApp.



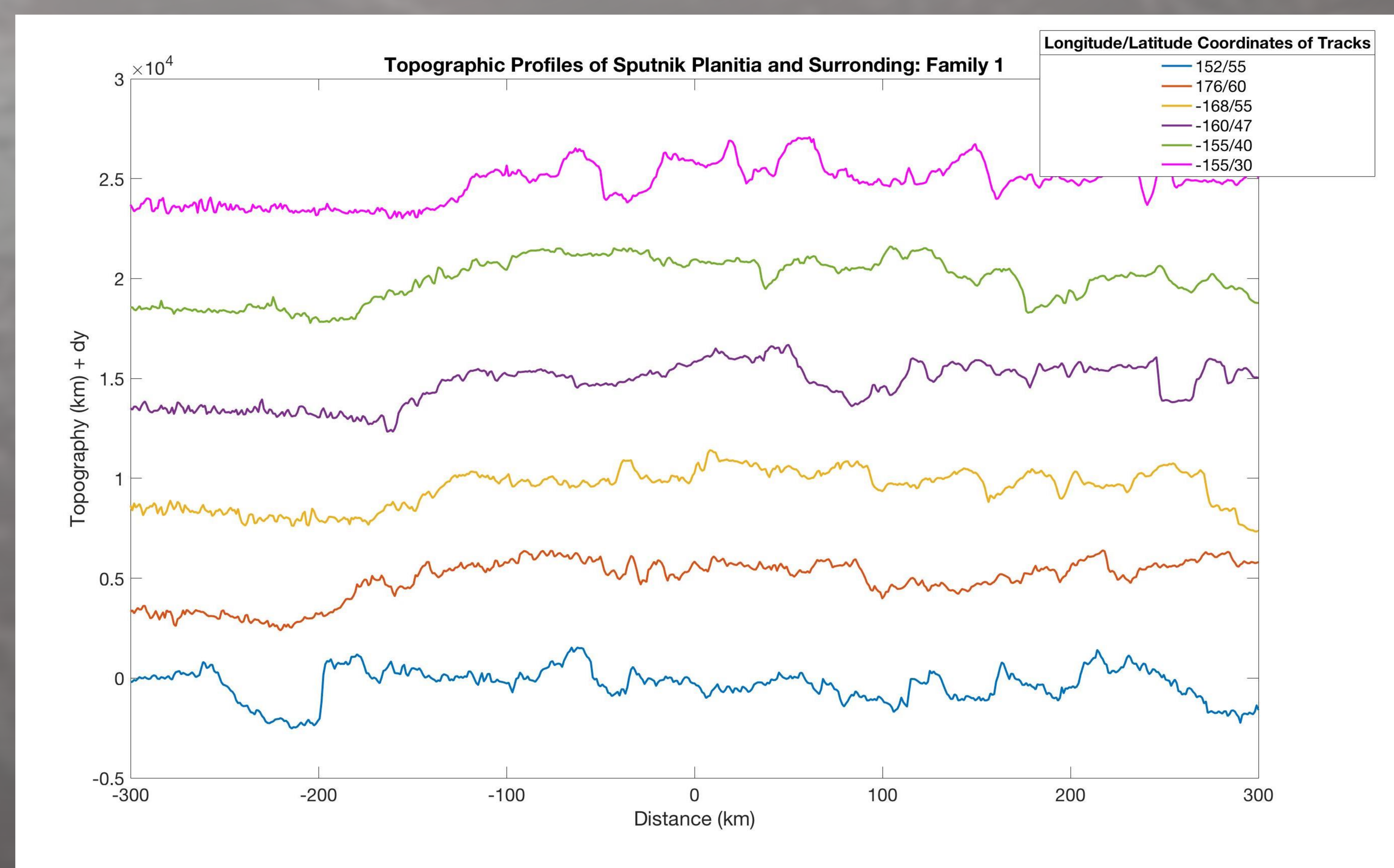
Theoretical model of elastic flexure

- An elastic plate subjected to a line load is deflected downward, forming a moat.
- A bulge develop due to the restoring force acting against the deflected plate.

