Origin of the amplitude of the Hawaiian swell

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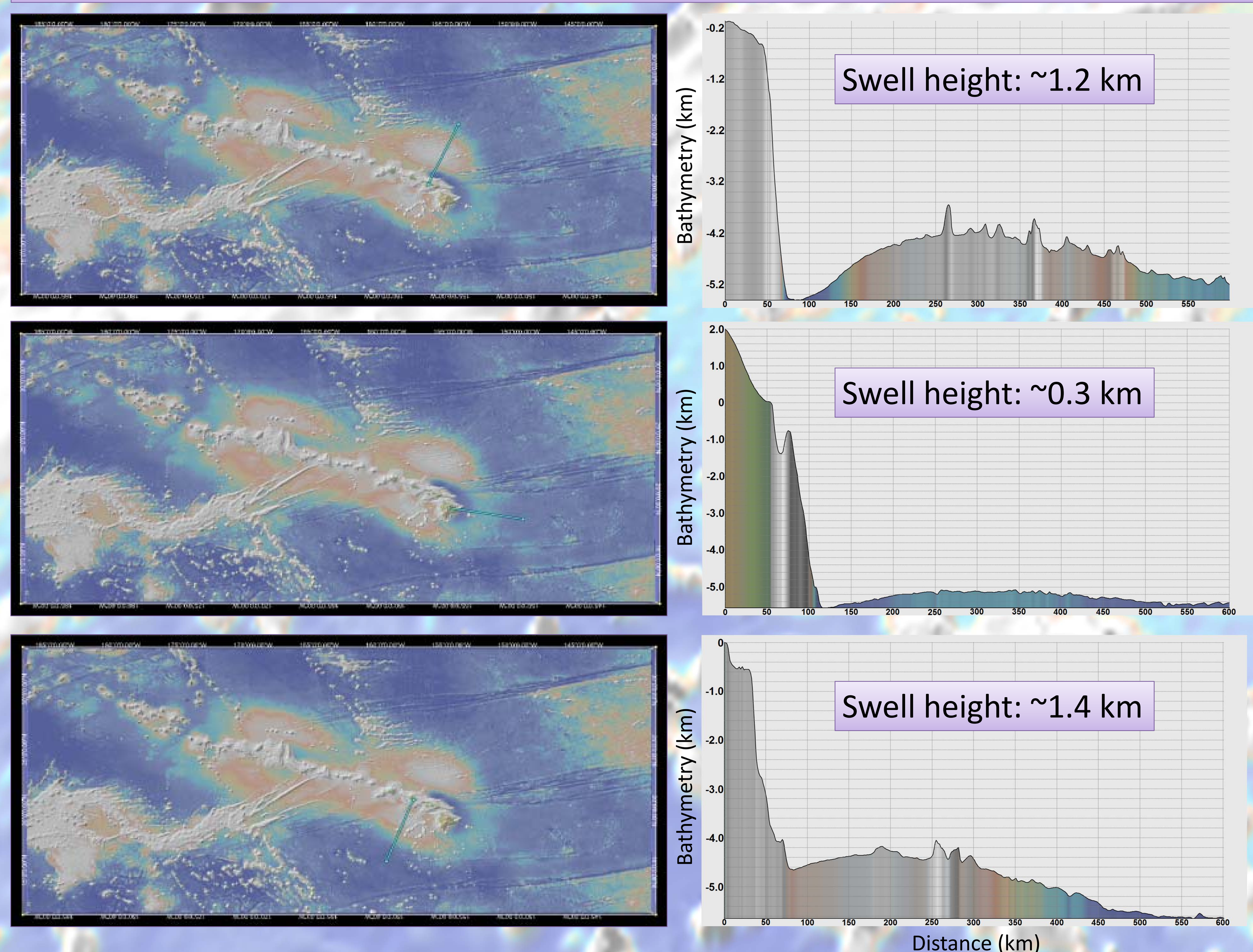
Introduction and Hypotheses