Sputnik Planitia



False-color elevation map of Sputnik Planitia with twenty 600 km long tracks approximately perpendicular to the edge of the basin.



Selected topographic profiles across the edge of Sputnik Planitia. Each profile is offset for clarity. Some profiles show a clear topographic bulge while others show apparently random variations.

Flexural modeling

Flexure model

The flexure of an elastic plate in two dimensions obeys Equation 1

$$D\frac{d^4w}{dx^4} + (\rho_m - \rho_c)gw = V_0 \qquad \textbf{Equation 1.}$$

where $\alpha = \left[\frac{4D}{(\rho_m - \rho_c)g}\right]^{1/4}$ is the flexural parameter, $D = \frac{E*h^3}{12(1-v^2)}$ is the flexural rigidity, ρ_m is the density of the underlying layer of the ice shell, ρ_c is the density of the ice shell. g is the gravity on Pluto, E is the Young's modulus of water ice, h is the elastic thickness, and ν is Poisson's ratio

For a single vertical load at $x = 0$, the boundary conditions are $D\frac{d^3w}{dx^3} = \frac{1}{2}V_0$ and $\frac{dw}{dx} = 0$

The deflection due to several line loads is found by superposition:

$$w = \sum_i \frac{V_i\,\alpha^3}{8D}\left(\sin\frac{x-x_i}{\alpha} + \cos\frac{x-x_i}{\alpha}\right)\exp\left(-\frac{x-x_i}{\alpha}\right) \qquad \textbf{Equation 2.}$$

Where $\{V_i\}$ is the magnitude of the loads at position $\{x_i\}$

Inversion

The load vector $\mathbf{V}$ that produces topography $\mathbf{w}$, is found by least squares optimization:

$$\mathbf{V} = (\mathbf{M}'\mathbf{M} + \mathbf{C}_m)^{-1}(\mathbf{M}'\mathbf{w})$$

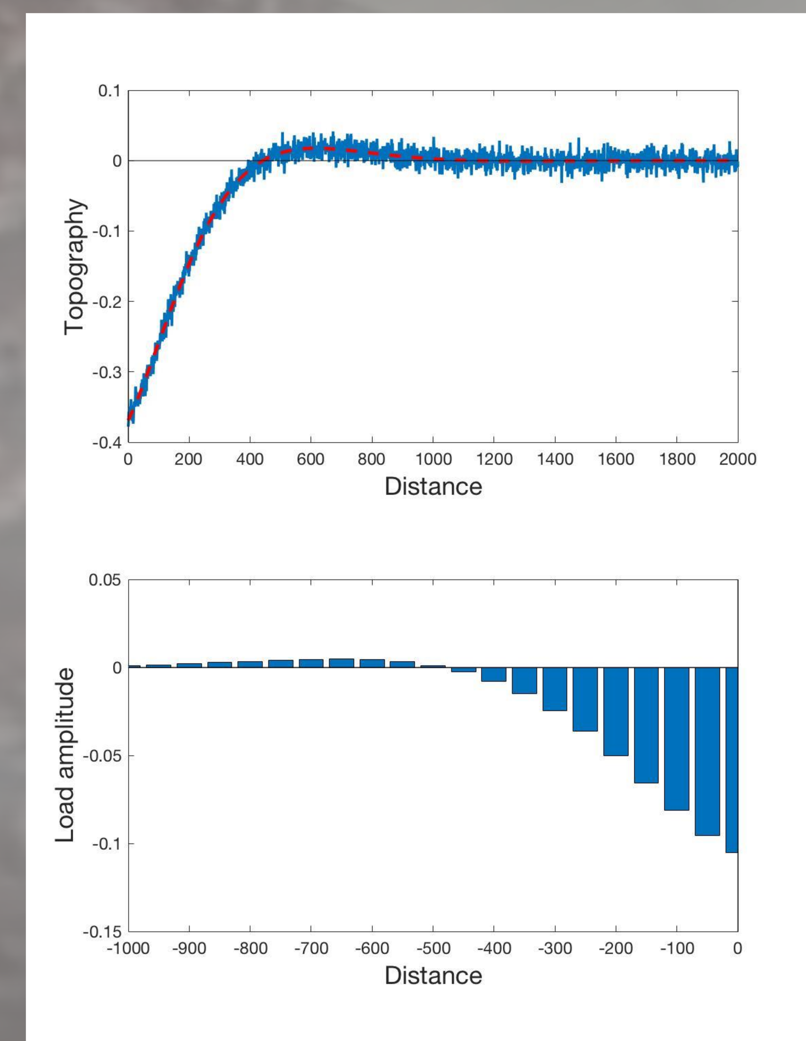
where **M** is the operator matrix that links a load at position x_j to deflection at a point x_i

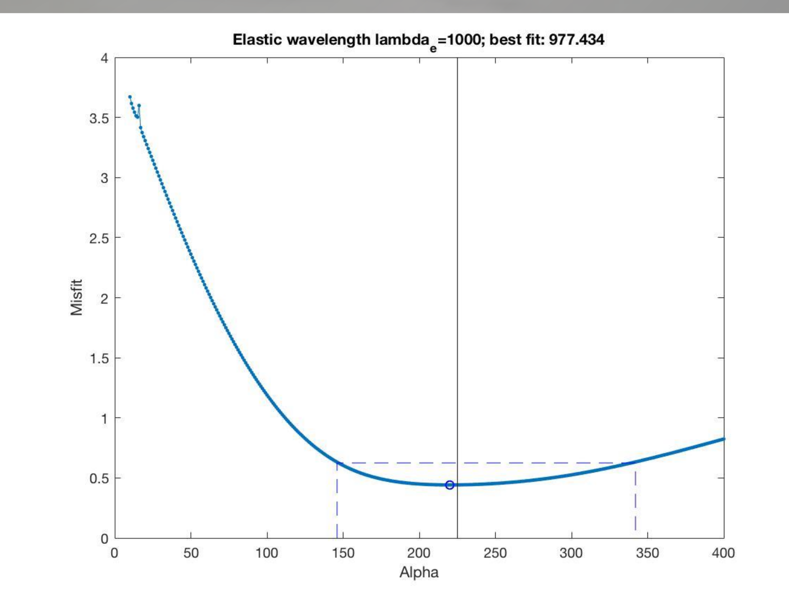
$$\mathbf{M}_{ij} = \frac{\alpha^3}{8D}\left(\sin\frac{x_i - x_j}{\alpha} + \cos\frac{x_i - x_j}{\alpha}\right)\exp\left(-\frac{x_i - x_j}{\alpha}\right)$$

and $\mathbf{C}_m = \sigma_t^{-2}\,\delta_{ij}$ is the covariance matrix, whose components are linked to the noise of the topographic data, σ_t

Synthetic Model

The inversion method was tested using two synthetic profiles generated by Dr. Montesi. I inverted the profiles using many test values of and recorded the misfit. The smallest misfit was found for $\alpha = 45 \pm 32$ (1σ value) when the synthetic profile was built with $\alpha = 45$ and for $\alpha = 224 \pm 160$ (1σ value) when the synthetic profile was built with $\alpha = 225$ (shown in figure below).





Left image. The top panel displays an elastic flexure signal under a line load along with the curve of best fit that was produced with the misfit test. The bottom panel displays the mass distribution. The model is unitless. **Right image.** The flexural parameter, alpha, and misfit values displayed as a graph with the best misfit value highlight with a dot.

Plans for GEOL 394

- Conduct the analysis of the inverse theory on the 20 profiles tracked for Sputnik Planitia and its surrounding.
- Find if the elastic thicknesses and load magnitude for all 20 profiles are consistent.
- Time permitting, create a 2D finite element model that uses the geometry of Sputnik Planitia to also find the best elastic thickness and load magnitude.
- The values for elastic thickness and load size will indicate if the interior is warm or cold, which helps constrain the timing of the load. This model may also allow the evaluation if a subsurface ocean is needed as that will change the flexibility of Pluto's ice shell.