A swell of unknown origin has formed around the Hawaiian Islands. The amplitude of the swell is greater along the side of the swell than at the head of the island to the southeast. I propose two hypothesis as to the origin of the swell:

- Hypothesis 1: *The smaller amplitude of the Hawaiian swell to the southeast of the islands compared to the sides of the chain is due to different geometry of the flexural response: 2D response along the side, axisymmetric to the southeast of the chain.*
- Hypothesis 2: *The smaller amplitude of the Hawaii swell to the Southeast of the islands compared to the sides of the chain is due to deflection of the plume toward the northwest, following plate motion: heat delivery to the southeast of the chain is less than on the sides of the chain.*

Bathymetric Profiles

Bathymetric profiles of the swell show the variation in swell amplitude. As seen in the figures below the swell to the northeast and the southwest, along the sides of the island chain, is much larger than at the head of the chain to the southeast. Measurements indicate that the sides are up to four times larger in amplitude. Topographic profiles are ~145”0’0.00”W to ~185”0’0.00”W and ~15”0’0.00”N to ~30”0’0.00”N.



Flexure Analytical Model

Turcotte and Schubert (2002) defines flexure as "the bending of the lithosphere to known surface load.“The general equation for the deflection of an elastic plate is provided by Watts (2001):

$$\nabla^4 w + \beta^{-4} w = q/D$$

Where *w* is the deflection of the plate and *D* and β are parameters given by:

$$D = \frac{EH^3}{12(1-\nu^2)} \qquad \beta = \left[\frac{D}{(\rho_m - \rho_{infill})g}\right]^{1/4}$$

With *E* Young’s modulus, ν Poisson’s ratio, *H* the thickness of the plate, ρ_m and ρ_{infill} the density of the mantle and infilling material (water) and g the acceleration of gravity

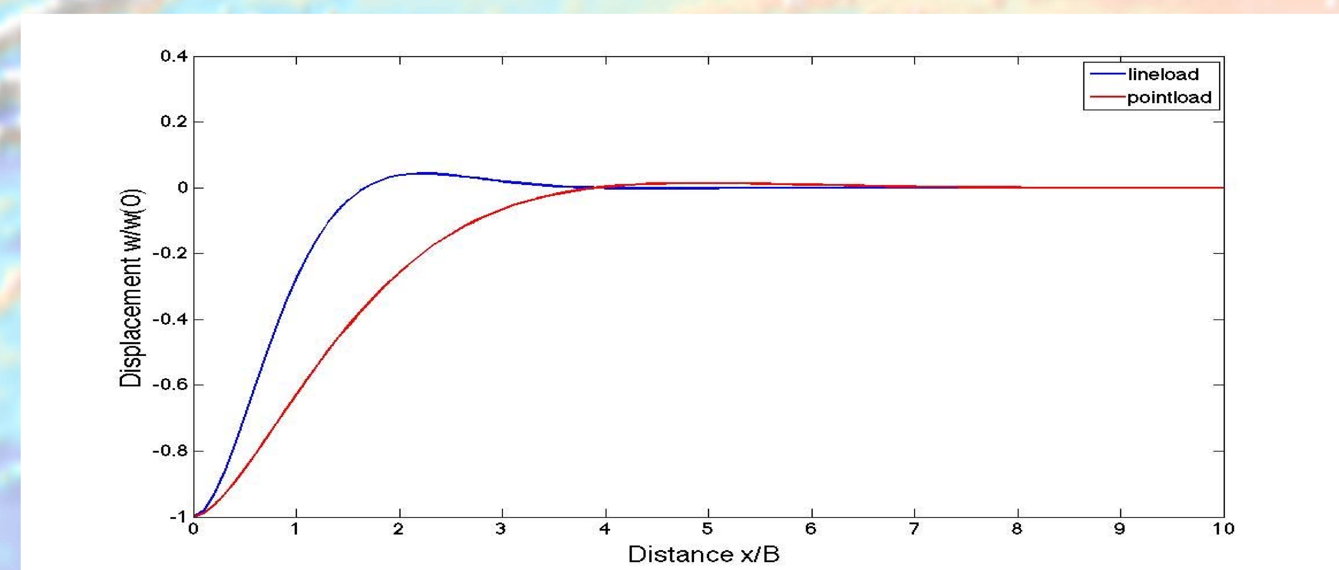
Different flexural responses occur under different load geometries. For an infinite line, flexure is given by:

$$w = w_0 e^{-\lambda x}[cos\lambda x + sin\lambda x]$$

Where $\lambda=(4)^{1/4}/\beta$. In the presence of a point the solution depends on *r*, the distance from the load and the Kelvin function *kei:*

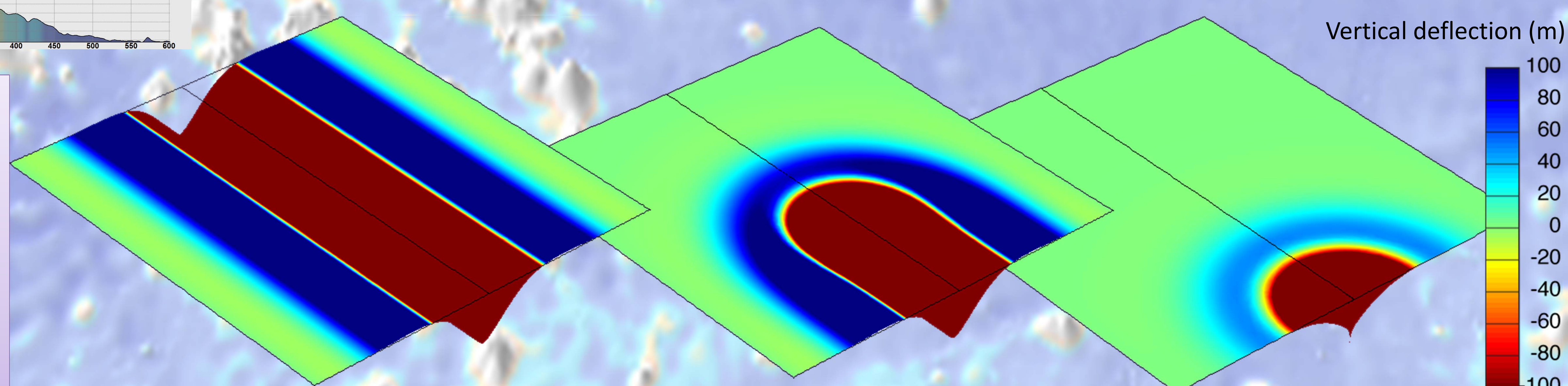
$$w = w_k kei\left(\frac{r}{\beta}\right)$$

This diagram compares the solutions for a point load (red) and a line load (blue) the swell produced by a line load is greater than with a point load, motivating hypothesis one.



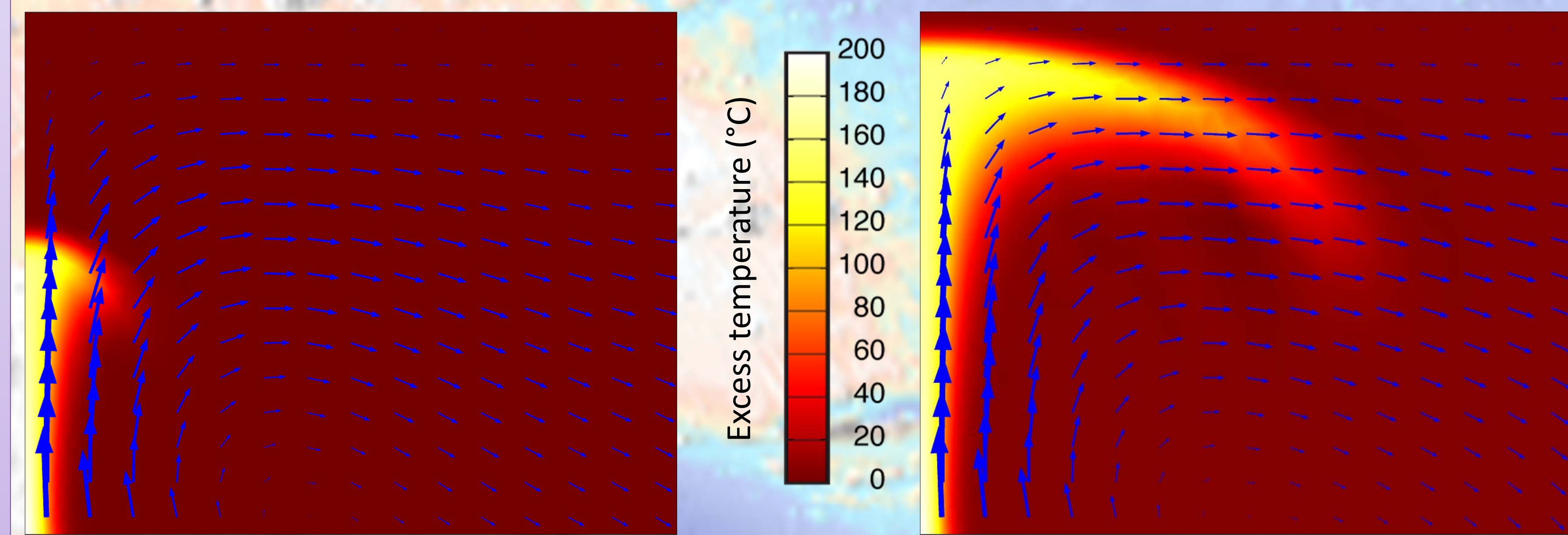
Flexural Numerical Models

These models below represent the flexure response of an elastic plate under various load geometries solved using COMSOL Multiphysics®. The bulge produced by a line load (left) is greater than the bulge caused by the point load (right), as in the analytical models. For a truncated (center) representing an island chain, the swell is greater along the sides of the line than at the point of truncation. This is similar to the swell around the Hawaiian islands.
The dimensions of the models are 1500 km by 1500 km. Displacement is -8000 m at the load. The displacement is exaggerated 25 times.



Plume Model

Below is an axisymmetric two-dimensional model of buoyancy-driven mantle flow. It shows the development of a plume over 20Ma driven by a thermal anomaly at the base of the model. As the plume reaches the top of the model, it flattens along the base of the plate.
Further models for GEOL394 will be constructed in three dimensions and include plate motion. The plume should deflect in the direction of plate motion (Sleep 1990). Heat transfer to the crust causes it to be more buoyant and to rise, according to isostasy. This adjustment should be reduced to the southeast of Hawaii because of plume deflection.



The dimensions of these plume models are 400 km by 500 km.
The physics necessary for the plume models include creeping flow and heat transfer.
Mantle flow is controlled by the incompressibility equation:

$$\nabla \cdot V = 0$$

And Stokes equation:

$$\nabla P + \eta \Delta V = \rho g$$

which represents momentum balance in a fluid, neglecting inertia. The first term is gradient of dynamic pressure P, the second term viscous stresses, where η is the viscosity and v the velocity of the fluid. The right-hand side represents the buoyancy of the fluid, with ρ the density and g the acceleration of gravity.
The temperature anomaly θ obeys an advection/diffusion heat equation, given by:

$$\frac{\partial\theta}{\partial t} + v \cdot \nabla v = \kappa \Delta\theta$$

The three terms of this equation represent temperature change, advection and diffusion of heat, respectively, with κ the heat diffusivity given by $=k/\rho_0 C_p$, where *k* the heat conductivity and C_p is the heat capacity of the fluid.

GEOL394 Time Line

Beginning in August, I will develop more flexural models and evaluate their uncertainty. Then in early October, I will develop plume models. Evaluation of plume model uncertainty will be finished in late October. After these models are completed I will compare the amplitude of the swell to the amplitude of the models.

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